EPJ Web of Conferences **109**, 06005 (2016)
DOI: 10.1051/epjconf/201610906005

Linking neutrino oscillations to the nucleosynthesis of elements

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Abstract. Neutrino interactions with matter play an important role in determining the nucleosynthesis outcome in explosive astrophysical environments such as core-collapse supernovae or mergers of compact objects. In this article, we first discuss our recent work on the importance of studying the time evolution of collective neutrino oscillations among active flavors in determining their effects on nucleosynthesis. We then consider the possible active-sterile neutrino mixing and demonstrate the need of a consistent approach to evolve neutrino flavor oscillations, matter composition, and the hydrodynamics when flavor oscillations can happen very deep inside the supernovae.

1 Introduction

The formation of the elements in the Universe is closely related to the weak interactions between neutrinos and matter. From the light elements made in the Big-Bang nucleosynthesis to the heavy elements that are made by the neutron-capture processes in explosive astrophysical environments, neutrino interactions can interchange protons and neutrons (free or inside the nucleus) and play an important role in shaping the neutron-richness of the baryonic matter. In the explosive astrophysical events such as core-collapse supernovae and mergers of binary neutron stars or a neutron star with a black hole, the role of neutrinos in determining the property of the ejected matter and the outcome of the associated nucleosynthesis has been extensively studied, mostly using hydrodynamical simulations with detailed neutrino transport equations and post-processing the nucleosynthesis by an extended nuclear reaction network (e.g., [1, 2]).

However, an important aspect that currently cannot be modelled in the hydrodynamical simulations is the quantum phenomenon of neutrino flavor oscillations. As neutrinos with different flavors interact differently with matter, any mechanism that alters the flavor of the neutrinos after their production can potentially affect the prediction of the matter property and the outcome of the nucleosynthesis (e.g., [3, 4]).

Neutrino flavor oscillations which arise from the mixing between their flavor eigenstates and the mass eigenstates have successfully accounted for the results of terrestrial and Solar neutrino experiments. In fact, nearly all the mixing parameters are precisely measured, except for the sign of the atmospheric mass-squared difference and the CP violating phase(s) [5]. However, in the extreme astrophysical environments, due to the high baryonic density which can be higher than the nuclear saturation density and the high temperature, large neutrino number density which is comparable to the densities of baryons and electrons is typically present. As a result, neutrino self-interactions that

Article available at http://www.epj-conferences.org or http://dx.doi.org/10.1051/epjconf/201610906005

must be considered in modelling their flavor oscillations lead to a non-linear coupling between the different neutrino quantum states. Despite numerous works studying neutrino flavor oscillations in those environments during the past decade (see e.g., [6, 7] for reviews and the references therein), it remains a challenging and exciting problem to be solved in order to fully appreciate the role of neutrinos in supernova explosions and in the nucleosynthesis of elements. In this article, we discuss some of our recent works along this direction in improving the link between neutrino flavor oscillations and the nucleosynthesis of elements, particularly in core-collapse supernovae.

2 Neutrino flavor oscillations in medium

For neutrino flavor oscillations in the dilute gas limit such that neutrinos kinematically decouple from matter, the equation of motion for the neutrino density matrix $\varrho(t, \mathbf{x}, \mathbf{p})$ is given by [8]

$$\frac{\partial \varrho(t, \mathbf{x}, \mathbf{p})}{\partial t} + \hat{\mathbf{v}} \cdot \nabla \varrho(t, \mathbf{x}, \mathbf{p}) = -i[H(t, \mathbf{x}, \mathbf{p}), \varrho(t, \mathbf{x}, \mathbf{p})]. \tag{1}$$

