Transition from direct to sequential two-proton decay in s–d shell nuclei

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ABSTRACT

The transition between different mechanisms of two-proton decay in exotic nuclei and the governing physical conditions are generally studied. To perform this large-scale survey, the direct-decay model, which is widely used, was improved by generalizing the semi-analytical approaches used before. The improved model provides a flawless phenomenological description of three-body correlations in 2p decays, as is demonstrated by several examples of decays of low-lying states in 16 Ne. Different transition mechanisms are shown to occur beyond the proton dripline for s–d shell nuclei. It is found that the transition dynamics of 2p-emitters allows the extraction of the width of the ground-state resonance of the core + p subsystem. The practical application of the method is illustrated by determining properties of the 14 Ne ground state derived from the 15 Ne → 13 O + 2 p decay data, and of the 34 Cl ground state derived from the 39 Ar → 38 S + 2 p decay data.

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1. Introduction

Two-proton (2p) radioactivity has attracted a large interest in theory and experiment in recent years. This peculiar radioactive decay mode was predicted for even-proton systems in the year 1960 [1] and experimentally discovered in 2002 [2,3]. Since then, important advances in the studies of two-proton (2p) radioactivity, and, more generally speaking, of true two-proton decays [4] were made. These studies comprise part of a general effort to extend our knowledge to the limits of existence of nuclei, up to the neutron and proton driplines and as far beyond them as possible. It is known that the true 2p decay mechanism, where both protons are emitted simultaneously, is realized under certain energetic conditions, in particular when the relation between one- and two-proton separation energies (S1 and S2p, respectively) makes sequential decay (two single-proton emissions separated in time) impossible: S(1) < 0 and S(2p) > 0, where A denotes the mass number of the mother nucleus. However, in light proton-rich and in neutron-rich nuclei, where the potential barriers governing the decay are relatively small, the genuine three-body decay mechanism can materialize in the form of a democratic decay [5]; these democratic decays exhibit distinct correlations between the decay products and follow a different systematic of lifetimes compared to true 2p decays [4]. Together, and at variance to the sequential decay mechanism, which requires the population of intermediate narrow states, the true 2p and democratic decays are often characterized as direct decays. A schematic view of the various decay mechanisms and their energy balance is shown in Fig. 1 (a,b,d,e).

The transition from the true 2p mechanism to the democratic decay dynamic is typically governed by the relation of the two quantities, Γr (the width of the resonance ground state) and Er (the ground state decay energy in the core + p subsystem of the three-body system core + p + p). The characteristic of this transition and the underlying physics have so far not been considered.

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systematically. This is the purpose of the present Letter, and the motivation will be further detailed in the following.

In general, the decay dynamics of 2p emission is fully characterized by three parameters: (i) the 2p decay energy \( E_T = -S(A)^{(2p)} \), (ii) the ground state (g.s.) decay energy in the core + p subsystem \( E_r = -S_0^{(A-1)} = S_0^{(A)} - S_{2p}^{(A)} \), and (iii) the width \( \Gamma_r \) of the intermediate state. As the width \( \Gamma_r \) is a function of \( E_r \), the delimitation of different 2p decay mechanisms can be depicted by a kind of “phase diagram” in the \((E_T, E_r)\) plane, see Fig. 1 (c).

The transitions between different mechanisms (or decay dynamics) were rudimentary mentioned in previous works [4,6–8]. Here, Fig. 1 provides an overall view of the transition dynamic in 2p decays. At first glance, one would expect that the direct 2p and the sequential decay are separated by the line where \( E_r = E_T \). However, it was shown in Refs. [6,7] that this transition happens at lower values \( E_T = (0.75–0.85)E_r \). Similarly, it was found experimentally for decays of \(^{16}\text{Be} \) and \(^{36}\text{Ne} \) in Refs. [9,10] that the transition “Democratic 2p” \( \leftrightarrow \) “Sequential 2p” does not even happen for energies \( E_T \) considerably higher than \( E_T = 2E_r \); this issue remains an open question. In the present work we investigate systematically the transition regime “True 2p” \( \leftrightarrow \) “Sequential 2p”, and, while moving along this line, we obtain detailed quantitative results for the transition region “True 2p” \( \leftrightarrow \) “Democratic 2p” in Section 5. It is demonstrated that this transition region is often involved in ground-state decays of \( s-d \) shell nuclei beyond the proton dripline.

