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Equation of state dependence of directed flow in a microscopic transport model

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ABSTRACT

We study the sensitivities of the directed flow in Au+Au collisions on the equation of state (EoS), employing the transport theoretical model JAM. The EoS is modified by introducing a new collision term in order to control the pressure of a system by appropriately selecting an azimuthal angle in two-body collisions according to a given EoS. It is shown that this approach is an efficient method to modify the EoS in a transport model. The beam energy dependence of the directed flow of protons is examined with two different EoS, a first-order phase transition and crossover. It is found that our approach yields quite similar results as hydrodynamical predictions on the beam energy dependence of the directed flow; Transport theory predicts a minimum in the excitation function of the slope of proton directed flow and does indeed yield negative directed flow, if the EoS with a first-order phase transition is employed. Our result strongly suggests that the highest sensitivity for the critical point can be seen in the beam energy range of $4.7 \le \sqrt{s_{NN}} \le 11.5$ GeV.

One of the most challenging problems in high energy heavy ion collisions is to map out the QCD phase diagram from low to high baryon densities. In particular, the main interest is to discover a first and/or second order phase transition together with the critical point of QCD matter at finite baryon chemical potentials, which was predicted by several effective models based on QCD [1]. In order to explore QCD matter in the full range of temperatures and baryon densities, it is necessary to measure various observables for a wide range of beam energies. A non-trivial beam energy dependence of various observables such as the K^+/π^+ ratio [2], higher order of the net-proton number cumulants [3,4] and the slope of the directed flow have been reported from the NA49 Collaboration and the STAR beam energy scan (BES) program [5,6]. We note that such a beam energy dependence usually is not explained by the standard hadronic transport models. There have been attempts to take into account a change of degrees of freedom (into quarks and gluons) [9,10] in transport models, or transport + hydrodynamics hybrid models [7]. For instance, hybrid models usually show an improved description of strange particle ratios and collective flow [8].

The collective transverse flow developed in the heavy ion collisions is considered to be sensitive to the equation of state (EoS) of QCD matter, since it reflects the pressure gradients in the early stages of the reaction. Especially, the softest region in the EoS with a (first-order) phase transition is expected to have significant influence on the directed flow $v_1 = \langle \cos\phi \rangle$ of nucleons [11,12]. Hydrodynamic predictions have shown that the excitation function of directed flow exhibits a local minimum at a specific beam energy characteristic for the first-order phase transition [11,13,15]. Furthermore, the slope of the directed flow of nucleons becomes negative at the softest point of the EoS [13,14,16,17]. Thus the collapse of directed flow may be a signature of the QCD first-order phase transition at high baryon densities. Indeed, such a behavior has been observed in the BES program by the STAR Collaboration [18–20] but no microscopic hadronic transport model is yet able to quantitatively explain the negative slope of protons at $\sqrt{s_{NN}} = 11.5$ and 19.6 GeV [21–23]. Only at higher beam energies, the microscopic transport models RQMD [24], UrQMD [25], and PHSD/HSD [21] show a negative slope of proton directed flow without a phase transition. We would like to note that studies including a hadronic mean field interaction also have not been able to describe the negative slope of the proton directed flow [26,27]. The excitation function of the directed flow has also been exam-

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ined by other approaches such as the UrQMD hybrid model [23], three fluid models [28], and JAM with an attractive scattering style [29], but none of the models is able to give a quantitative satisfactory description of the STAR data. Therefore it is still premature to make unambiguous interpretations of the STAR data, and more reliable models have to be developed in order to simulate the spacetime evolution of the system created in heavy ion collisions at intermediate beam energies and to understand the underlying collision dynamics.

