# Magneto-hydrodynamic simulations of Heavy Ion Collisions with ECHO-QGP

**G Inghirami[1,2,3,4], L Del Zanna[5,6,7], A Beraudo[8], M Haddadi Moghaddam[9], F Becattini[5,7] and M Bleicher[1,2,3,4]**

[1] Frankfurt Institute for Advanced Studies, Ruth-Moufang-Straße 1, 60438 Frankfurt am Main, Germany.
[2] Institute for Theoretical Physics, Goethe-Universität, Max-von-Laue-Straße 1, 60438 Frankfurt am Main, Germany.
[3] GSI Helmholtzzentrum für Schwerionenforschung GmbH, Planckstraße 1, 64291 Darmstadt, Germany.
[4] John von Neumann Institute for Computing, Forschungszentrum Jülich, 52425 Jülich, Germany.
[5] Dipartimento di Fisica e Astronomia, Università di Firenze, Via G. Sansone 1, I-50019 Sesto F.no (Firenze), Italy.
[6] INAF - Osservatorio Astrofisico di Arcetri, L.go E. Fermi 5, I-50125 Firenze, Italy.
[7] INFN - Sezione di Firenze, Via G. Sansone 1, I-50019 Sesto F.no (Firenze), Italy.
[8] INFN - Sezione di Torino, Via P. Giuria 1, I-10125 Torino, Italy.
[9] Department of Physics, Hakim Sabzevari University, P. O. Box 397, Sabzevar, Iran.

E-mail: inghirami@fias.uni-frankfurt.de

**Abstract.** It is believed that very strong magnetic fields may induce many interesting physical effects in the Quark Gluon Plasma, like the Chiral Magnetic Effect, the Chiral Separation Effect, a modification of the critical temperature or changes in the collective flow of the emitted particles. However, in the hydrodynamic numerical simulations of Heavy Ion Collisions the magnetic fields have been either neglected or considered as external fields which evolve independently from the dynamics of the fluid. To address this issue, we recently modified the ECHO-QGP code, including for the first time the effects of electromagnetic fields in a consistent way, although in the limit of an infinite electrical conductivity of the plasma (ideal magnetohydrodynamics). In this proceedings paper we illustrate the underlying 3+1 formalisms of the current version of the code and we present the results of its basic preliminary application in a simple case. We conclude with a brief discussion of the possible further developments and future uses of the code, from RHIC to FAIR collision energies.

## 1. Introduction

Heavy ion collisions represent an excellent method to study strongly interacting matter at high energy densities [1, 2]. In turn, relativistic hydrodynamics has proved to be a reasonable model of the expanding Quark Gluon Plasma fireball that it is expected to form at high collision energies [3, 4]. In order to provide a better explanation of certain experimental observables, in particular the collective anisotropic flows, viscous effects have been taken into account [5, 6, 7, 8] and recent developments involve also anisotropic hydrodynamics [9, 10, 11]. However, only in the last years it has been realized that the magnetic fields generated by the fast moving charges contained in the colliding nuclei might be strong enough [12, 13, 14] to produce measurable effects, like the chiral magnetic effect [15], the chiral magnetic wave [16, 17], the chiral vortical effect [18], changes in the QGP EoS [19] and modifications of the collective flows [20, 21, 22]. In the first studies of their influence on the dynamical evolution of the QGP fireball, the magnetic fields have been modeled as external entities not coupled with the medium [23, 24, 25]. To overcome this limitation, we modified the ECHO-QGP code [26] to consistently take into account the interplay between the plasma and the electromagnetic field, although in the approximation of an infinite electrical conductivity of the medium (ideal relativistic magnetohydrodynamics). In this proceedings paper we will briefly review the formalism and present some preliminary results.