Besides the general interest, there is another motivation for the present study, which has practical impact. Any phenomena of the transition type are expected to be prone to abrupt changes of its properties in response to minor variation of the parameters. Here even small variations of the characteristic decay parameters can lead to strong modifications of the decay properties and observed quantities (such as branching ratios, lifetimes, etc.). In turn, this sensitivity can be used for a precise derivation of the nuclear parameters from measurements or for imposing limitations on their relations. Among the relevant parameters \((E_T, E_r, \Gamma_r)\), the most interesting one is the sensitivity to \( \Gamma_r \). While resonance energies can be determined with rather good precision from available experimental data, much better resolution and/or statistics is required to derive the width of a state. The latter conditions are presently often not attainable for exotic dripline systems. Moreover, there is a rather large experimental “blind spot” with respect to the g.s. resonance widths, which covers the interval from \( \sim 1 \) meV to \( \sim 10 \) keV (corresponding to resonance lifetimes ranging from \( \sim 10^{-19} \) s to \( \sim 1 \) ps): the invariant and missing mass methods using spectrometers or kinematical complete measurements can be applied when resonance widths exceed \( \sim 10 \) keV (i.e., resonance lifetimes \( \lesssim 10^{-19} \) s), while other techniques (such as the in-flight decay method, plunger, etc.) are applicable only for long-lived states with lifetimes exceeding a few picoseconds (i.e., widths \( \lesssim 1 \) meV). Therefore, we derive here an indirect method to obtain information on the widths \( \Gamma_r \) of the proton subsystems in the transition regimes between different mechanisms of two-proton emission. The practical feasibility of this method is demonstrated with the examples of \(^{14}\text{F} \) g.s. populated in the 2p decay of the \(^{19}\text{Ne} \) g.s. [11] and of \(^{20}\text{Cl} \) g.s. populated in the 2p decay of \(^{30}\text{Ar} \) [12].

2. Dynamics of three-body systems near the proton dripline

As mentioned above, the relationship of \( S_p^{(A)} \) and \( S_{2p}^{(A)} \) is one of the decisive conditions for different forms of nuclear dynamics in the even-proton systems in the proximity of the proton dripline. To illustrate this behavior, the systematics of the \( p \) and 2p separation energies for Ne and Ar isotopes near the proton dripline is shown in Fig. 2. These nuclides represent the lower and the upper parts of the \( s-d \) shell, respectively. One can see that the evolution of \( S_p^{(A)} \) and \( S_{2p}^{(A)} \) values is different, and the corresponding curves intersect near the proton dripline, leading to rapid changes of the dynamical properties of nuclides in this region. Particle-stable nuclei that are closest to the dripline typically exhibit a so-called Borromean structure (this denotes the phenomena that removal of one of the three bodies, either one of the two protons or the core, leads to disintegration of the whole three-body system) with the condition \( S_p^{(A)} > S_{2p}^{(A)} \). Particle-unstable nuclei located just beyond the dripline often undergo direct decays, i.e. either 2p decay by the true \( (S_{2p}^{(A)} < 0, S_p^{(A)} > 0.25S_{2p}^{(A)}) \) or by the democratic \( (S_{2p}^{(A)} < 0, S_p^{(A)} \sim 0) \) mechanism; in addition, a transition situation is possible.
ble. Nuclei located further beyond the dripline pass the transition area and arrive either at a domain of democratic 2p decays (very likely for the lightest nuclei) or at a domain of sequential 2p decays (rather expected for the heavier, higher-Z systems). Thus, the transition dynamics investigated here is a quite likely situation in the area between the direct (true or democratic) and sequential 2p-decay dominance.

3. Improved direct 2p-decay model

Simplified analytical models of 2p decay are useful theoretical tools, which can substitute complicated and time-consuming three-body calculations when “quick” estimates, systematic studies, or just a phenomenological comparison with experimental observations is needed. The derivation of the direct decay model for 2p emission was described in detail in Refs. [7,14]. The model is based on the utilization of simplified three-body Hamiltonians, which allow a factorization of the Green’s function. This approach can be traced back to the ideas of Galitsky and Cheltsov [15]. The expressions originating from the resonant R-matrix-type approximation [14] look very similar to those arising from the quasi-classical approximation [16], later used for 2p emission in a number of works [17,18, and references therein]. However, the models have important differences, which are discussed for example in [4,14,19].

The direct decay models of Refs. [7,14] have some disadvantages, which are overcome in the present approach. Namely, the decay amplitudes must be factorized either in the “V” system [see Fig. 3 (a), or in one of the “Y” systems [see Fig. 3 (b,c)]. In the “V” system, the anti-symmetrization between identical protons is straightforward. However, the assumption of an infinite core mass is required in this case. Also the interaction of the protons (including Coulomb repulsion) is neglected, which leads to some overestimation of the width. This type of approximation is called “no p–p Coulomb” (NPP), see Eq. (21) of Ref. [7]. Working in the “Y” systems, the p–p Coulomb interaction can be effectively considered by taking the Z/2 term of the Coulomb interaction in the X dimension as Z/2, while in the Y coordinate as Z/1. Unfortunately, a consistent anti-symmetrization is complicated in the “Y” system, and one of the core + p resonances is not treated appropriately. That is because the X coordinate is exactly the vector between core and p, but the Y coordinate is only approximately equal to the vector between the core and another p. The latter approximation becomes exact in the limit Ac → ∞ (in this limit “Y” → “V”). This approximation is called “effective p–p Coulomb” (EFC), see Eq. (22) of Ref. [7].