There is a long history of implementing an EoS into transport theoretical approaches. The Boltzmann–Uehling–Uhlenbeck (BUU) model [30] was developed first followed by quantum molecular dynamics (QMD) [31] to test various EoS in a transport model that includes nuclear mean field potentials. Later a relativistic extension of both BUU and QMD was developed which are called RBUU [32] and RQMD [33]. It is also known that the EoS can be controlled by changing the scattering style in the two-body collision term [34–36]. For example, choosing a repulsive orbit in two-body collisions can simulate the effect of a repulsive potential. Later, effects of a phase transition have been investigated by including a mean field interaction [37–39] or using attractive orbits in two-body collisions [29,40].

The theoretically favored method is to include mean fields to vary the EoS, and in principle any mean field can be incorporated provided that the interaction Lagrangian is known. It is, however, practically very challenging to implement a mean field in a dynamical simulation, and one often relies on the simplified phenomenological form of potentials. On the other hand, an advantage of the second approach is that we can include any kind of EoS computed by other methods such as effective models or lattice QCD. In this work, we shall follow the second approach to examine effects of the EoS on the directed flow within the microscopic transport approach JAM by modifying the scattering style in the two-body collision term. We do not include explicitly partonic degree of freedom, as for example in AMPT [9] or PHSD [10], since the transition from one set of degrees of freedom is difficult and usually introduced new uncertainties in the model. See also Ref. [41] for a recent attempt to implement a first order phase transition into a parton cascade model.

In JAM [42], particle production is modeled by the excitation and decay of resonances and strings similar to the other models [43–45]. Secondary products from decays can interact with each other via binary collisions. A detailed description of the JAM model can be found in Refs. [42,46].

In the standard cascade approach the two body collision term is implemented so that it does not generate additional pressure. Namely, the azimuthal angle in the two-body collision is randomly chosen. It is well known that the pressure can be controlled by changing the scattering style. Here we employ a method similar to that proposed in Ref. [40] in which the momentum change in each two-body collision between particle i and j at the space–time coordinates of q_i and q_j is related to the pressure $\Delta P = P - P_f$, where P_f is the free streaming part of the local pressure. Motivated by the virial theorem [47], The formula is given by

$$\Delta P = -\frac{\rho}{3(\delta\tau_i + \delta\tau_j)}(p'_i - p_i)^\mu (q_i - q_j)_\mu, \quad (1)$$

where ρ is the Lorentz invariant local particle density obtained by $\rho = N^\nu u_\nu$, $N^\nu = \sum_h \int \frac{d^3p}{p^0} p^\nu f_h(x,p)$, $\delta\tau_i$ is the proper time interval between successive collisions, and $p'_i - p_i$ is the energy–momentum change of the particle i. P_f can be computed from the energy–momentum tensor $T^{\mu\nu}$: $P_f = -\frac{1}{3}\Delta_{\mu\nu}T^{\mu\nu}$, where $T^{\mu\nu} = \sum_h \int \frac{d^3p}{p^0} p^\mu p^\nu f_h(x,p)$ with the projector of $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$, where u^ν is a hydrodynamics velocity defined by the Landau and Lifshitz's definition. In this way, we control the pressure such that P coincides with a given EoS at an energy density $e = u_\mu T^{\mu\nu} u_\nu$.

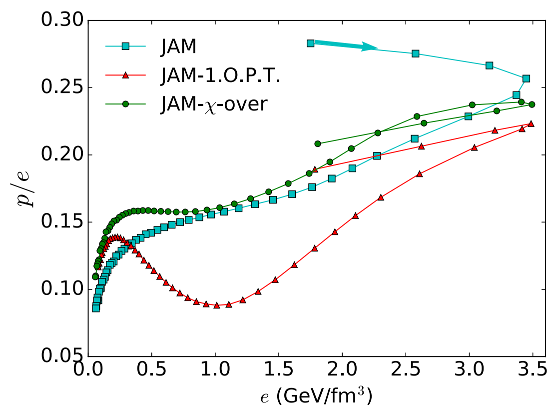

Fig. 1. Time evolution of isotropic pressure p and energy density e in mid-central Au+Au collisions (10–40%) at $\sqrt{s_{NN}} = 11.5$ GeV from JAM cascade mode (squares), JAM with first-order EoS (triangles) and with crossover EoS (circles). Initial point corresponds to the time 0.65 fm/c after touching of two nuclei, and time interval is 0.25 fm/c. Arrow indicates the direction of the reaction. Pressure and energy density are averaged over collision points in a cylindrical volume of transverse radius 3 fm and longitudinal distance of 2 fm centered at the origin.