The Wigner-transformed density matrix $\varrho(t, \mathbf{x}, \mathbf{p})$ can be explicitly written in the flavor basis for the active neutrinos:

$$\varrho(t, \mathbf{x}, \mathbf{p}) = \begin{bmatrix} \varrho_{ee} & \varrho_{e\mu} & \varrho_{e\tau} \\ \varrho^*_{e\mu} & \varrho_{\mu\mu} & \varrho_{\mu\tau} \\ \varrho^*_{e\tau} & \varrho^*_{\mu\tau} & \varrho_{\tau\tau} \end{bmatrix}, \tag{2}$$

where the diagonal terms $\varrho_{\alpha\alpha}(t, \mathbf{x}, \mathbf{p}) = f_{\nu_\alpha}(t, \mathbf{x}, \mathbf{p})$ are the statistical phase-space distribution functions of neutrinos with flavor α. The off-diagonal (correlation) terms encode the information of neutrino flavor mixing. The Hamiltonian $H(t, \mathbf{x}, \mathbf{p}) = H_{\rm vac}(p) + H_m(t, \mathbf{x}) + H_{\nu\nu}(t, \mathbf{x}, \mathbf{p})$ contains the contribution from the vacuum neutrino mixing, neutrino forward-scattering with matter [9, 10], and neutrino forward-scattering among themselves [11–13]. $H_{\rm vac}(p) = UM^2U^\dagger/2p$ where U is the unitary mixing matrix, $M = {\rm diag}(m_1, m_2, m_3)$ with m_i being the mass of the ith neutrino mass eigenstate. $H_m(t, \mathbf{x}) = \sqrt{2}G_F[n_e(t, \mathbf{x}){\rm diag}(1, 0, 0) - n_n(t, \mathbf{x})\mathbb{I}_{3\times3}/2]$, $n_e(t, \mathbf{x})$ is the net electron number density and $n_n(t, \mathbf{x})$ is the neutron number density. The ν-ν Hamiltonian

$$H_{\nu\nu}(t, \mathbf{x}, \mathbf{p}) = \frac{\sqrt{2}G_F}{(2\pi)^3} \int d^3q (1 - \hat{\mathbf{p}} \cdot \hat{\mathbf{q}}) \{\varrho(t, \mathbf{x}, \mathbf{q}) - \bar{\varrho}(t, \mathbf{x}, \mathbf{q}) + {\rm Tr}[\varrho(t, \mathbf{x}, \mathbf{q}) - \bar{\varrho}(t, \mathbf{x}, \mathbf{q})]\mathbb{I}_{3\times3}\}, \tag{3}$$

where $\bar{\varrho}$ is the density matrix for antineutrinos defined in the same way as in Eq. (2). In the above equations, we have neglected the sub-leading terms in the Hamiltonian which can cause the helicity coherence [8] and the beyond-mean-field correlations [14].

The above formulation can be easily generalized to describe the flavor mixing between active neutrinos and the sterile neutrinos (ν_s) by enlarging ϱ to include the components $\varrho_{\alpha s}$ that however do not contribute to the Hamiltonian $H_{\nu\nu}$ in the leading order. Therefore, the only change to the Hamiltonian is the addition of vacuum mixing entries in $H_{\rm vac}$.

3 Collective neutrino oscillations and supernova nucleosynthesis

To apply the above formalism in the astrophysical environments such as supernovae, we first assume that all neutrinos decouple from matter kinematically at a sharp neutrinosphere $r = R$. We further make the assumptions that the supernova environment is spatially spherically-symmetric, temporally stationary during the time of neutrino propagation, and the flavor evolution of neutrinos preserves these symmetries. In this case, Eq. (1) can be reduced to :

$$\frac{d\varrho(t, r, u, E)}{dr} = -i[H(t, r, u, E), \varrho(t, r, u, E)], \tag{4}$$

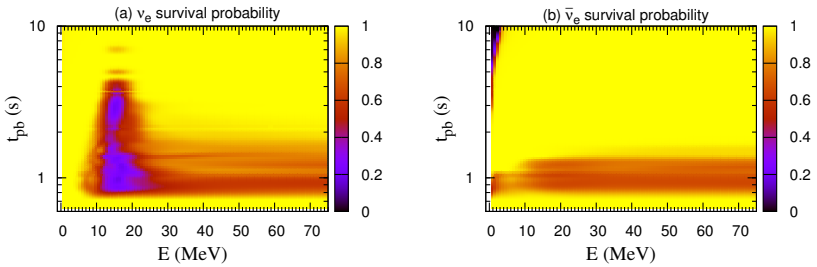

Figure 1. The angle-averaged survival probabilities of (a) ν_e and (b) $\bar{\nu}_e$ as functions of the neutrino energy E and the post-bounce time $t_{\rm pb}$. (Reprinted figure from [15]; copyright (2015) by the American Physical Society.)