In the following, an improved direct decay model (IDDM) is developed which is based on simple R-matrix-type analytical approximations for amplitudes; it combines the positive features of both approximations, NPP and EFC. The general expression derived in Ref. [7] [see Eqs. (30)–(32) of this work], however with a modification, which accounts in a correct way for the angular momentum coupling of \{j_1,l_1\} and \{j_2,l_2\} of the first and the second core + p subsystems, is:

\[
\frac{d\Gamma(E_{2p})}{d\Omega} = \frac{8E_{2p}}{\pi(2J+1)} \sum_{\ell J} \sum_{\gamma} A_{\gamma J}^{M_1}(E,\Omega_1,\Omega_2)^2, \tag{1}
\]

where \(J\) is the total angular momentum, \(\gamma = \{j_1,l_1,j_2,l_2\}\), \(\Omega_{\gamma} = \{\gamma,\Omega_1,\Omega_2\}\), and the angles \(\Omega_\gamma\) are related to the vectors \(r_\gamma\) (see Fig. 3). The amplitude \(A_{\gamma J}^{M_1}\) is defined as

\[
A_{\gamma J}^{M_1}(\Omega_{\gamma}) = \frac{j_1 \otimes j_2}{\sqrt{v_1v_2}} \int_0^R dr_1 \int_0^R dr_2 \varphi_{j_1l_1}(k_1r_1) \times \varphi_{j_2l_2}(k_2r_2) \Delta V(r_1',r_2') \psi_{\gamma}(r_1',r_2'), \tag{2}
\]

where \(\psi_{\gamma}\) is assumed to be the radial part of the three-body resonant wave function (WF) in \(jj\) coupling. The nature of the short-range potential \(\Delta V\) and its derivation and the underlying approximations are discussed in detail in Ref. [7]. The amplitude \(A_{\gamma J}^{M_1}\) does not depend on the parameter \(R\) when it is chosen beyond the range of \(\Delta V\). The integral in Eq. (2) can be evaluated by replacing the continuum WFs with the quasi-stationary WF in proximity of the two-body resonance energies:

\[
\varphi_{\gamma}(kr) = \frac{\sqrt{\Gamma(E)}}{2} A_{\gamma J}(E) \hat{\psi}_{\gamma}(E_r,r), \tag{3}
\]

where \(k, v, E\) are the related momentum, velocity, and energy in the two-body channel. The pure radial function \(\hat{\psi}_{\gamma}(E_r,r)\) is the quasi-bound WF: it has a resonant boundary condition at the two-body resonant energy \(E_r\) and is normalized to unity in the internal domain as

\[
\hat{\psi}_{\gamma}(E_r,r > R) \propto G_{\gamma}(kr), \quad \int_0^R dr \left| \hat{\psi}_{\gamma}(E_r,r) \right|^2 = 1,
\]

where \(G_{\gamma}\) is irregular at the origin Coulomb WF with angular momentum \(l\). In this definition, the value of parameter \(R\) should be chosen around the classical outer turning points for the core + p Coulomb barriers. The amplitude \(A_{\gamma J}\) of the resonance with the parameters \(E_r\) and \(\Gamma_{\gamma}\) is \(\Gamma_{\gamma}(E_r)\) is defined as

\[
A_{\gamma J}(E) = \frac{\sqrt{\Gamma(E)}}{E_r - i\Gamma(E)/2} + A_{\gamma J}^{(p)}(E). \tag{4}
\]

The width as a function of energy is defined by the standard R-matrix expression

\[
\Gamma(E) = 2 - \frac{\theta^2}{2Mr_{cp}^2} P_l(Z_1, Z_2, r_{cp}, E), \tag{5}
\]

via the penetrability function \(P_l\), channel radius \(r_{cp}\), and reduced width \(\theta^2\). In Eq. (4), the resonant term typically used in such calculations is augmented by taking into account the “potential scattering” contribution \(A_{\gamma J}^{(p)}\). This allows additional tests of reliability of the obtained results. The potential scattering term can be approximated reasonably well by the scattering off a solid sphere with radius \(r_{sp}\):

\[
A_{\gamma J}^{(p)}(E) = -\frac{2F_{1}(kr_{sp}) G_{1}(kr_{sp}) - iF_{2}(kr_{sp})}{\sqrt{\Gamma(E)}} \frac{F_{2}(kr_{sp})}{F_{1}(kr_{sp})} + G_{1}^2(kr_{sp}). \tag{6}
\]