## 2. Essential theory and numerical implementation

### *2.1. The basic formalism*

We consider a system composed by a dominant species determining a main *fluid* current, associated to the net-baryon current, and by a secondary species at the origin of the conduction current [27, 28]. We impose the standard conservation laws for this fluid current $N^\mu$ and for the *total* (matter and fields) energy-momentum tensor of the plasma $T^{\mu\nu}$:

$$d_\mu N^\mu = 0, \tag{1}$$
$$d_\mu T^{\mu\nu} = 0, \tag{2}$$

with $d_\mu$ being the covariant derivative. Then, we consider that the electromagnetic field obeys Maxwell's equations:

$$d_\mu F^{\mu\nu} = -J^\nu \quad (d_\mu J^\mu = 0), \tag{3}$$
$$d_\mu F^{\star\mu\nu} = 0, \tag{4}$$

where $F^{\mu\nu}$ is the Faraday tensor and $F^{\star\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\lambda\kappa}F_{\lambda\kappa}$ is its dual. We neglect any possible polarization and magnetization effect of the plasma and we assume to have local thermal equilibrium. With all these assumptions, we can define a single *fluid four-velocity* $u^\mu$ ($u_\mu u^\mu = -1$) and, in its comoving frame, the electric and magnetic fields as: $e^\mu = F^{\mu\nu}u_\nu$, ($e^\mu u_\mu = 0$) and $b^\mu = F^{\star\mu\nu}u_\nu$, ($b^\mu u_\mu = 0$). In general, the Ohm law which relates the current with the fields has a tensorial structure, nevertheless in our ideal MHD approximation we assume that the plasma has an infinite electrical conductivity and, in this special case, it is possible to show that $e^\mu = 0$ is implied and that the electromagnetic energy-momentum tensor can be expressed as

$$T^{\mu\nu} = (e + p + b^2)u^\mu u^\nu + (p + \tfrac{1}{2}b^2)g^{\mu\nu} - b^\mu b^\nu \tag{5}$$

where $e$ is the energy density, $p$ the thermal pressure, $b^2 = b_\mu b^\mu$ (all measured in the comoving frame of the fluid) and $g^{\mu\nu}$ is the metric tensor (we use either Minkwoski or Milne coordinates). Eventually, the system of ideal RMHD equations which we solve is

$$d_\mu(nu^\mu) = 0, \tag{6}$$
$$d_\mu[(e + p + b^2)u^\mu u^\nu + (p + \tfrac{1}{2}b^2)g^{\mu\nu} - b^\mu b^\nu] = 0, \tag{7}$$
$$d_\mu(u^\mu b^\nu - u^\nu b^\mu) = 0, \tag{8}$$

plus an equation of state (EoS) to close it, in the unknowns $n$, $e$, $p$, $u^\mu$, and $b^\mu$.

### *2.2. The RMHD module in ECHO-QGP*

To numerically solve the system of ideal RMHD equations, we need to separate the time and space components (3 + 1 split) [29] and rewrite the equations in *conservative form* [30]. We refer to other articles [31, 32, 7, 26] to get more details about the procedure, here we report just the final form of the equations:

$$\partial_0 \mathbf{U} + \partial_i \mathbf{F}^i = \mathbf{S}, \tag{9}$$

where

$$\mathbf{U} = |g|^{\frac{1}{2}} \begin{pmatrix} \gamma n \\ S_j \equiv T^0_j \\ \mathcal{E} \equiv -T^0_0 \\ B^j \end{pmatrix}, \mathbf{F}^i = |g|^{\frac{1}{2}} \begin{pmatrix} \gamma n v^i \\ T^i_j \\ S^i \equiv -T^i_0 \\ v^i B^j - B^i v^j \end{pmatrix}, \mathbf{S} = |g|^{\frac{1}{2}} \begin{pmatrix} 0 \\ \frac{1}{2} T^{ik} \partial_j g_{ik} \\ -\frac{1}{2} T^{ik} \partial_0 g_{ik} \\ 0 \end{pmatrix} \tag{10}$$

are respectively the set of *conservative variables*,the *fluxes* and the *source terms*.
The components of $T^{\mu\nu}$ shown in the previous equations are:

$$S_i = (e+p)\gamma^2 v_i + \varepsilon_{ijk} E^j B^k, \tag{11}$$

$$T_{ij} = (e+p)\gamma^2 v_i v_j + (p + u_{\rm em}) g_{ij} - E_i E_j - B_i B_j, \tag{12}$$