There are three ways to satisfy the constraint Eq. (1) to control the pressure in two-body collisions;

1. change the scattering angle,
2. change the azimuthal angle,
3. change the magnitude of the relative momentum by modifying the masses of the outgoing particles.

In Ref. [40], an additional elastic scattering is generated after any of the standard two-body scatterings, and it was found that more elliptic flow is generated. It is known that the shear viscosity is sensitive to the scattering angle [48]. Thus, it is unclear whether the EoS alone is responsible for the modification of elliptic flow, since transport coefficients are also modified at the same time by additional extra elastic scatterings. We therefore avoid changing the scattering angle in this work in order to model the EoS and effectively keeping the transport coefficients as unchanged as possible. The idea of changing the outgoing mass is attractive in the sense that it may effectively simulate a dropping mass due to chiral symmetry restoration (CSR). But it is not straightforward how to relate it to CSR within our approach and hadrons would eventually have to retain their vacuum masses before escaping the dense system. Thus, in this work, we adapt the method of only changing the azimuthal angle in the two-body collision so that the constraint Eq. (1) is fulfilled.

We test two types of EoS. For the first-order phase transition, the hadron resonance gas phase contains all hadronic resonances up to 2 GeV, and a baryon density ρ_B dependent vector type repulsive mean field $V(\rho_B) = \frac{1}{2}K\rho_B^2$ with $K = 0.45$ GeV fm^3 is included as described in [49]. The QGP phase is modeled by the ideal gas of massless quarks and gluons with a bag constant of $B^{1/4} = 220$ MeV which leads to a phase transition temperature of $T_c = 156$ MeV at vanishing net baryon density. For the crossover EoS, we use the chiral model EoS from Ref. [50] in which EoS at vanishing and finite baryon density is consistent with a smooth crossover transition, as found in lattice QCD results.

In Fig. 1, the time evolution of the local isotropic pressure and energy density, averaged over events, in mid-central Au+Au collision at $\sqrt{s_{NN}} = 11.5$ GeV are compared for the different EoS used. The JAM standard EoS is the stiffest, which is due to the chemical non-equilibrium in the initial pre-equilibrium evolution as pointed

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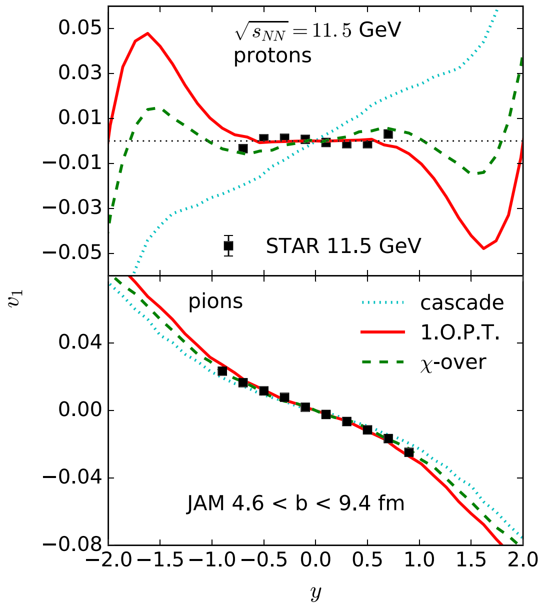

Fig. 2. Rapidity dependence of directed flow v_1 for protons and pions in semi-central Au+Au collisions for $\sqrt{s_{NN}} = 11.5$ GeV from JAM simulations with different EoSs. The solid line presents the JAM with the EoS with 1st order phase transition, and the dashed line presents the JAM with the crossover EoS. The dotted line is for the standard JAM result. The STAR data are taken from Ref. [18].