From Eq. (3) the amplitude is factorized into a momentum-dependent term and a radial integral.
\[ A^{LM}_{j_f}(\Omega_x) = \frac{V_j^f}{4} \{j_1 \otimes j_2\}_{LM} A_{j_1j_2}(E_1) A_{j_2j_2}(E_2), \]
\[ V_j^f = \int_0^R \int_0^R dr_1 \int_0^R dr_2 \langle \psi_{j_11}(E_1, r_1', r_1') | \psi_{j_22}(E_2, r_2', r_2') \rangle \times \Delta V(r_1', r_2') \varphi_{j_f} / \varphi_{j_1}(r_1', r_2'). \]  

(7)

Because the spin degrees of freedom cannot be observed even in the most sophisticated modern radioactive-ion-beam experiments, the summation over the final state spin quantum number is performed. For systems with zero core spin there are two terms to be treated. The first type is with \( S = 0 \) where \( J = L \), and the second is with \( S = 1 \) where \( L = |J - 1, J, J + 1| \). After averaging over \( M_J \), it is obtained that
\[ \frac{d\Gamma(E_{3\gamma})}{d\Omega_{3\gamma}} = \sum_{LS} \frac{E_{3\gamma}}{2\pi(2L+1)} \left| \sum_J A^{LM}_{3\gamma}(\Omega_x) \right|^2, \]
\[ A^{LM}_{3\gamma}(\Omega_x) = C^{LS}_{j_1j_2} V_j^f \left[ \{l_1 \otimes l_2\}_{LM} A_{j_1j_2}(E_1) A_{j_2j_2}(E_2), \right. \]
\[ \left. \{l_1 \otimes l_2\}_{LM} \right] \sum_{m_1m_2} \left( \frac{E_{L-1}}{E_{L2}} \right)^2 \langle \psi_{j_1m_1}(E_1, \hat{r}_1) | \psi_{j_2m_2}(E_2, \hat{r}_1) \rangle \].

(8)

The coefficient \( C^{LS}_{j_1j_2} \) provides the \( jj \) to \( LS \) re-coupling for two protons in the orbitals \( j_1 \) and \( j_2 \):
\[ C^{LS}_{j_1j_2} = \frac{l_1 l_2 L}{1/2 1/2 S} \frac{l_1 l_2 J}{j_1 j_2 J}. \]

The potential matrix elements \( V_j^f \) can be reasonably approximated [7,10] as
\[ V_j^f = c_j^f \sqrt{(E_{L1} + E_{L2} - E_T)^2 + \left[ \Gamma_1(E_{L1}) + \Gamma_2(E_{L2}) \right]^2/4}, \]
where \( c_j^f \) are phenomenological complex coefficients, all normalized to unity, as \( \sum_j |c_j^f|^2 = 1 \). The expression in Eq. (9) uses intermediate states in the subsystems with appropriate sets of quantum numbers and correctly couples them to the total spin of the system. However, there are important drawbacks in the approximation which do not allow the reproduction of realistic momentum distributions of fragments in \( pp \) decays.

Therefore, a phenomenological improvement of Eq. (9) is proposed by replacing the amplitude with a more realistic one. The latter uses terms composed in different Jacoby sectors instead of those in the “\( V \)“ system. For the case without potential scattering contribution in Eq. (4), it reads
\[ A^{LM}_{3\gamma}(\Omega_x) \rightarrow \hat{O}_S \left( \left[ \frac{l_1^x}{l_1^x} \otimes \frac{l_1^y}{l_1^y} \right]_{LM} A_{j_1j_2}(E_1) \sqrt{\Gamma_2(E_2)} \right) \]
\[ + \left[ \frac{l_2^x}{l_2^x} \otimes \frac{l_2^y}{l_2^y} \right]_{LM} \Gamma_1(E_1) A_{j_1j_2}(E_2) \right) \times \frac{\Gamma_1(E_1) + \Gamma_2(E_2)}{E_{L1} - E_{L2} + i \left[ \Gamma_1(E_{L1}) + \Gamma_2(E_{L2}) \right]/2}. \]

(10)

The permutation operator \( \hat{O}_S \) provides the correct symmetry of the WFs with respect to proton permutations. As the isospin WFs of two protons are symmetric, it should be used as
\[ \hat{O}_0 = S, \quad \hat{O}_1 = A. \]

The amplitude \( A^{(pp)}_S \) introduces corrections for the \( pp \) final state interactions both for the \( S = 0 \) and \( S = 1 \) states of the two protons. The following expression is used:

\[ A^{(pp)}_S(E_{T_f}) = \frac{N}{v_{kp}} \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{d\phi}{\phi} \left[ \frac{1}{\phi^{1/2}} \right] \phi(r) \exp \left( -\frac{3r^2}{4a_{pp}^2} \right). \]

