Weak decays of mesons with charm and meson distribution amplitudes*

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Exclusive processes and Distribution Amplitudes

The aim of this project is to analyze data which will hopefully be obtained by $\overline{\text{P}}$ANDA with first principle QCD. This chapter aims at explaining what we mean with this statement.
Although QCD is the exact theory of strong interactions and as such subject to intense investigations since decades many of its aspects are still only poorly understood. In fact, the applicability of QCD to any given problem has to be based on exact factorization proofs, Operator Product Expansion analysis (OPE) etc. Whenever it is applied to problems for which these prerequisites are not fulfilled one is only developing a model rather than performing a first principle theory calculation. This fact greatly limits the speed with which the range of phenomena accessible by state of the art QCD can grow. Presently hard exclusive reactions that can be described by Distribution Amplitudes (DAs) are one of the "new territories" in this sense.
For the heavy, potentially exotic mesons which are planned to be produced at FAIR the information one will get are primarily masses and decay characteristics for various channels. This information boils down to just a few numbers which can only characterize some parameters of DAs.
The nice thing about decay reactions for, e.g., XYZ mesons is that the relatively large charm quark mass allows to use perturbative QCD techniques, though it is not large enough to justify a purely leading twist, leading order treatment. We are working on state of the art QCD calculations taking also such corrections into account. The most important theoretical tool to do so are DAs, not to be confused with parton distribution functions (PDFs). Very roughly speaking a PDF is related to the square of a parton wave-function with all other partons as well as any transverse momentum being integrated out. Therefore, leading twist, leading order PDFs can be interpreted as probabilities. In contrast DAs are roughly speaking parton wave functions of only the leading Fock state of a hadron with any transverse momentum being integrated out. Therefore it is a wave function not a probability and not positive definite even at leading order and leading twist.
The hard scale is crucial because the coupling to all higher Fock states is suppressed by additional hard propagators, see Fig.1. To the extend that the K^- DA is known, the decay rate depends only on the D structure. In a completely analogous manner one can extract information on the structure of unknown mesons from their semi-leptonic decay rates into known hadrons and also in principle from their decay rates into pairs of known hadrons.
More formally the difference between the DAs and the more well known PDFs, GPDs, TMDs etc. can be described as follows: The latter parametrize matrix elements of the form (with two, possibly identical, hadrons H and H' with momenta P, P' and spin S, S' and some reaction-specific operator $\mathcal{O}$

$$\langle H'(P',S')|\mathcal{O}|H(P,S)\rangle \tag{1}$$

while DAs parametrize matrix elements of the form

$$\langle 0|\mathcal{O}|H(P,S)\rangle \quad . \tag{2}$$

Obviously, both types of matrix elements test the hadronic wave functions in complementary ways. One has to determine as many of them as possible to pin down the latter better and better. It is a bit of an anomaly that up to now far less theoretical effort was invested into the exploration of DAs than for PDFs, GPDs, etc. which on the other hand means that presently it is relatively easy to make progress. Technically speaking the consistent QCD treatment at the medium large energies which are typical for hadron physics is in both cases very demanding but possible.

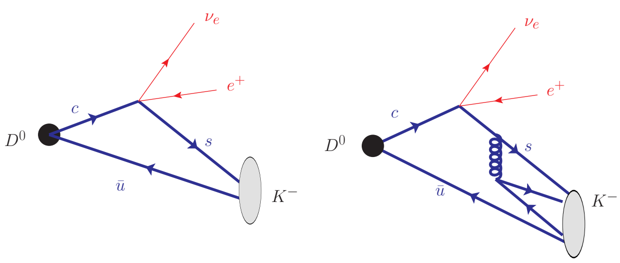

Figure 1: Contributions to the decay matrix element for $D^0 \to K^- e^+ \nu_e$.

Determination of DAs

We perform extensive calculations aiming at the systematic determination of hadron DAs, using perturbative QCD (pQCD), light-cone sum rules (LCSRs) and Lattice QCD (LQCD). To proceed, the DAs are expanded into Gegenbauer polynomials (which are the eigenfunctions of the leading order evolution equations) and the different investigations can then be related by combining all results for the expansion parameters. The resulting parametrisation provides then the input for the determination of decay rates. As

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we do not yet have results from the present project, which aims at describing, e.g., the semi-leptonic decay of the D_s into a pion pair, we present in the following as illustration some results from projects funded by other sources.