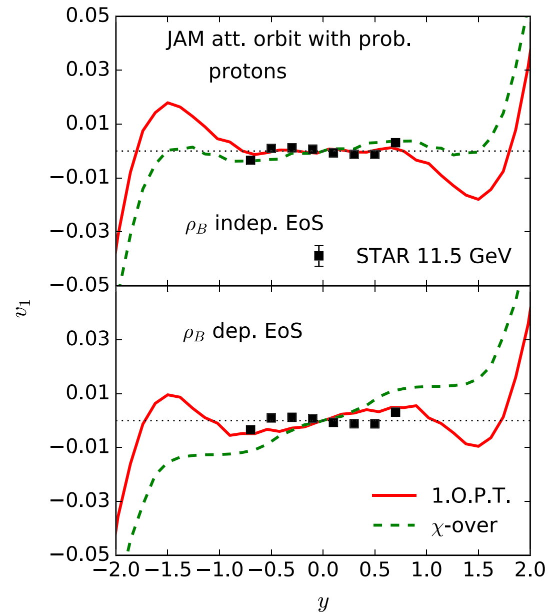

Fig. 3. Rapidity dependence of directed flow v_1 for protons in semi-central Au+Au collisions for $\sqrt{s_{NN}} = 11.5$ GeV from JAM simulations by choosing attractive orbit with a probability given by Eq. (2). The solid line presents the JAM with the EoS with 1st order phase transition, and the dashed line presents the JAM with the crossover EoS. Upper panel: the results from baryon density independent EoS used in Ref. [29]. Lower panel: the results from baryon density dependent EoS. The STAR data are taken from Ref. [18].

out in Ref. [29]. A pronounced softening can be seen in the case of the first order phase transition at around the energy density of $e \approx 1$ GeV/fm^3 as we expected. On the other hand, the effect of softening is weaker for the crossover EoS. In the construction of the crossover EoS from a chiral model, correct nuclear saturation properties at zero temperature were required, resulting in the stiff hadronic EoS, compared to the standard JAM EoS which is expected to be close to an ideal hadron resonance gas EoS. This property leads effectively to the inclusion of repulsive potential in the hadronic phase in JAM simulations.

We now compute the directed flow in mid-central Au+Au collisions, and compare the two EoS described above. In the simulation, we choose the impact parameter range $4.6 < b < 9.4$ fm for mid-central collisions. In Fig. 2, we show the rapidity dependence of the proton (upper panel) and pion (lower panel) directed flows in mid-central Au+Au collision at $\sqrt{s_{NN}} = 11.5$ GeV. The pion directed flow is not sensitive to the EoS, all models are able to reproduce the pion data, which means the pion directed flow is not useful to probe the early time dynamics of the collision. As we showed in Ref. [29], the JAM cascade calculation for protons shows a significantly larger flow than the STAR data. It is seen that the proton directed flow from JAM with both the first order phase transition and crossover EoS are significantly reduced at mid-rapidity, and close to the data, indicating the importance of the EoS effects in the early phase. However, both EoS still predict a positive v_1-slope contrary to the STAR data.