(11)

where \( k_{T_f} \) and \( v_{T_f} \) are momentum and velocity associated with the \( p-p \) energy \( E_{T_f} \). The meaning of such an approximation is to treat the protons as being emitted from a broad spatial region defined by the function \( \phi(r) \). The source function \( \phi(r) \) with a Gaussian type formfactor is used
\[ \phi(r) = \frac{1}{a_{pp}^2} \left[ \frac{54}{\pi} \right] \exp \left( -\frac{3r^2}{4a_{pp}^2} \right). \]

The radius parameter \( a_{pp} \) in the above expression is equal to the root mean square radius of the function \( \phi(r) \). The singlet \( p-p \) potential \( V(r) = -V_0 \exp[-(r/r_0)^2] \) with \( V_0 = -31 \) MeV and \( r_0 = 1.8 \) fm is used to define the s-wave proton scattering WF \( \psi^{(pp)}_s \). The zero nuclear interaction is assumed for the \( p-p \) motion \( \psi^{(pp)}_s \). The normalization coefficient \( N \) is chosen such that for a fixed three-body decay energy \( E_T \)

\[ \int_0^1 d\phi \phi \left| A^{(pp)}_S \right|^2 = \pi / 8. \]

In the calculations, \( a_{pp} = 4 \) fm is used, for which \( |A^{(pp)}_S(E)|^2 \) closely resembles the free \( p-p \) scattering cross section profile; a difference appears at small \( E \) values, where \( A^{(pp)}_S(E) \) has the correct asymptotic behavior.

The IDDM unifies the advantages of both the NPP and EFC models and also goes beyond, thus providing a reliable tool for quantitative estimates and a phenomenological description of \( pp \) decays.

(i) It can be shown, that Eq. (10) returns exactly Eq. (9) in the approximation \( A_c \gg 1, Z_c \gg 1, A^{(pp)}_S \rightarrow 1 \). Thus, the improved expression shows the proper transition to the previously used models.

(ii) The spins of the resonances in the subsystems and total angular momentum coupling schemes are correctly accounted in the proposed expressions as well as the proper symmetries of two-proton amplitudes.

(iii) In the present model two resonances can be treated simultaneously in two core + \( p \) subsystems without the infinite-core-mass assumption.

(iv) The proton–proton final state interaction is treated in a phenomenologically sufficient way.

4. Application to \(^{16}\)Ne decays

In the following, the IDDM is applied to the examples of the \(^{16}\)Ne 0\(^+\) g.s. and the 2\(^+\) first excited state, both decaying to \(^{14}\)O + \( p + p \), and its properties and performance is illustrated. In Refs. [10,20], the \(^{16}\)Ne decay data have been compared with sophisticated full-scale three-body calculations. The comparison yields quantiative agreement, therefore the results of the detailed three-body calculations are used as a reference for the results obtained in the present work. This allows a direct and general comparison and avoids the use of Monte-Carlo (MC) simulations. The latter are needed for a realistic comparison of the semi-analytical model results with the data, which comprise the bias of the particular detector setup. In Fig. 4 the energy and angular distributions are presented, and widths are given in Table 1. It is assumed that a single configuration is involved in the decay: \( |s_1^f j_0 \rangle \) for the 0\(^+\) state and \( |s_1^f j_2 d^2 \rangle \) for the 2\(^+\) state. The corresponding \( (E_{T_f}, \Gamma_f) \)
sets in $^{15}\text{F}$ are $[1.405, 0.7]$ for $s_{1/2}$ and $(2.8, 0.37)$ for $d_{5/2}$. The contributions of the $|s_{1/2}\rangle_0$ and $|s_{1/2}/d_{5/2}\rangle_2$ configurations in the structure of $^{16}\text{Ne}$ are taken as 67% [21] and 16% [10].

The different models yield the following results given in Table 1. The NPP model tends to overestimate the decay width [14]. The results of the IDDM lie in between the NPP and EFC width values. The NPP and EFC models also have different deficiencies in describing the momentum distributions of the $2p$ decay. The IDDM model reproduces all features of the momentum distributions qualitatively correct, see Fig. 4. Also a quantitative agreement with the full-scale three-body calculations is largely obtained. For example, one should note the correct positions and intensity-ratio of the two peaks in Fig. 4 (c). For the $2^+$ state, however, a complete agreement with full-scale three-body calculations is not expected, as several quantum configurations are needed for description of the data [10].

As an intermediate result, the example of $^{16}\text{Ne}$ decays shows that the proposed IDDM combines the advantages of the NPP and EFC models, and it provides results which are close to the sophisticated three-body calculations. This makes the proposed approach a reasonable substitute for complicated three-body calculations, at least for exploratory and systematics studies.