$$\mathcal{E} = (e+p)\gamma^2 - p + u_{\rm em}, \tag{13}$$

where we have defined the electromagnetic energy density $u_{\rm em} = \frac{1}{2}(E^2 + B^2)$.
The $v^i$ are the contravariant 3-vector components of the fluid velocity which are related to the previous 4-vector by $u^\mu = (\gamma, \gamma v^i)$, where $\gamma = (1 - v^2)^{-1/2}$ is the Lorentz factor of the bulk flow and $v^2 = v_k v^k$. The electric and magnetic field are now 3-vectors measured in the laboratory frame which are related to the previous 4-vectors by $e^\mu = (\gamma v_k E^k, \gamma E^i + \gamma \varepsilon^{ijk} v_j B_k)$ and $b^\mu = (\gamma v_k B^k, \gamma B^i - \gamma \varepsilon^{ijk} v_j E_k)$, where $\varepsilon_{ijk}$ is the Levi-Civita pseudo-tensor of the spatial three-metric, namely $\varepsilon_{ijk} = |g|^{\frac{1}{2}}[ijk]$, with $g = \det\{g_{\mu\nu}\} = -\det\{g_{ij}\} < 0$ and $[ijk]$ the usual alternating symbol of three-dimensional space with values $\pm 1$ or 0. In this $3D+1$ scheme, the ideal Ohm law $e^\mu = 0$ translates to $E_i = -\varepsilon_{ijk} v^j B^k$. This means that the components of the electric field are not independent anymore, but they are derived from the $v^i$ and $B^i$ dynamical variables. From a numerical point of view, it is also necessary to enforce the solenoidal (i.e. "null-B divergence") condition $\partial_i(|g|^{\frac{1}{2}} B^i) = 0$. This problem can be managed in different ways [30]; for its simplicity, we adopt the "general Lagrange multipliers" (known also as Dedner's) method [33, 34].
ECHO-QGP is based on finite difference schemes, we invite to read Refs. [32, 7, 26] for details. Here we just mention that we use the HLL (Harten-Lax-Van Leer) [35] approximate solver to solve the Riemann problem and a second or third order Runge Kutta algorithm to perform time integration.

## 3. A preliminary application

### *3.1. Setup*

After testing the code [26], we applied it in a quite typical and simple case of heavy ion collisions. We performed a $2D+1$ simulation of two gold nuclei colliding at $\sqrt{s}_{\rm NN} = 200$GeV with an impact parameter $b = 10$fm, adopting geometrical Glauber initial conditions[36] to determine the spatial energy density distribution at $\tau = 0.4$fm/c. We computed the spatial distribution of the magnetic field as in Ref. [14], assuming that the nuclei moved in a medium with constant electrical conductivity $\sigma = 5.8$MeV until the system reached the condition of local thermal equilibrium at $\tau = 0.4$fm/c, when we started the simulation. During the magnetohydrodynamical evolution we assumed that the medium had infinite electrical conductivity, consistently with our model, and we used the massless particles EoS $p = e/3$. We chose 150MeV/fm$^3$ as freeze-out energy density and we computed the elliptic flow of positive pions using the Cooper-Frye prescription [37].

### *3.2. Results*

In Fig. 1, we compare the time evolution of the magnetic field in the center of mass of the system in the ideal RMHD approach (i.e. fluid with an infinite electrical conductivity, coupled with the magnetic field) versus the evolution of the magnetic field in a medium with constant electrical conductivity, decoupled from the dynamics of the fluid. We can notice that the expansion on the transverse plane makes the magnetic field to decay faster compared to the Bjorken flow case (line $a$ vs line $e$), nevertheless the decay is still much slower than in the case of a magnetic field generated by electric charges moving in the void (line $c$).

In Fig. 2, we compare the final $v_2(p_T)$ of pions with and without an initial magnetic field. In the context of our setup, we don't observe any significant difference between these two cases.

