Comparison of the beam mixing models proposed by Lam and Zaider & Rossi and a derived D_t extension for the Zaider & Rossi model *

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In the radiotherapy of cancer with heavy ions, treatment planning systems like TRiP [1] use externally calculated tables for the biological effect of monoenergetic ion beams and derive the effect of a therapeutic mixed beam from these data by means of a beam-mixing model. In TRiP, the effects of monoenergetic beams are predicted by the Local Effect Model (LEM) [2]. The full simulation approach of this model [3] predicts ion and energy dependent threshold values, D_t, for the transitions between linear-quadratic and pure linear part of dose-effect curves (Linear-Quadratic-Linear (LQL) model), but in the previously used version, the single particle approximation, such individual D_ts could not be estimated. Instead, as an approximation, an independence of the D_t value from the beam quality was assumed. This could be exploited by TRiP by using the very efficient beam-mixing model proposed by Zaider & Rossi [4]. In principle this model does not include a D_t and an extension is not straight-forward [5]. Nevertheless, for a constant D_t, the model could be extended by applying the D_t threshold after the actual beam-mixing ("constant-D_t extension"). As this approach could not be used for the varying D_ts of the full-simulation method, the much more flexible beam-mixing method proposed by Lam [6] has to be introduced. However, this model is conceptually different from the Zaider & Rossi approach and the predicted RBE-weighted doses (Relative Biological Effectiveness) deviate by a few percent. A theoretical understanding of the differences is therefore highly interesting, especially for the comparison with previous TRiP/LEM results.

In the LQ model (LQL without D_t threshold) dose-effect curves, $\epsilon(D)$, are described by $\epsilon = \alpha D + \beta D^2$. Formally, this could be separated in a linear effect $\epsilon_\alpha = \alpha D$ and a quadratic effect $\epsilon_\beta = \beta D^2$. As the Lam method can handle any dose-effect curve, the method could, formally, be applied to both effect-curves separately, resulting in $\overline{\epsilon_\alpha}$ and $\overline{\epsilon_\beta}$, and providing a new mixed effect $\bar{\epsilon} = \overline{\epsilon_\alpha} + \overline{\epsilon_\beta}$. This $\bar{\epsilon}$ is not the mixed-beam prediction of the original Lam model but, interestingly, it could be proven, that $\bar{\epsilon}$ is identically to the result predicted by the Zaider & Rossi model. In addition to a theoretical description of the model differences, this immediately leads to a D_t extension of the Zaider & Rossi method: For monoenergetic ϵ including a D_t, this threshold can be moved to the β part by using $\epsilon_\alpha = \alpha D$ and $\epsilon_\beta = \epsilon - \epsilon_\alpha$.

In this extension, individual ion and energy dependent D_t thresholds can be used, but for constant D_t this model does not exactly lead to the previously used constant-D_t method. This is shown in the figure, where relative differences between RBE-weighted doses calculated by TRiP/LEM for the new D_t extension and the constant-D_t method are plotted. Significant differences could mainly be found inside the SOBP (Spread-Out Bragg Peak) and only if the planned dose was above the photon D_t (blue/green curves in the figure: planned doses below/above $D_{t,\gamma}$).

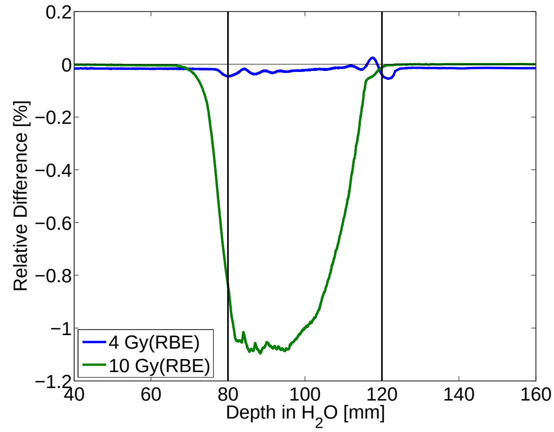

Figure: Relative differences of RBE-weighted doses between Zaider & Rossi D_t-extensions ((new − old)/old) predicted by TRiP/LEM (single particle approximation) for 4 Gy(RBE) (blue) and 10 Gy(RBE) (green) in the SOBP (marked). Carbon beam, $\alpha_\gamma = 0.313$ Gy^{-1}, $\beta_\gamma = 0.0615$ Gy^{-2}, $D_{t,\gamma} = 7.5$ Gy.

Conclusion: The beam-mixing models proposed by Zaider & Rossi and Lam could be expressed in the same mathematical framework which shows that the difference between both methods is completely related to the strict separation between linear and quadratic part of the dose-effect curves assumed by the Zaider & Rossi model. For instance, a high α value of a contributing monoenergetic beam can induce an increased mixed-beam β in the Lam model but not in the Zaider & Rossi model.

The derived D_t extension of the Zaider & Rossi model does not directly extend the constant-D_t extension used so far. However, differences are small (up to a few percent depending on the irradiated cell line), mostly seen in the SOBP, and generally negligible for RBE-weighted doses below the D_t of the reference photon irradiation.

References

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