Table 1

<table>
<thead>
<tr>
<th>$\Gamma$ (keV)</th>
<th>NPP</th>
<th>EFC</th>
<th>IDDM</th>
<th>3-body</th>
<th>Exp.</th>
</tr>
</thead>
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<tr>
<td>$0^+$</td>
<td>3.4</td>
<td>4.6</td>
<td>4.1</td>
<td>3.1</td>
<td>$&lt; 80$ [20]</td>
</tr>
<tr>
<td>$2^+$</td>
<td>14</td>
<td>11</td>
<td>12</td>
<td>56</td>
<td>175 ± 75 [10]</td>
</tr>
</tbody>
</table>

5. Transition patterns in the $s$–$d$ shell nuclei

In this Section, the IDDM will be applied more generally to several $s$–$d$ shell nuclei and the decay dynamics is studied. The first case is $^{30}\text{Ar}$ decaying to $^{28}\text{S} + p + p$ via the $|s_{1/2}\rangle_0$ configuration, see Fig. 5. In the calculation, the $|E_T, s\rangle$ plane is scanned for a fixed proton resonance energy $E_R(32\text{Cl}) = 1.8$ MeV and several decay widths $\Gamma_r, (32\text{Cl}) = 60, 170, 330, 500$ keV. In the considered “Y” Jacobi system $\mathcal{E} = E_{\text{core}+p}/E_T$ is the relative energy content of the core + $p$ subsystem.

Transitions between different decay mechanisms become visible and the following evolution pattern can be recognized in Fig. 5 (a). At low $E_T$ values, the true $2p$ decay mechanism prevails, which is characterized by a relatively narrow, bell-shaped distribution centered at $\mathcal{E} \sim 1/2$ (i.e., where the energies of both protons are equal). For $E_T > E_r$, two new peaks, associated with the sequential emission of protons, appear on the rising and falling edge of the central bell-shaped distribution. At an energy of $E_T \sim 1.2E_r \sim 2.2$ MeV, the sequential-emission peaks become higher than the central peak around $\mathcal{E} \sim 1/2$, and this value marks the transition point to the sequential decay mechanism (this was already sketched in Fig. 1 using the label “True 2p”→“Sequential 2p” transition).

Overall, the triangular structure reflects the evolution of the double-peak sequential decay pattern as a function of $E_T$. At the top of the triangle, at $E_T = 2E_r$, the two peaks belonging to the sequential emission overlap; at still higher $E_T$ they split up again.

Fig. 5 (b,c,d) shows the results for different, larger widths $\Gamma_r$, which is achieved by variation of the reduced width $\theta^2$ parameter in Eq. (5). At first glance, similar patterns are obtained as for case (a). In detail, one can see that with increasing $\Gamma_r$, the triangular sequential-decay pattern arises at higher energy $E_T$. However, it gradually smears out, and finally vanishes with increase of $\Gamma_r$. This qualitative difference reflects the transition to the democratic decay mechanism: the evolution observed in the decay patterns from (b) to (d) corresponds to the “Sequential 2p”→“Democratic 2p” transition regime (which is also indicated in Fig. 1).

The panels (a) and (b) in Fig. 5 correspond to the $\theta^2$ values of 0.5 and 2, respectively. The first value should correspond to the structure of a nucleus with strong configuration mixing. The second value is the upper limit of the admissible range for $\theta^2$, which corresponds to a pure single-particle structure. The difference between the discussed $E_T$ evolution patterns is quite significant. For instance, the transition “True 2p”→“Sequential 2p” in the calculation (b) takes place at an $E_T$ value, which is about 300 keV larger than that in the case (a). Unrealistically large $\theta^2$ values of 4 and 6 are used in cases (c) and (d), respectively. They are considered for the study of the transition to the democratic $2p$-decay mechanism. From these and analogous evaluations it is found that the “True 2p”→“Democratic 2p” transition takes place when the width $\Gamma_r$ is described by the relation

$$\Gamma_r = \Gamma_r (\mathcal{E}_r) \sim (0.2–0.3)E_r,$$

for this, see also Fig. 1.

It is obvious that the charge of $^{30}\text{Ar}$ is too large (hence the $^{30}\text{Cl}$ g.s. width is too small) to allow the democratic decay mechanism to materialize with the $E_r = 1.8$ MeV value used. The evolution of $2p$-decay mechanisms similar to the one shown in Fig. 5 (a)→(d) can be achieved by variation of a charge of the $2p$ precursor or/and energy $E_r$. Fig. 6 illustrates this issue by presenting three examples of the calculated transition patterns ranging from light to heavy $s$–$d$ shell $2p$ emitters. In the calculated cases of $^{15}\text{Ne}$, $^{26}\text{S}$, and $^{34}\text{Ca}$ g.s. decays, it can be seen that the observed ($^{15}\text{Ne}$) as well as the evaluated ($^{26}\text{S}$) separation energies provide the g.s. location near the transition points. This means that it can be expected that the latter cases exhibit a strong sensitivity of the decay correlation patterns to the parameters of the involved nuclear states. In the following section, these findings will be utilized to study the core + $p$ subsystem of a core + $p + p$ nucleus.