We now check if the microscopic details are important for the generation of directed flow. In order to see systematics on the modeling in controlling a EoS, we show in Fig. 3, the rapidity dependence of proton directed flow at $\sqrt{s_{NN}} = 11.5$ GeV from the method of choosing attractive orbit in two-body scattering with the probability:

$$p_{\text{attractive}} = \max\left(0, 1.3\frac{P_f - P(e)}{P(e)}\right), \qquad (2)$$

as proposed in Ref. [29], where $P(e)$ is a pressure as a function of energy density e from a given EoS. As we simply choose attractive orbit according to the probability given by Eq. (2) in this method, pressure at each two-body collision is not equal to the pressure of a given EoS, and it fluctuates a lot. Only average pressure of the system coincides with the given EoS. With this approach, one sees that proton directed flow is less suppressed compared with our new method. We also show the directed flow obtained by the baryon density independent EoS as used in Ref. [29] in the upper panel of Fig. 3. Baryon density dependent EoS used in this work yields less suppression of the proton directed flow as we expected, since our EoS predicts less softening at finite baryon densities. In this work, we estimate the energy–momentum tensor by including constituent quarks. Technically it is done by counting leading hadrons that have original constitute quarks within a formation time multiplying the reduction factor of one-third (two-third) or one-half depending on the number of constituent quarks inside baryons or mesons. In the previous calculations in Ref. [29], we did not take into account the reduction factors in leading hadrons which leads to larger energy densities.

To see the sensitivities of the different assumptions in defining the energy density within our model, the proton directed flow from the calculation in which only formed hadrons are taken into account for the estimation of energy density [51] is presented in Fig. 4. In this case, energy density corresponds to hadronic energy density, and it is less than the default calculation, in which the contributions from constituent quarks are included, and as a result, we have more softening of EoS and the slope of proton directed flow at $\sqrt{s_{NN}} = 11.5$ GeV becomes negative. However, we do not see any sensitivities between the first order phase transition and the crossover EoS at $\sqrt{s_{NN}} = 11.5$ GeV. We also note that pion directed flow from JAM deviates from the data when we use hadronic energy density.

In the following we present the beam energy dependence of the slope of the directed flow. In Fig. 5 we show the slope of directed flow dv_1/dy of protons in mid-central collisions from the standard JAM cascade and JAM with a first-order phase transition and with

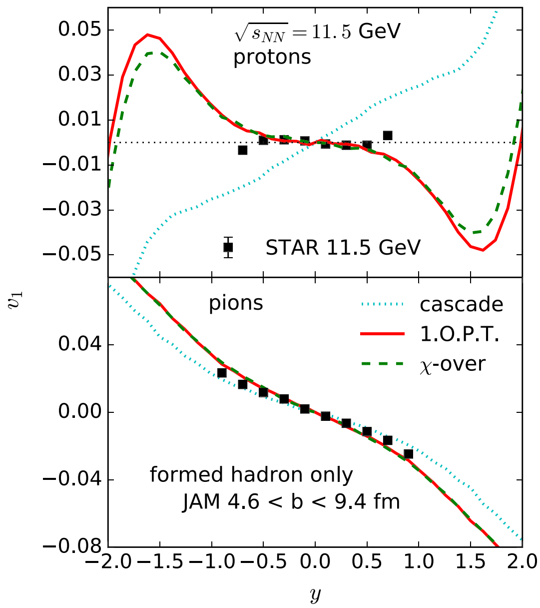

Fig. 4. Same as Fig. 2, but only formed hadrons are taken into account for the estimation of energy density.