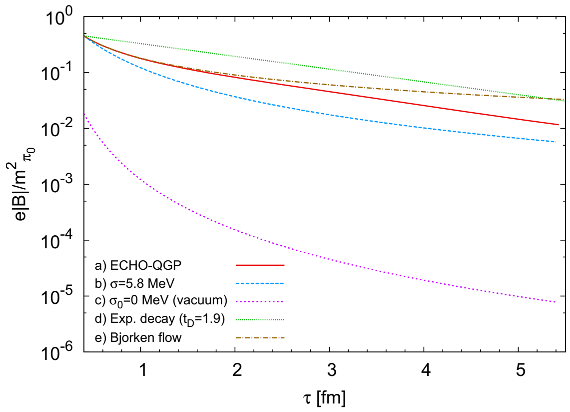


**Figure 1:** Comparison of the time evolution of the magnitude of the magnetic field (in neutral pion mass units squared) at the center of the grid in five different cases: a) ECHO-QGP 2D+1 RMHD evolution b) time evolution of the magnetic (as in Ref. [14]) in a medium with uniform and constant electrical conductivity $s = 5.8$ MeV c) same as in case b), but assuming zero electrical conductivity $s = 0$ MeV (vacuum) d) exponential decay of magnetic field as modeled in Ref. [25], with $t_D = 1.9$ e) Bjorken flow [38]. The parameters are for the reaction Au+Au, b=10 fm at $\sqrt{s_{NN}}$=200 GeV.

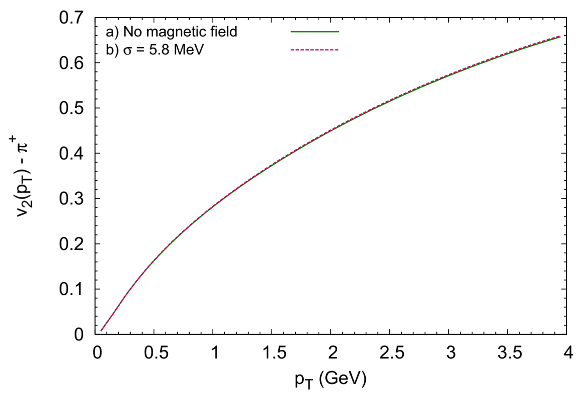


**Figure 2:** $v_2$ of $\pi^+$ in two cases: a) Without magnetic field, b) With an initial magnetic field computed assuming $\sigma = 5.8$ MeV. The parameters are for the reaction Au+Au, b=10 fm at $\sqrt{s_{NN}}$=200 GeV.

## 4. Conclusions and future perspectives

The present work represents a necessary, but limited step toward a realistic dynamical description of the Quark Gluon Plasma coupled with electromagnetic fields. Short term plans involve 3D+1 simulations with a lattice QCD + Hadron gas EoS, studying collisions at different energies and impact parameters, while long term plans will probably focus on resistive RMHD [39, 40, 41] and magnetic fields of chiral origin [42]. Hadronic interactions and particle decays at the end of the hydro phase will have to be taken into account, as well. Some effects that might be partially due to magnetic fields [43], like the splitting between the polarization of $\Lambda$ and $\overline{\Lambda}$ [44], are more evident at low collision energies, close to the regime of the upcoming FAIR CBM experiment. Unfortunately, at such low collision energies hydrodynamic simulations can provide a realistic description of the QGP only for a short and delayed time, especially when focusing on the most peripheral events, in which magnetic fields are expected to be strongest. Nevertheless, even in the case of CBM, a consistent treatment of the magnetic fields in the hydro phase will certainly contribute to provide a more accurate and complete physical model of the QGP evolution in heavy ion collisions.