6. Decay of the $^{15}\text{Ne}$ ground state

The first illustration, where the $2p$-decay transition dynamics is used to study the core + $p$ subsystem, is provided by the $^{15}\text{Ne}$ g.s. decay. One can see in Fig. 6 (a) that the $2p$-decay energy of $^{15}\text{Ne}$ g.s., $E_T(\exp) = 2.52$ MeV [11], is located in the region where the width of the $\mathcal{E}$-distribution strongly varies with $E_T$. The shape of the $\mathcal{E}$-distribution by itself can be seen as a not very reliable, model-dependent indicator. However, the width of the $^{15}\text{Ne}$ g.s. is
Fig. 5. (Color online.) Energy correlations as a function of the 2p-decay energy $E_r$ calculated by IDDM in the "Y" Jacobi system for $^{30}$Ar. The $^{29}$Cl 1/2+ g.s. energy is $E_r = 1.8$ MeV. The panels (a, b, c, d) correspond to the assumed $\Gamma_r$($^{29}$Cl) values of 60, 170, 330, 500 keV, respectively. The distributions for each energy $E_r$ are normalized to unity maximum value.

Fig. 6. (Color online.) Energy correlations as a function of the 2p-decay energy $E_r$ calculated by IDDM in the "Y" Jacobi system for $^{15}$Ne, $^{26}$S, and $^{34}$Ca. The corresponding $(E_r, \Gamma_r)$ values are [1.56, 0.9] (both values taken from the experiment in Ref. [22]), [0.12, 1.5], [0.25, 1.4] (E from [11]). $\Gamma_r$ calculated assuming an $s$-wave decay with $\varphi^2 = 2$, which is the single-particle limit. The vertical dotted lines mark the $E_r$ values from the experiment for $^{15}$Ne [11] as well as from the systematics [13] for the cases $^{26}$S and $^{34}$Ca. The thin dotted lines for $^{15}$Ne indicate the $E_r$ region within experimental value of the g.s. width $\Gamma = 0.59$ MeV [11]. The distributions for each energy $E_r$ are normalized to unity maximum value.

Fig. 7. (Color online.) Energy distributions of the 2p decay products of $^{15}$Ne (dots, with statistical uncertainties, in the "Y" Jacobi system [11]) calculated at three $E_r$ values (in MeV) of the $^{15}$Ne g.s. The $^{14}$F g.s. width is assumed to be either 0.7 MeV (a) or 1.1 MeV (b).

Fig. 8. (Color online.) Energy distributions of the 2p-decay products of $^{30}$Ar, calculated in the "Y" Jacobi system for the two $E_r$ values shown in panels (a) and (b). The $E_r = 1.8$ MeV value is fixed, while the curves represent different $\Gamma_r$ values. In panels (c) and (d), the experimental angular distribution between core and one of the protons measured for $^{30}$Ar g.s. decay in Ref. [12] (histograms) is compared with those from the respective theoretical distributions shown in panels (a) and (b). The experimental bias is taken into account in the Monte-Carlo simulations.

Quite broad, $\Gamma(\exp) = 0.59$ MeV, and one may in addition investigate the evolution of the $\varepsilon$ distribution as function of $E_r$ depending on model parameters. In Fig. 7 the "Y" system $\varepsilon$-distributions are calculated for different decay energies of $^{15}$Ne within the g.s. resonance peak (namely, $E_r(\exp)$ and $E_r(\exp) \pm \Gamma(\exp)/2$, and for different widths of the $^{14}$F g.s. within the experimental uncertainty for this value. Ref. [22] provides $E_r(\exp) = 1.56(4)$ MeV and $\Gamma_r(\exp) = 0.92(5)$ MeV for $^{14}$F g.s., so the values $\Gamma_r = 0.7$ and $\Gamma_r = 1.1$ are used for calculations in Fig. 7.

As expected, the $\varepsilon$-distributions, calculated for the centroid ($E_r$) and for the limiting cases ($E_r \pm \Gamma/2$) of the $^{15}$Ne g.s. resonance, differ from each other. In particular, the differences are quite pronounced with respect to the different values of the $^{14}$F g.s. widths $\Gamma_r(\varepsilon)$, which, in turn, allows to extract some information on the widths $\Gamma_r$ from the $^{15}$Ne decay correlation data. The data for $^{15}$Ne g.s. measured in Ref. [11] are shown in Fig. 7 (a). A direct quantitative comparison is not appropriate, since the measured distribution is broadened by the experimental resolution and the counts is not sufficient to study the $E_r$ evolution in detail. However, we would like to mention that such an analysis is possible with better statistics and better experimental resolution, as can
be seen in the similar example of the $^{16}$Ne g.s. decay [20]. The precision of the latter experiment seems to be sufficient to distinguish the solid and dotted curves in Fig. 7 (a) or dashed and solid curves in Fig. 7 (b). So, the proposed analysis is in principle feasible.