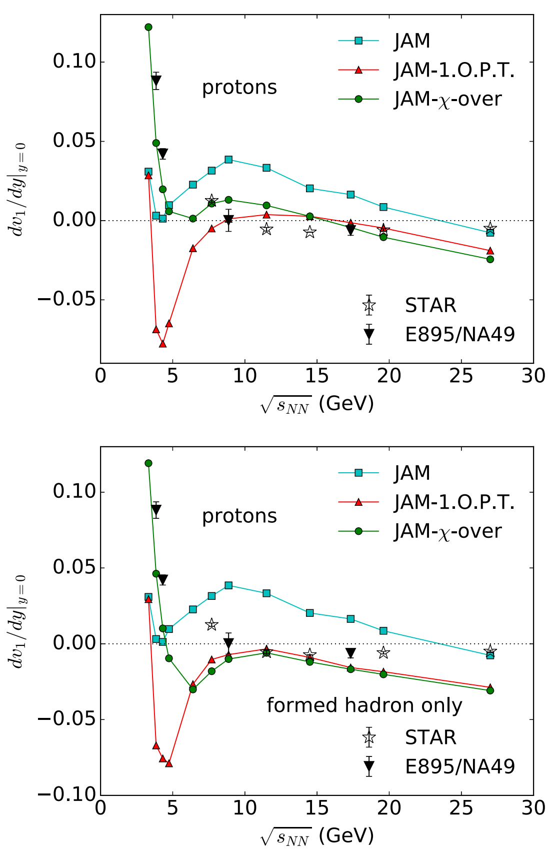

Fig. 5. Beam energy dependence of the slope of the directed flows of protons in mid-central Au+Au collisions (10–40%) from JAM cascade mode (squares), JAM with first-order EoS (triangles) and with crossover EoS (circles) in comparison with the STAR/NA49/E895 data [18,19,52–54]. Local energy densities are computed by taking into account the contributions of constituent quarks in the upper panel, while only formed hadrons are included in the lower panel.

a crossover EoS, in Au+Au collisions in comparison with the data from STAR, NA49 and E895 Collaborations [18,52–54]. The slope is obtained by fitting the rapidity dependence of v_1 to a cubic equation $v_1(y) = Fy + Cy^3$ in the rapidity interval $-0.8 < y < 0.8$.

The standard JAM cascade calculation predicts a minimum at around AGS energies which was first reported in Ref. [55] within the UrQMD approach. The decrease of the directed flow in the standard cascade approach can be understood by the rapid change in degree of freedoms due to the excitation of hadronic resonances up to 2 GeV. Note that PHSD does not have such minimum [21], since there are only a few number of hadronic resonances included in the PHSD model. It is important to notice that the slope of the proton directed flow obtained by the cascade model is still positive at the minimum point. At higher energies up to $\sqrt{s_{NN}} \approx 20$ GeV, the JAM standard model overestimates the slope of the proton directed flow as already reported in Ref. [29].

In the case of the EoS with the first order phase transition (JAM-1.O.P.T.), we also see the minimum in the excitation function of the proton directed flow at almost the same beam energy as the cascade model. In addition, the slope is now negative as predicted by hydrodynamical approaches. The beam energy dependence is very similar to the pure hydrodynamical simulation except that the magnitude of the minimum in JAM-1.O.P.T. is about a factor 5 smaller than the ideal hydrodynamical prediction [23], which may be related to the finite viscosity in the transport approach. The local minimum predicted by the JAM-1.O.P.T. is located at a slightly higher beam energy than in the one-fluid model prediction ($\sqrt{s_{NN}} \approx 4$ GeV) in which the strong coupling of the fluids leads to an almost instantaneous full stopping and maximum energy deposition, different than in the three-fluid model in Ref. [13] in which finite stopping power of nuclear matter is taken into account. We note that the local minimum predicted by the three-fluid model in Ref. [28] shows $\sqrt{s_{NN}} = 6.5$ GeV. Probably the location of minimum depends both on the EoS and the degree of stopping and its modeling. In Ref. [28], the EoS in the QGP phase is modeled by a quasi-particle approximation with mean field potential [56]. It is interesting to notice that the ART BUU approach with the first order phase transition also exhibits minimum with a negative slope [37] which supports that our method effectively handles the effect of the EoS.