where E is the neutrino energy, $u = \cos\theta_{\rm em}$ with $\theta_{\rm em}$ being the emission angle of the neutrinos from the neutrinosphere w.r.t to the radial direction. The initial conditions are given by setting the non-zero diagonal elements of $\varrho(t, r, u, E)$ equal to the neutrino distribution function $f_{\nu_\alpha}(t, r, u, E)$ which can be parametrized or given by supernova simulations with detailed neutrino transport. Supplied with the density profiles $n_e(t, r)$ and $n_n(t, r)$, Eq. (4) can then be solved for each given t.

We performed a set of comprehensive numerical calculations to map out the flavor conversion probabilities $P_{\nu_\alpha\to\nu_\beta}(t, r, u, E)$ between active flavors [15] for the supernova models of [16]. We found that within the first $\sim$ 500 km, the neutrino flavor conversion is dominated by the ν-ν contribution. This leads to the so-called "collective neutrino oscillations" [6, 7], which may give rise to a sharp transition of $P_{\nu_\alpha\to\nu_\beta}(t, r, u, E)$ at some specific E (the "spectral splits/swaps"). Fig. 1 shows the angle-averaged survival probabilities for the initial ν_e and $\bar{\nu}_e$ as functions of E and the time post supernova core-bounce, $t_{\rm pb}$, at r = 500 km where the collective neutrino oscillations have ceased for the 18 $M_\odot$ supernova. We see that the feature of the spectral splits is indeed present in our calculations. More importantly, the survival probabilities change substantially as the supernova evolves with time. At the later stage, the flavor conversions between $\nu_e \leftrightarrow \nu_{\mu,\tau}$ and $\bar{\nu}_e \leftrightarrow \bar{\nu}_{\mu,\tau}$ are more suppressed.

We stress here that this time evolution has important consequences for supernova nucleosynthesis in both the ν-driven wind and the ν(-induced) nucleosynthesis in the supernova envelopes. For the ν-driven wind which may be a site of heavy element formation, the time scale relevant for the neutrino-matter interactions to change the ejecta composition can last a few seconds if it is initially proton-rich such that the νp process can occur [17, 18]. Similarly, for the ν(-induced) nucleosynthesis (e.g. [19, 20]), it is the total exposure of nuclei to the neutrino fluence that determines the final yields. In this study, we found that due to the suppression of flavor conversion in the $\bar{\nu}$ sector, collective neutrino flavor oscillations have little impact on the νp process. As substantial flavor conversion occurs in the ν sector, the production of rare nuclei such as ^{138}La and ^{180}Ta may be enhanced by the flavor conversion of $\nu_e \leftrightarrow \nu_{\mu,\tau}$. Nevertheless, we note that these results are subject to change once the effect of symmetry breaking of neutrino flavor oscillations (see e.g., [7] and references therein) can be self-consistently taken into account in the future.

4 The interplay between flavor oscillations and hydrodynamics

The discussion in the last section is based on the assumption that neutrino flavor oscillations do not change the hydrodynamic variables such as the baryonic density ρ, the temperature T, the fluid velocity v, and the matter composition. However, if neutrino flavor oscillations happen deep enough inside

supernovae where the neutrino-matter interactions are still important in setting up the hydrodynamic properties of the environment, one has to evolve the flavor equations along with the hydrodynamic equations. In principle, for a fully consistent derivation, one needs to extend Eq. (1) to full quantum kinetic equations [8] by including the collision terms and couple them with the hydrodynamic equations. However, as a first step, we make the assumption that the neutrino interactions with matter are weak enough such that the collision terms can still be neglected for flavor evolution while those interactions may be strong enough to change the hydrodynamic properties.

