Bayesian analysis of the EC-decay rate oscillations - Part II *

N. Winckler[1] *and the Two-Body Weak Decay collaboration*[1]
[1]GSI, Darmstadt

Bayesian model selection

We present the results of the model probabilities and Bayes factor of the model and data defined in Ref. [1, 2]. The table 1 shows the results obtained for uniform priors and different data sets. In table 1, the uniform prior of ω is defined in $[0, 3]$, which exclude the contribution of the global maximum likelihood at $\omega = 5.35$.

	EC data (245 MHz res.)	β^+ data (245 MHz res.)	EC data (cap. pick-up)
Sample size N	3616	2912	2989
Range (s)	[6,60]	[10,60]	[6,60]
$P(M_0\|\text{data})$	66.3%	84.94%	0.03%
$P(M_1\|\text{data})$	33.7%	15.06%	99.97%
B_{01}	1.97	5.64	0.0003
B_{10}	0.5	0.17	3645.4

Table 1: Model probabilities and Bayes factors results.

Prior sensitivity analysis

In order to study the the prior sensitivities, other prior distributions have been used in the analysis . For example, increasing the range of the uniform prior of the angular frequency to $[0, 7]$ increases the probability of M_1 to about 43%, as larger frequency components in the likelihood function are taken into account.

The use of gaussian priors with mean and width obtained from the 2007 experimental results – i.e. $a = 0.23(4)$, $\omega = 0.88(3)$, $\phi = -1.6(5)$ – have shown, for the model M_1, small probabilities of about 0.3%. This small probability can be explained when considering the likelihood function. The maximum contribution of the likelihood function is found for $a = 0.09(2)$ in the resonator data. The strength of the likelihood is reduced for amplitudes outside the range $a = 0.09 \pm 0.02$. Using a gaussian prior $N_a(0.23, 0.04)$ results in a posterior with small weights in the $a = 0.09 \pm 0.02$ region, reducing the M_1 posterior, and hence the Bayes factor. As a consequence, we note that previous measurements are not supported by the present analysis of the resonantor data, and that this latter does not exhibit a sufficiently strong likelihood at $a = 0.09 \pm 0.02$ to overcome the $N_a(0.23, 0.04)$ subjective prior.

Discussion

According to Jeffreys' scale, the Bayes factor for the 245 MHz resonator β^+- and EC- data have no support for the modulated decay model. This result is independent on the chosen priors. However the EC-data set obtained from the capacitive Schottky pick-up present decisive Bayesian evidence for the modulated decay, which is in complete contradiction with results obtained in resonator data.

As mentioned in Ref. [2], the observable of the pick-up and resonator data are not the same. The pick up data consist of three components, i.e. the decay time, the delay required to electron-cool the ion, and the systematic error in the determination of the observable. The decay time + delay observable has been analyzed as well in the resonator data. The obtained posterior probability is about 33% for the oscillation model, which a priori exlude the delay distribution as being responsible for the difference in the pick-up and resonator data. The main difference remaining between these two data sets are the distribution of the systematic errors in the determination of the observables. These systematic errors are assumed to be small enough to be neglected in the resonator data because of the large signal-to-noise ratio of the ion signatures in the time resolved spectra. This assumption may not be valid for the pick-up time resolved spectra, which present poor signal-to-noise ratio.

Conclusion

We have shown that Bayesian model selection methods do not support the oscillation model in the 245 MHz resonator data but support, nevertheless, oscillation in the pick-up data. This conclusion is corroborated by the results of frequentist hypothesis testing and likelihood analysis [2], by Bayesian information criterion [3] and by an independent Bayesian analysis using the binned likelihood method and the nested sampling algorithms [4]. We note that AIC analysis presents oscillation in the resonator data as well. However, it has been shown [5] that the associated decision making in the AIC framework has a 99% Type I Error rate, discrediting its reliability in our analysis.

Bayesian and frequentist analysis of the decay time + delay in the resonator data have been performed, and do not support the oscillation model. Since the main difference between the significant and non significant data set remains in the systematic error distributions, this result point out to possible uncontrolled systematic effects in the pick-up data.

References

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