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Research Article

Analysis of CP Violation in $D^0 \longrightarrow K^+K^-\pi^0$

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We study the CP violation induced by the interference between two intermediate resonances $K^*(892)^+$ and $K^*(892)^-$ in the phase space of singly-Cabibbo-suppressed decay $D^0 \longrightarrow K^+K^-\pi^0$. We adopt the factorization-assisted topological approach in dealing with the decay amplitudes of $D^0 \longrightarrow K^\pm K^*(892)^\mp$. The CP asymmetries of two-body decays are predicted to be very tiny, which are $(-1.27\pm0.25)\times10^{-5}$ and $(3.86\pm0.26)\times10^{-5}$, respectively, for $D^0 \longrightarrow K^+K^*(892)^-$ and $D^0 \longrightarrow K^-K^*(892)^+$, while the differential CP asymmetry of $D^0 \longrightarrow K^+K^-\pi^0$ is enhanced because of the interference between the two intermediate resonances, which can reach as large as 3×10^{-4}. For some NPs which have considerable impacts on the chromomagnetic dipole operator O_{8g}, the global CP asymmetries of $D^0 \longrightarrow K^+K^*(892)^-$ and $D^0 \longrightarrow K^-K^*(892)^+$ can be then increased to $(0.56\pm0.08)\times10^{-3}$ and $(-0.50\pm0.04)\times10^{-3}$, respectively. The regional CP asymmetry in the overlapped region of the phase space can be as large as $(1.3\pm0.3)\times10^{-3}$.

1. Introduction

Charge-Parity (CP) violation, which was first discovered in K meson system in 1964 [1], is one of the most important phenomena in particle physics. In the Standard Model (SM), CP violation originates from the weak phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix [2, 3] and the unitary phases which usually arise from strong interactions. One reason for the smallness of CP violation is that the unitary phase is usually small. Nevertheless, CP violation can be enhanced in three-body decays of heavy hadrons, when the corresponding decay amplitudes are dominated by overlapped intermediate resonances in certain regions of phase space. Owing to the overlapping, a regional CP asymmetry can be generated by a relative strong phase between amplitudes corresponding to different resonances. This relative strong phase has nonperturbative origin. As a result, the regional CP asymmetry can be larger than the global one. In fact, such kind of enhanced CP violation has been observed in several three-body decay channels of B meson [4–7], which was followed by a number of theoretical works [8–19].

The study of CP violation in singly-Cabibbo-suppressed (SCS) D meson decays provides an ideal test of the SM and exploration of New Physics (NP) [20–23]. In the SM, CP violation is predicted to be very small in charm system. Experimental researches have shown that there is no significant CP violation so far in charmed hadron decays [24–33]. CP asymmetry in SCS D meson decay can be as small as

$$A_{CP} \sim \frac{\left|V_{cb}^*V_{ub}\right|}{\left|V_{cs}^*V_{us}\right|}\frac{\alpha_s}{\pi} \sim 10^{-4}, \tag{1}$$

or even less, due to the suppression of the penguin diagrams by the CKM matrix as well as the smallness of Wilson coefficients in penguin amplitudes. The SCS decays are sensitive to new contributions to the $\Delta C = 1$ QCD penguin and chromomagnetic dipole operators, while such contributions can affect neither the Cabibbo-favored (CF) ($c \longrightarrow s\bar{d}u$) nor the doubly-Cabibbo-suppressed (DCS) ($c \longrightarrow d\bar{s}u$) decays [34]. Besides, the decays of charmed mesons offer a unique opportunity to probe CP violation in the up-type quark sector.

Several factorization approaches have been wildly used in nonleptonic B decays. In the naive factorization approach [35, 36], the hadronic matrix elements were expressed as a product of a heavy to light transition form factor and a decay constant. Based on Heavy Quark Effect Theory, it is shown in the QCD factorization approach that the corrections to the hadronic matrix elements can be expressed in terms of short-distance coefficients and meson light-cone distribution amplitudes [37, 38]. Alternative factorization approach based on QCD factorization is often applied in study of quasi two-body hadronic B decays [19, 39, 40], where they introduced unitary meson-meson form factors, from the perspective of unitarity, for the final state interactions. Other QCD-inspired approaches, such as the perturbative approach (pQCD) [41] and the soft-collinear effective theory (SCET) [42], are also wildly used in B meson decays.