Based on the above, we have coupled the further reduced flavor evolution equation using the so-called "single-angle approximation" [6] which assumes that the flavor evolution history is independent of the neutrino emission angles:

$$\frac{d\varrho(r,E)}{dr} = -i[H(r,E),\varrho(r,E)], \tag{5}$$

with the steady-state hydrodynamic equations that may adequately describe the physical conditions in the ν-driven wind from the proto-neutron star (PNS) [21, 22]:

$$\dot{M} = 4\pi r^2 \rho v y, \tag{6a}$$

$$\frac{1}{y}\frac{dy}{dr} + \frac{1}{\varepsilon + P}\frac{dP}{dr} = 0, \tag{6b}$$

$$\frac{d\varepsilon}{dr} - \frac{\varepsilon + P}{\rho}\frac{d\rho}{dr} - \rho\frac{\dot{q}_\nu}{vy} = 0, \tag{6c}$$

where $\dot{M}$ is the constant mass outflow rate of the ejecta, v is its radial velocity, $y^2 = (1 - 2GM/r)/(1 - v^2)$, G is the Newtonian gravitational constant, M is the mass of the PNS, ε is the total energy density, P is the pressure, and $\dot{q}_\nu$ is the net energy gain/loss rate per unit mass by ν heating and cooling. For $\dot{q}_\nu$, we have included the charged-current ν absorption, $\nu\bar{\nu}$ annihilation and their reverse reactions, and ν scattering with $e^\pm$ and nucleons. Detail expressions will be reported in a forthcoming publication [23].

For the matter composition of the wind, we consider the phase when the temperature is still high enough so that matter consists of free protons, neutrons, and $e^\pm$. In this case, the matter composition is determined by the electron number fraction $Y_e = n_e/(\rho/m_u)$ where m_u is the atomic mass unit and the evolution of Y_e is governed by

$$(vy)\frac{dY_e}{dr} = (\lambda_{\nu_e n} + \lambda_{e^+ n})(1 - Y_e) + (\lambda_{\bar{\nu}_e p} + \lambda_{e^- p})Y_e, \tag{7}$$

where $\lambda_{\nu_e n}$, $\lambda_{e^+ n}$, $\lambda_{\bar{\nu}_e p}$ and $\lambda_{e^- p}$ are the corresponding charged-current reaction rates.

We consider here in particular the reduced flavor subspace of ν_e ($\bar{\nu}_e$) and ν_s ($\bar{\nu}_s$), with a mass-squared difference $\delta m^2 \approx 1.75\ \mathrm{eV}^2$ and the vacuum mixing angle corresponding to $\sin^2 2\theta \approx 0.1$, as indicated by the neutrino anomalies [24, 25]. Such an active-sterile flavor conversion for the initial ν_e and $\bar{\nu}_e$ can happen at $Y_e \approx 1/3$ such that both $\dot{q}_\nu$ and Y_e are greatly influenced as suggested by previous studies without considering the feedback of flavor oscillations on hydrodynamics [26–28].

Eqs. (5)–(7) can be solved given the boundary conditions below:

- The ν luminosity and the temperature, L_{ν_α} and T_{ν_α}, at the PNS surface $r = R$, assumed to be given by a Fermi-Dirac distribution with zero chemical potential. This specifies the neutrino density matrix element $\varrho_{\alpha\alpha}(R,E) = f_{\nu_\alpha}(E) = 1/(1 + e^{E/T_{\nu_\alpha}})$.
- The hydrodynamic conditions at the PNS surface, $T(R) = T_{\nu_e}$, $\dot{q}(R) = 0$ and $\dot{Y}_e(R) = 0$.

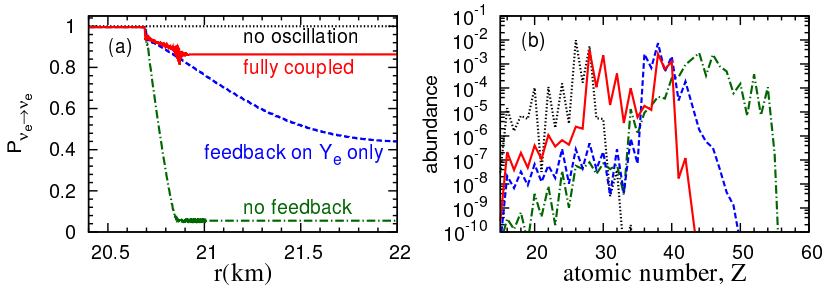

Figure 2. (a) The average ν_e survival probability as a function of radius for different cases discussed in the text, and (b) the corresponding elemental abundances of nucleosynthesis as a function of atomic number.