On the other hand, in the case of a crossover EoS (JAM-χ-over), there is still a local minimum at $\sqrt{s_{NN}} \approx 6$ GeV, but it is not so pronounced. Thus our approach supports the idea that a large negative slope of the proton directed flow would be a good observable to identify a strong softening in the early phase of the collision due to the first order phase transition. However, the minimum observed from our approach is located at a beam energy similar to that predicted by hydrodynamical simulations [13] and much lower than the minimum measured by the STAR. If the EoS we employ is close to the true EoS in nature, we do not see the connection between the softest point of the EoS and the STAR data. In order to describe the minimum at a larger beam energy one would need a very soft EoS at very high baryon densities, much higher than what is found in the currently used EoS, in order to shift the minimum to the higher beam energies. Within our analysis, the reduction of the proton directed flow at $\sqrt{s_{NN}} > 10$ GeV is essentially related to the early pressure in the pre-equilibrium stages of the reaction. The reason why JAM-1.O.P.T. yields stronger flow than JAM-χ-over at $\sqrt{s_{NN}} > 10$ GeV is simply because of the fact that JAM-1.O.P.T. in the current study assumes the massless ideal quark-gluon EoS at the QGP phase, while crossover EoS is consistent with the lattice QCD data at the vanishing baryon chemical potentials. Even at the finite baryon chemical potentials, our crossover EoS is softer than the massless ideal quark-gluon EoS at high energy densities. Since a massless ideal QGP EoS is not supported by the lattice QCD calculations, we need to test more realistic EoS in order to see the difference between a first order phase transition and crossover transition in the future.

Also it has been pointed out that the usual description of baryon stopping in string models in a transport model assumes essentially an instant deceleration of the leading baryons because strings are immediately decayed into hadrons with a formation time. In [59] it was argued that a constant deceleration would lead to a very different initial density profile in coordinate space, i.e. a smaller density at mid-rapidity, which may transform into a different time evolution of the particle flow.

Besides an uncertainty of the EoS, there is a model uncertainty regarding the handling of static equilibrium EoS $P(e)$ within a non-equilibrium dynamical framework in which only effective local energy density can be obtained. To check how the directed flow is affected by the different definition of the energy density, we compute the slope of the proton directed flow by using hadronic energy density by counting only formed hadrons in the estimation of energy–momentum tensor which may be regarded as a possible lower limit of the energy density in the model. The results from JAM-1.O.P.T. and JAM-χ-over simulations are shown in the lower panel of Fig. 5. Overall tendency does not change with this approach, but the slopes of proton directed flow at higher beam energies become negative compatible with the data up to $\sqrt{s_{NN}} = 19.6$ GeV except the point at $\sqrt{s_{NN}} = 7.7$ GeV. We demonstrate that the directed flow of proton is highly sensitive to the EoS at the beam energy range of $4 \leq \sqrt{s_{NN}} \leq 10$ GeV, where no experimental data exists between 4.7 and 7.7 GeV. It is utmost important to measure the directed flow in this range in order to reveal the phase structure of QCD at the highest baryon densities.

In summary, we have investigated the EoS dependence of the excitation function of the directed flow of protons within a microscopic transport approach by employing an energy density dependent EoS modified collision term. We showed that our approach provides an efficient method to control the EoS in a microscopic transport model, and our result yields qualitatively similar result as the pure hydrodynamical predictions. As predicted by hydrodynamical approach, the minimum of the directed flow, from a strong first order phase transition, is located at much lower beam energy than the STAR data. In order to distinguish between a first order phase transition and crossover transition in the directed flow data, we need to examine different EoS from models consistent with the lattice QCD up to finite baryon chemical potentials accessible in the current lattice QCD calculations. A systematic study including strange hadrons should be addressed since strange hadrons follow a more complex pattern due to the different cross sections [57]. In the future, it is interesting to analyze STAR data by explicitly implementing the mean field with phase transition into a transport model. It is also interesting to investigate the effect of dropping of hadron mass due to CSR on the directed flow.

We suggest to study the directed flow in the beam energy region of $4.7 < \sqrt{s_{NN}} < 11.5$ GeV which may give suitable signatures to study the properties of EoS at highest baryon density, where the softening effect could be best manifested. Future experiments such as the BES II at RHIC [58], FAIR [60], NICA [61], and J-PARC [62] should provide important information on the properties of high dense matter created in the heavy ion collisions.

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