Unbinned likelihood Analysis of the EC-decay rate oscillations - Part II *

N. Winckler¹ and the Two-Body Weak Decay collaboration¹
¹GSI, Darmstadt

Likelihood Ratio Profile

A likelihood ratio profile w.r.t. the angular frequency $\omega$ (c.f. fig. 1) have shown that the global maximum in the range $\omega \in [0, 7]$ in the EC data is found at $\omega = 5.35(1)$. This maximum remains in the complete set of data (sample size of 8663 EC-decays) but is absent in the EC-data obtained with the capacitive pick-up and in the $\beta^+$-data of the 245 MHz resonator. The absence of the frequency at 5.35 in the capacitive pick-up data is expected, since the observable of these data correspond to the decay time + the systematic uncertainty + the distribution of the delay required to electron-cool the ion, washing out high frequencies by convolution. We can observe a third maximum at $\omega = 4.68(2)$. The strength of the three observed maxima is sensitive to systematic effects. For example, decreasing the time interval of the data from [6,60] to [10,60] have shown that these frequencies, though remaining, fluctuate in their relative strength, putting on the same footing.

![Projection of profile of $-\log(\text{likelihood})$](image)

Figure 1: Likelihood ratio profile for the oscillation frequency in the EC-data set. The global maximum is found at 5.35. Two more maxima of smaller strength are obtained with this method at $\omega = 0.88$ and 4.68.

Likelihood ratio test

In order to determine the significance of the modulation, a likelihood ratio test (LRT) was performed. The distribution of the LRT have been obtained by Monte Carlo simulation [1]. The results of the hypothesis tests are shown in table 1.

We note, for the 245 MHz resonator EC-data, that a significance of 1.3 sigma is not sufficient to reject the pure exponential decay. On the other hand the capacitive pick-up data present a significance greater than 3.7 sigma. The boundary in the capacitive pick-up data is due to the sample size of the simulated likelihood ratio distribution. As mentioned above, the observable of the pick-up data corresponds to the decay time + a delay (and a systematic error). This observable has been evaluated as well for the 245 MHz resonator data and therefore has been analyzed for comparison. The obtained LRT statistics is 14 which is comparable to the one found for the decay times without delay. This result exclude oscillation in the decay time + delay as well. Note that the significances are obtained from sampled LRT distribution under a clean null hypothesis, i.e. without systematics, which may result in an overestimation of the significance.

Table 1: Likelihood ratio test results. Only the capacitive pickup data present significant results.

<table>
<thead>
<tr>
<th>Sample size N</th>
<th>Time interval (s)</th>
<th>$\beta^+$ data (245 MHz Res.)</th>
<th>EC data (cap.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[245 MHz Res.]</td>
<td>[6.0 : 60.0]</td>
<td>[6.0 : 60.0]</td>
</tr>
<tr>
<td>LRT statistics</td>
<td>14.8</td>
<td>6.8</td>
<td>35</td>
</tr>
<tr>
<td>LRT p-value</td>
<td>9.4%</td>
<td>97.5%</td>
<td>&lt; 0.0102%</td>
</tr>
<tr>
<td>significance</td>
<td>1.32 sigma</td>
<td>≈ 0</td>
<td>&gt; 3.7 sigma</td>
</tr>
</tbody>
</table>

Conclusion

For the resonator data we have shown that an amplitude estimate of 9 (2) % is compatible with a pure exponential decay, that the likelihood $\omega = 0.88$ is not the global maximum, and that the hypothesis test do not reject the pure exponential decay for the EC- and $\beta^+$ decay time data. These results are consistent with a pure exponential decay but contradict the pick-up data which present, despite a smaller sample size, a much higher significance for the oscillation model. The analysis of the decay time + delay in the 245 MHz resonator is also consistent with the null hypothesis. Accordingly, the delay distribution cannot explain this difference.

Assuming that the physics signatures are better observed with the 245 MHz resonator data, these results might point out possible systematic effects that are not under control or artefact in the capacitive pick-up data. A Bayesian analysis corroborate this result [2].

References


* This work is part of the EMMI Rapid Reaction Task Force presentation given in Jena, July 2014