However, for D meson decays, such QCD-inspired factorization approaches may not be reliable since the charm quark mass, which is just above 1 GeV, is not heavy enough for the heavy quark expansion [43, 44]. For this reason, several model-independent approaches for the charm meson decay amplitudes have been proposed, such as the flavor topological diagram approach based on the flavor $SU(3)$ symmetry [44–47] and the factorization-assisted topological-amplitude (FAT) approach with the inclusion of flavor $SU(3)$ breaking effect [48, 49]. One motivation of these aforementioned approaches is to identify as complete as possible the dominant sources of nonperturbative dynamics in the hadronic matrix elements.

In this paper, we study the CP violation of SCS D meson decay $D^0 \longrightarrow K^+K^-\pi^0$ in the FAT approach. Our attention will be mainly focused on the region of the phase space where two intermediate resonances, $K^*(892)^+$ and $K^*(892)^-$, are overlapped. Before proceeding, it will be helpful to point out that direct CP asymmetry is hard to be isolated for decay process with CP-eigen-final-state. When the final state of the decay process is CP eigenstate, the time integrated CP violation for $D^0 \longrightarrow f$, which is defined as

$$a_f \equiv \frac{\int_0^\infty \Gamma\left(D^0 \longrightarrow f\right) dt - \int_0^\infty \Gamma\left(\overline{D}^0 \longrightarrow f\right) dt}{\int_0^\infty \Gamma\left(D^0 \longrightarrow f\right) dt + \int_0^\infty \Gamma\left(\overline{D}^0 \longrightarrow f\right) dt}, \quad (2)$$

can be expressed as [34]

$$a_f = a_f^d + a_f^m + a_f^i, \quad (3)$$

where a_f^d, a_f^m, and a_f^i are the CP asymmetries in decay, in mixing, and in the interference of decay and mixing, respectively. As is shown in [34, 50, 51], the indirect CP violation $a^{\text{ind}} \equiv a^m + a^i$ is universal and channel-independent for two-body CP-eigenstate. This conclusion is easy to be generalized to decay processes with three-body CP-eigenstate in the final state, such as $D^0 \longrightarrow K^+K^-\pi^0$. In view of the universality of the indirect CP asymmetry, we will only consider the direct CP violations of the decay $D^0 \longrightarrow K^+K^-\pi^0$ throughout this paper.

The remainder of this paper is organized as follows. In Section 2, we present the decay amplitudes for various decay channels, where the decay amplitudes of $D^0 \longrightarrow K^\pm K^*(892)^\mp$ are formulated via the FAT approaches. In Section 3, we study the CP asymmetries of $D^0 \longrightarrow K^\pm K^*(892)^\mp$ and the CP asymmetry of $D^0 \longrightarrow K^+K^-\pi^0$ induced by the interference between different resonances in the phase space. Discussions and conclusions are given in Section 4. We list some useful formulas and input parameters in the Appendix.

2. Decay Amplitude for $D^0 \longrightarrow K^+K^-\pi^0$

In the overlapped region of the intermediate resonances $K^*(892)^+$ and $K^*(892)^-$ in the phase space, the decay process $D^0 \longrightarrow K^+K^-\pi^0$ is dominated by two cascade decays, $D^0 \longrightarrow K^+K^*(892)^- \longrightarrow K^+K^-\pi^0$ and $D^0 \longrightarrow K^-K^*(892)^+ \longrightarrow K^-K^+\pi^0$, respectively. Consequently, the decay amplitude of $D^0 \longrightarrow K^+K^-\pi^0$ can be expressed as

$$\mathcal{M}_{D^0\longrightarrow K^+K^-\pi^0} = \mathcal{M}_{K^{*+}} + e^{i\delta}\mathcal{M}_{K^{*-}} \quad (4)$$

in the overlapped region, where $\mathcal{M}_{K^{*+}}$ and $\mathcal{M}_{K^{*-}}$ are the amplitudes for the two cascade decays and δ is the relative strong phase. Note that nonresonance contributions have been neglected in (4).