- An outer boundary temperature T_b at some large radius $r \gg R$.

We have performed such a calculation with the parameters $(L_{\nu_e}, L_{\nu_e}, L_{\nu_{\mu,\tau}}) = (1.67, 2.01, 2.58) \times 10^{51}$ erg/s, $(T_{\nu_e}, T_{\nu_e}, T_{\nu_{\mu,\tau}}) = (2.68, 3.78, 3.71)$ MeV, $M = 1.282\ M_\odot$, $R = 18.07$ km, and $T_b = 0.12$ MeV at $r = 10^3$ km. Those values are taken with the guide of a supernova simulation [1]. To understand the role of the convoluted feedback of the composition and hydrodynamic changes on the flavor oscillations, we have performed additional calculations as follows:

1. No feedback: Eq. (5) decoupled from Eq. (6) and Eq. (7), i.e., we first derive $Y_e(r)$ and $\rho(r)$ from Eq. (6) and Eq. (7) without considering flavor oscillations. We then evolve Eq. (5) with the derived $Y_e(r)$ and $\rho(r)$.

2. Feedback on Y_e only: Eq. (5) coupled with Eq. (7) but decoupled from Eq. (6), i.e., we first derive $\rho(r)$ from Eq. (6) and Eq. (7) without considering flavor oscillations. We then evolve Eq. (5) and Eq. (7) with the derived $\rho(r)$.

In Fig. 2(a), we show the energy-averaged ν_e survival probability $P_{\nu_e \to \nu_e}$ as a function of radius for all three cases described above. It clearly shows that both Y_e and hydrodynamic evolution have strong impact on the neutrino flavor conversion. In the case of "no feedback", nearly all the initial ν_e are converted to ν_s, due to the so-called "matter-neutrino resonance" (MNR) mechanism [29, 30]. For the "feedback on Y_e only" case, ν_e are less converted to ν_s but the flavor conversion process takes place up to a much larger radius. As for the fully coupled case, only $\sim 15\%$ of ν_e are converted. The main differences between those results arise mainly because MNR can be extended to a much larger radius once Y_e is affected by ν interactions. However, this extended resonance may not remain stable, depending on the velocity of the ejecta, which is affected by the change of $\dot{q}_\nu$. The details will be reported in a forthcoming publication [23].

In Fig. 2(b), we further show the resulting elemental abundances as a function of the atomic number Z. Because $P_{\nu_e \to \nu_e}$ differs when a different level of coupling among Eqs. (5)–(7) is employed, the nucleosynthesis outcome can be dramatically different. As expected, when ν_e are converted more to ν_s, the matter composition becomes more neutron-rich, which results in the production of heavier elements.

5 Summary

In this article, we have discussed two aspects in connecting neutrino flavor oscillations to the nucleosynthesis of elements. We have shown in Sec. 3 that due to the time evolution of the neutrino

characteristics and the density structure above the PNS in supernovae, it is important to include this time-dependence when studying the collective neutrino flavor oscillations and their impact on the nucleosynthesis in the ν-driven wind and in the supernova envelopes. In Sec. 4, we have shown that when neutrino flavor oscillations take place very close to the PNS, which may happen when considering the possible active-sterile neutrino flavor transformation, the change of the matter composition and hydrodynamic quantities may have a large impact on the ν flavor evolution histories, thereby complicating the determination of the nucleosynthesis outcome in the ν-driven wind.

Acknowledgements

This work was partly supported by the Helmholtz Association (HGF) through the Nuclear Astrophysics Virtual Institute (VH-VI-417), the Helmholtz International Center for FAIR within the framework of the LOEWE program launched by the state of Hesse and by the US DOE grant DE-FG02-87ER40328 at UMN.

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