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## References

[1] Csernai L P 1994 *Introduction to relativistic heavy ion collisions*
[2] Vitev I 2005 *Int. J. Mod. Phys.* **A20** 3777–3782
[3] Maruhn J A 1991 *Int. Rev. Nucl. Phys.* **5** 81–178
[4] Kolb P F, Sollfrank J and Heinz U W 2000 *Phys. Rev.* **C62** 054909
[5] Romatschke P and Romatschke U 2007 *Phys. Rev. Lett.* **99** 172301
[6] Bozek P 2012 *Phys. Rev.* **C85** 034901
[7] Del Zanna L, Chandra V, Inghirami G, Rolando V, Beraudo A, De Pace A, Pagliara G, Drago A and Becattini F 2013 *Eur. Phys. J.* **C73** 2524
[8] Karpenko I, Huovinen P and Bleicher M 2014 *Comput. Phys. Commun.* **185** 3016–3027
[9] Martinez M, Ryblewski R and Strickland M 2012 *Phys. Rev.* **C85** 064913
[10] Tinti L 2015 *Phys. Rev.* **C92** 014908
[11] Molnar E, Niemi H and Rischke D H 2016 *Phys. Rev.* **D93** 114025
[12] Kharzeev D E, McLerran L D and Warringa H J 2008 *Nuclear Physics A* **803** 227 – 253 ISSN 0375-9474
[13] Tuchin K 2013 *Adv. High Energy Phys.* **2013** 490495
[14] Tuchin K 2013 *Phys. Rev.* **C88** 024911
[15] Fukushima K, Kharzeev D E and Warringa H J 2008 *Phys. Rev. D* **78**(7) 074033
[16] Kharzeev D E and Yee H U 2011 *Phys. Rev. D* **83**(8) 085007
[17] ALICE Collaboration (ALICE Collaboration) 2016 *Phys. Rev. C* **93**(4) 044903
[18] Kharzeev D, Liao J, Voloshin S and Wang G 2016 *Progress in Particle and Nuclear Physics* **88** 1 – 28 ISSN 0146-6410
[19] Bali G S, Bruckmann F, Endrödi G, Katz S D and Schäfer A 2014 *JHEP* **08** 177
[20] Gürsoy U, Kharzeev D, Marcus E and Rajagopal K 2016 *Nucl. Phys.* **A956** 389–392
[21] Moghaddam M H, Azadegan B, Kord A F and Alberico W M 2017 (*Preprint* 1705.08192)
[22] Das A, Dave S S, Saumia P S and Srivastava A M 2017 (*Preprint* 1703.08162)
[23] Deng W T and Huang X G 2012 *Phys. Rev.* **C85** 044907
[24] Roy V and Pu S 2015 *Phys. Rev.* **C92** 064902
[25] Pang L G, Endrődi G and Petersen H 2016 *Phys. Rev.* **C93** 044919
[26] Inghirami G, Del Zanna L, Beraudo A, Moghaddam M H, Becattini F and Bleicher M 2016 *Eur. Phys. J.* **C76** 659
[27] Bekenstein J D and Oron E 1978 *Phys. Rev. D* **18**(6) 1809–1819
[28] Anile A M 1989 *Relativistic fluids and magneto-fluids : with applications in astrophysics and plasma physics* (Cambridge ; New York : Cambridge University Press) ISBN 0521304067
[29] Gourgoulhon E 2007 (*Preprint* gr-qc/0703035)
[30] Font J A 2008 *Living Rev. Rel.* **11** 7
[31] Del Zanna L, Bucciantini N and Londrillo P 2003 *Astronomy & Astrophysics* **400** 397–414
[32] Del Zanna, L, Zanotti, O, Bucciantini, N and Londrillo, P 2007 *Astronomy & Astrophysics* **473** 11–30
[33] Dedner A, Kemm F, Kröner D, Munz C D, Schnitzer T and Wesenberg M 2002 *Journal of Computational Physics* **175** 645 – 673 ISSN 0021-9991
[34] Mignone A and Tzeferacos P 2010 *Journal of Computational Physics* **229** 2117 – 2138 ISSN 0021-9991
[35] Harten A, Lax P D and van Leer B 1983 *SIAM Rev.* **25** 35–61 ISSN 0036-1445; 1095-7200/e
[36] Glauber R J 1959 *Lectures in Theoretical Physics* (New York : WE Brittin and LG Dunham)
[37] Cooper F and Frye G 1974 *Phys. Rev. D* **10**(1) 186–189
[38] Roy V, Pu S, Rezzolla L and Rischke D 2015 *Physics Letters B* **750** 45 – 52 ISSN 0370-2693
[39] Dionysopoulou K, Alic D, Palenzuela C, Rezzolla L and Giacomazzo B 2013 *Phys. Rev.* **D88** 044020
[40] Bucciantini N and Del Zanna L 2013 *Mon. Not. Roy. Astron. Soc.* **428** 71
[41] Del Zanna L, Papini E, Landi S, Bugli M and Bucciantini N 2016 *Mon. Not. Roy. Astron. Soc.* **460** 3753–3765
[42] Hirono Y, Hirano T and Kharzeev D E 2014 (*Preprint* 1412.0311)
[43] Becattini F, Karpenko I, Lisa M, Upsal I and Voloshin S 2017 *Phys. Rev.* **C95** 054902
[44] Adamczyk L *et al.* (STAR) 2017 (*Preprint* 1701.06657)