One of the main issues in this field are the BaBar and BELLE data on another exclusive process, namely the $\gamma^*\gamma\pi$ form factor. The experimental data favor rather strange looking pion DAs, see Fig.2, with enhancements close to the end points. The leading Fock state of a pion consists of just a quark antiquark pair with longitudinal momentum fractions x and $1-x$. Therefore, the pion DA is a function of x and the momentum scale only. The leading deviation from the asymptotic form is parametrized by the parameter a_2. A recent large scale lattice simulation of our

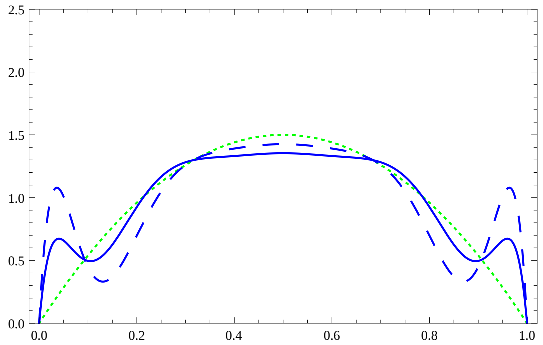

Figure 2: Pion DAs extracted from an analysis of the BaBar (dashed) and combined BaBar and Belle data in comparison to the asymptotic distribution amplitude (green dots).

group [2] resulted in the value $a_2^{\overline{MS}} = 0.1364(154)(145)$ where the first error is the statistical and the second the systematic error. The different curves in Fig.2 correspond to $a_2 = 0$ (green dots), $A_2 = 0.10$ (solid line) and $a_2 = 0.14$ (dashed line). Obviously we are getting close to the precision at which we will be able to settle this problem. Let us add that the analysis which resulted in the solid line in Fig.2 and which was the technically most advanced one is also compatible with $a_2 = 0.14$, i.e. from the data alone both models are admissible.

We perform and performed similar analysis for the ρ, K, η and η' mesons, allowing to analyze information for all exclusive decays of new heavy meson states into these hadrons.

As another example we have calculated in [3] the branching ratios Eq.(3) showing that as soon as the experimental errors get smaller by a factor of two or so one will be able to pin down the purely gluonic contribution of the η, η' wave functions. (The experimental errors must become smaller than the potential gluonic contribution which is parametrized by the gluon Gegenbauer coefficient B_2^g.)

$$\begin{aligned}
\frac{\Gamma\left(D_s^+ \to \eta' e^+ \nu_e\right)}{\Gamma\left(D_s^+ \to \eta e^+ \nu_e\right)} &= 0.37 \pm 0.09\,(B_2^g) \pm 0.04 \\
Exp: &\ 0.36 \pm 0.14 \\
\frac{\Gamma\left(D^+ \to \eta' e^+ \nu_e\right)}{\Gamma\left(D^+ \to \eta e^+ \nu_e\right)} &= 0.16 \pm 0.06\,(B_2^g) \pm 0.02 \\
Exp: &\ 0.19 \pm 0.09 \\
\frac{\Gamma\left(B \to \eta' e^+ \nu_e\right)}{\Gamma\left(B \to \eta e^+ \nu_e\right)} &= 0.50 \pm 0.29\,(B_2^g) \pm 0.05 \\
Exp: &\ 0.67 \pm 0.24 \pm 0.1 \qquad (3)
\end{aligned}$$

Baryon DAs

Let us add that the same types of analysis can be performed for baryons and we do so for the complete SU(3) Octet. Though this is probably less relevant for $\overline{\text{P}}$ANDA we show the result for the DAs of the nucleon and the two lowest negative parity nucleon resonances. In this case the leading Fock state consists of three quarks with $x_1 + x_2 + x_3 = 1$ which explains the triangular shape of the plot.

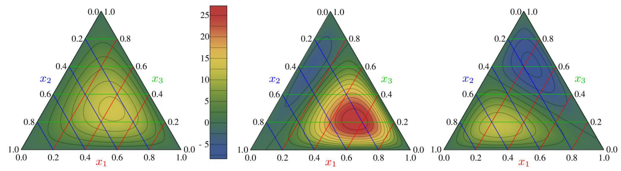

Figure 3: The nucleon, $N^*(1650)$ and $N^*(1535)$ distribution amplitude taken from [4].

Conclusions

We have tried to give a short introduction to the role the analysis of exclusive hadron decays at $\overline{\text{P}}$ANDA could play to pin down the quark gluon structure of known and new hadrons. As far as QCD is concerned all of these calculations touch the limits of what is technically feasible today and will require much more theory work to become more precise. However, what is missing most to put all of this to work is high-precision data from $\overline{\text{P}}$ANDA.

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