# Bayesian analysis of the EC-decay rate oscillations - Part I *

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## Introduction

In this contribution we perform a Bayesian analysis with the models and data presented in Ref. [1]. We first remind the definitions of the Bayesian quantities computed here, and present the results of parameter estimation analysis. The results of model selection and the conclusion are presented in the part II of this GSI scientific report.

## Computation

In the following $M_0$ denotes the pure exponential decay, and $M_1$ the modulated exponential decay. This analysis has been performed using the BAT [2] and CUBA [3] C++ packages. All probabilities have been computed from unbinned likelihood, reducing the information loss.

### *Parameter probabilities*

The posterior probability density functions of the parameter(s) of interest $\theta$ is given by :

$$P(\theta|\text{data}) = \frac{P(\text{data}|\theta)P(\theta)}{P(\text{data})}, \quad (1)$$

where $P(\theta)$ is the prior probability of $\theta$, $P(\text{data})$ the normalization term, and $P(\text{data}|\theta)$ the marginal likelilihood. The Metropolis algorithm has been used to sample the marginal likelihood distribution. The high resolution of the obtained posterior distributions, as the one shown in figure 1, have been achieved using 10 Markov chains, each with 10 millions iterations. The convergence has been reached after about 140 thousands iterations.

### *Model probabilities*

The posterior probability of a model $M_i$ given the data is given by

$$P(M_i|\text{data}) = \frac{P(\text{data}|M_i)\pi_i}{P(\text{data})}, \quad (2)$$

where $\pi_i$ is the prior probability of the model $M_i$, $P(\text{data})$ the normilzation term, and $P(\text{data}|M_i)$ the Bayesian Evidence :

$$P(\text{data}|M_i) = \int_{\vec{\theta_i}} P(\text{data}|\vec{\theta_i}, M_i)P(\vec{\theta_i}|M_i)d\vec{\theta_i}. \quad (3)$$

Given the models $M_0$ and $M_1$, the posterior odds is defined as :

$$\frac{P(M_0|\text{data})}{P(M_1|\text{data})} = B_{01}\frac{\pi_0}{\pi_1} \quad (4)$$

For equal model prior probabilities, the posterior odds reduces to the Bayes factor $B_{01}$. Both, model probabitlities and Bayes factors have been obtained using the VEGAS Monte Carlo algorithm for the integration of the Bayesian evidence. The obtained relative precisions of the integrals are smaller than $10^{-3}$. The results of the computed model probabilities and Bayes factor are presented in the Part II of this report.

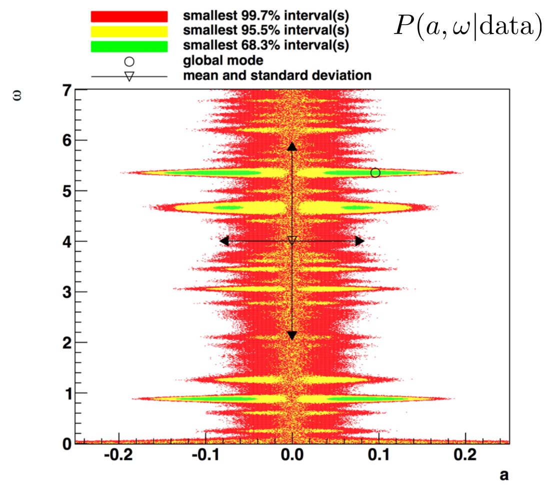


Figure 1: Two-dimensional posterior probability-density for the oscillation frequency $\omega$ and the oscillation amplitude $a$ in the 245 MHz resonator EC-data set. The global mode of the posterior $P(a, \omega|\text{data}, M_1)$ is found for $\omega = 5.35$ and $a = 0.09$.

## Bayesian parameter estimation

For all parameters, and for all possible combination of parameter pairs, the 1- and 2-dimensional posterior probabilities have been computed using the models $M_0$ and $M_1$ and the data presented in Ref. [1]. Different priors have been used. The posteriors computed with uniform prior distributions have shown, as expected, excellent agreements with the unbinned likelihood analysis (see e.g. figure 1 and Ref. [4]).

## References

[1] P. Kienle et al., "High-resolution measurement of the time-modulated orbital electron capture and of the $\beta^+$ decay of hydrogen-like $^{142}Pm^{60+}$ ions", PLB 726 (2013) 638

[2] A. Caldwell, D. Kollar, K. Kroeninger, "BAT - The Bayesian Analysis Toolkit", Comput. Phys. Commun. 180 (2009)

[3] T. Hanh, "Cuba—a library for multidimensional numerical integration" Comput. Phys. Commun. 168 (2005)

[4] N. Winckler, "Unbinned likelihood Analysis of the EC-decay rate oscillations - Part I and II", GSI Report 2014 - 2015

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