Unbinned likelihood Analysis of the EC-decay rate oscillations - Part I *

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Introduction

An unbinned likelihood analysis was performed on the three decay times of stored hydrogen-like $^{142}Pm^{60+}$ ions data sets presented in [1]. Unlike to usual binned statistics, such as e.g. χ^2 or binned likelihood, the unbinned likelihood statistics is free from information loss and arbitrariness of the binning procedure. This method was applied using two models [1], namely M_0 and M_1, the probability density function of a pure exponential (null hypothesis) and modulated exponential decay (alternative hypothesis), respectively.The computation of the corresponding unbinned likelihood functions L_0 and L_1 was performed using the RooFit C++ package [2].

From this analysis, the following quantities could be obtained : Maximum Likelihood Estimates (MLE), Likelihood Ratio Profile (LRP), Likelihood Ratio Test (LRT), Akaike and Bayesian Information Criteria (AIC and BIC). In this report we present the MLE results. The LRP and LRT analyses, as well as the conclusion, are presented in the part II. The AIC and BIC results are presented in Ref. [3, 4].

Maximum Likelihood Estimates

The results of the unbinned maximum likelihood estimation for different data sets are summarized in Table 1. The Figure 1 shows the fit functions with their 1-σ error bands (dashed line) obtained from the 245 MHz resonator EC-data and rescaled on binned data for presentation. The correlation coefficients are weaks, all having low values ($|C_{\theta_i,\theta_j}| < 0.03$) except for the angular frequency-phase coefficients, which amount to $C_{\omega,\phi} = -0.87$.

	EC data (245 MHz Res.)	β^+ data (245 MHz Res.)	EC data (cap.)
Sample size N	3616	2912	2989
Time interval (s)	[6.0 ; 60.0]	[10.0 ; 60.0]	[6.0 ; 60.0]
$-\log(L_0)$	14340.7	11354.6	11858.2
λ_0	0.013 (1)	0.011 (1)	0.013 (1)
$-\log(L_1)$	14333.3	11351.2	11840.7
λ_1	0.013 (1)	0.011 (1)	0.013 (1)
a_1	0.09 (2)	0.074 (26)	0.15 (2)
ω_1	0.88 (1)	3.09 (27)	0.88 (1)
ϕ_1	2.4 (5)	-2 (1)	1.6 (3)

Table 1: Results of the unbinned maximum likelihood method. The λ parameters denote the total decay rate (i.e. EC and β^+). The a, ω, and ϕ parameters denote the modulation parameters, that is, the amplitude, the angular frequency and the phase, respectively.

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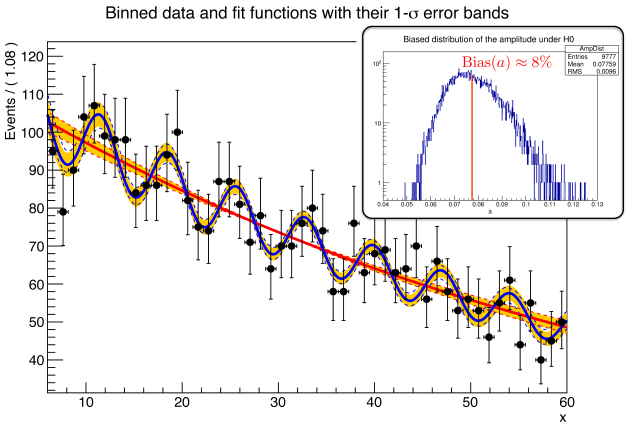

Figure 1: Number of EC decays per 1.08 s of H-like $^{142}Pm^{60+}$ ions, recorded by the 245 MHz resonator, vs. the time after injection of the ions into the storage ring ESR. Also shown are the fit functions obtained from unbinned Likelihood fits with models: M_0, a pure exponential decay, andM_1, with an additional superposed decay rate oscillation. The data are binned for display only. In this data set, the oscillation amplitude amounts to 9 (2) %. From a Monte Carlo study, we estimate the amplitude due to statistical fluctuations to be 8(1) %, see inset in fig.1.

Bias of the amplitude under H_0

It has been observed that the amplitude estimator is biased under the null hypothesis. The bias amount, e.g. for the sample size of the 245 MHz resonator data, to about $8(1)\%$ (c.f. inset in figure 1), which is compatible with the $9(2)\%$ amplitude found in real data. The bias increases with decreasing sample size, and therefore might contribute to the high significance observed in pick-up data (c.f. part II of this report).

The results of parameter estimations are in good agreement with those obtained with the χ^2-method used in [1]. However, note that, the results in Table 1, have ω constrained in range $[0, 4]$. A likelihood ratio profile showed other maxima depending on the ω or/and data range.

References

[1] P. Kienle et al. PLB 726 (2013) 638

[2] W. Verkerke and D. Kirkby arXiv:physics/0306116

[3] N. Winckler, et al., “Further insight into Bayesian and Akaike information criteria ”, GSI Report 2013 - 2014

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