7. Decay of the $^{30}$Ar ground state

The observation and study of the new isotopes $^{30}$Ar and $^{29}$Cl were reported recently [12]. It was concluded that the $2p$ decay of the $^{30}$Ar g.s. is located in the transition regime between true $2p$ and sequential $2p$ decays. The analysis was carried out in a simplified way with the decay energy as the only parameter. Here, the $^{30}$Ar data are re-analyzed using IDDM calculations.

Several calculated correlations and the respective “experimental” distributions (simulated by taking into account the setup response function, along the lines described in [12]) are shown in Fig. 8 together with the experimental data. Despite strong effects of the experimental setup, the measured angular distributions of the $2p$-decay products appear to be informative. One can see from the figure, that the specific patterns connected with the formation of sequential-decay correlations of the $^{30}$Ar g.s. produce sizable changes in the angular distributions $^{28}$S+$p$.

In order to find the best match with experimental data, the parameters $\{E_T, E_r, \Gamma_r\}$ were systematically varied; a Kolmogorov test provides the probability for agreement between simulation results and experimental histograms, see Fig. 9. In the previous study Ref. [12], the assigned parameters were $E_T = 2.25\pm0.15$ MeV, $E_r = 1.8(1)$ MeV. On the basis of the present, more refined analysis, whose results are shown in Fig. 9, a more precise value $E_T = 2.45^{+0.05}_{-0.10}$ MeV is inferred in agreement with our previous findings, and also the width of $^{29}$Cl g.s. is derived as $\Gamma = 85(30)$ keV. A pleasant feature of this analysis is that uncertainties of the major parameters are correlated, which can be used for imposing further limits. For example, it is shown in Fig. 9 (d) that a smaller $\Delta \Gamma \sim 15$ keV can be established for a fixed $E_T$ value.

It should be emphasized that the analysis performed is in the “proof of concept” stage, and should be validated by direct measurements.

8. Sensitivity of IDDM results

The reliability of the proposed width determination of a core+$p$ g.s. nuclear system from the energy distributions obtained in the “Y” coordinate system depends on the robustness of the distributions with respect to the model parameters used. Such a sensitivity was systematically investigated, and it was found that the only significant dependence results from the general decay parameters $\{E_T, E_r, \Gamma_r\}$. For instance, the transition patterns strongly depend on the interference of two-body amplitudes at resonant and near-resonant energy. However, Fig. 10 shows that a modification of amplitudes outside the resonance peak does not lead to significant changes of the characteristic behavior.

The other important assumption is that only one quantum configuration provides a dominating contribution to the decay width of the state. However, this is a natural situation for the transition dynamics, which is connected to the strong variation of the partial width just in one selected channel. This may be estimated in a forthcoming theoretical study.

9. Summary

Nearly half of the neutron-deficient $s$–$d$ shell nuclei located 1–2 neutron numbers beyond the proton drip line decay by $2p$ emission. This decay mode exhibits three different characteristic: true $2p$, democratic and sequential decay mechanisms. Their occurrence depends mainly on the relationship of the three parameters $E_T$, $E_r$, and $\Gamma$. The changeover from one decay mechanism to another, which is reflected by altered momentum distributions of the decay products, occurs within rather narrow transition areas, i.e. within small changes of the relevant $2p$-decay parameters. The behavior of the momentum distributions of $2p$-decay products in the transition regions can be described by the semi-analytical improved direct decay formalism, IDDM. At variance to previously used approximations, the IDDM depicts all qualitative features of $2p$-decay distributions in the whole kinematical space; in particular it covers the mentioned $2p$-decay transition regions. Even with moderate computing effort, this model yields reliable quantitative estimates and phenomenological evidence, ideally suited for large-scale systematic surveys. Further development and refinements of such a model are of importance for future studies. In the transition regions, the momentum distributions of the decay products exhibit a strong sensitivity with respect to the variation of the three general $2p$-decay parameters. Therefore experimental data from the $2p$ correlations in transition regions can be used to extract resonance parameters in the core+$p$ subsystem of the $2p$-decay
precursor. The practical feasibility of such an approach has been demonstrated here with the example of the $^{15}$Ne g.s. 2p-decay. By relying on the transition dynamics, improved constraints for the parameters $E_T$, $E_r$, and $\Gamma_r$ for the $^{30}$Ar and $^{29}$Cl g.s. can be set in comparison with the values derived in Ref. [12]. Overall, this will contribute to a better understanding of structure and decay dynamics of nuclei and resonances at and beyond the driplines.

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