The decay amplitude for the cascade decay $D^0 \longrightarrow K^+K^*(892)^- \longrightarrow K^+K^-\pi^0$ can be expressed as

$$\mathcal{M}_{K^{*-}} = \frac{\sum_\lambda \mathcal{M}^\lambda_{K^{*-}\longrightarrow K^-\pi^0} \cdot \mathcal{M}^\lambda_{D^0\longrightarrow K^{*-}K^+}}{s_{\pi^0K^-} - m^2_{K^{*-}} + im_{K^{*-}}\Gamma_{K^{*-}}}, \quad (5)$$

where $\mathcal{M}^\lambda_{K^{*-}\longrightarrow K^-\pi^0}$ and $\mathcal{M}^\lambda_{D^0\longrightarrow K^+K^{*-}}$ represent the amplitudes corresponding to the strong decay $K^{*-} \longrightarrow K^-\pi^0$ and weak decay $D^0 \longrightarrow K^+K^{*-}$, respectively, λ is the helicity index of K^{*-}, $s_{\pi^0K^-}$ is the invariant mass square of π^0K^- system, and $m_{K^{*-}}$ and $\Gamma_{K^{*-}}$ are the mass and width of $K^*(892)^-$, respectively. The decay amplitude for the cascade decay, $D^0 \longrightarrow K^-K^*(892)^+ \longrightarrow K^-K^+\pi^0$, is the same as (5) except replacing the subscripts K^{*-} and $K^\pm$ with K^{*+} and $K^\mp$, respectively.

For the strong decays $K^*(892)^\pm \longrightarrow \pi^0K^\pm$, one can express the decay amplitudes as

$$\mathcal{M}_{K^{*\pm}\longrightarrow\pi^0K^\pm} = g_{K^{*\pm}K^\pm\pi^0}\left(p_{\pi^0} - p_{K^\pm}\right)\cdot\varepsilon_{K^{*\pm}}\left(p,\lambda\right), \quad (6)$$

where p_{π^0} and $p_{K^\pm}$ represent the momentum for π^0 and $K^\pm$ mesons, respectively, and $g_{K^{*\pm}K^\pm\pi^0}$ is the effective coupling constant for the strong interaction, which can be extracted from the experimental data via

$$g^2_{K^{*\pm}K^\pm\pi^0} = \frac{6\pi m^2_{K^{*\pm}}\Gamma_{K^{*\pm}\longrightarrow K^\pm\pi^0}}{\lambda^3_{K^{*\pm}}}, \quad (7)$$

with

$$\lambda_{K^{*\pm}} = \frac{1}{2m_{K^{*\pm}}} \cdot \sqrt{\left[m^2_{K^{*\pm}} - \left(m_{\pi^0} + m_{K^\pm}\right)^2\right]\cdot\left[m^2_{K^{*\pm}} - \left(m_{\pi^0} - m_{K^\pm}\right)^2\right]}, \quad (8)$$

and $\Gamma_{K^{*\pm}\longrightarrow K^\pm\pi^0} = \text{Br}(K^{*\pm} \longrightarrow K^\pm\pi^0)\cdot\Gamma_{K^{*\pm}}$. The isospin symmetry of the strong interaction implies that $\Gamma_{K^{*\pm}\longrightarrow K^\pm\pi^0} \simeq (1/3)\Gamma_{K^{*\pm}}$.

The decay amplitudes for the weak decays, $D^0 \longrightarrow K^+K^*(892)^-$ and $D^0 \longrightarrow K^-K^*(892)^+$, will be handled with

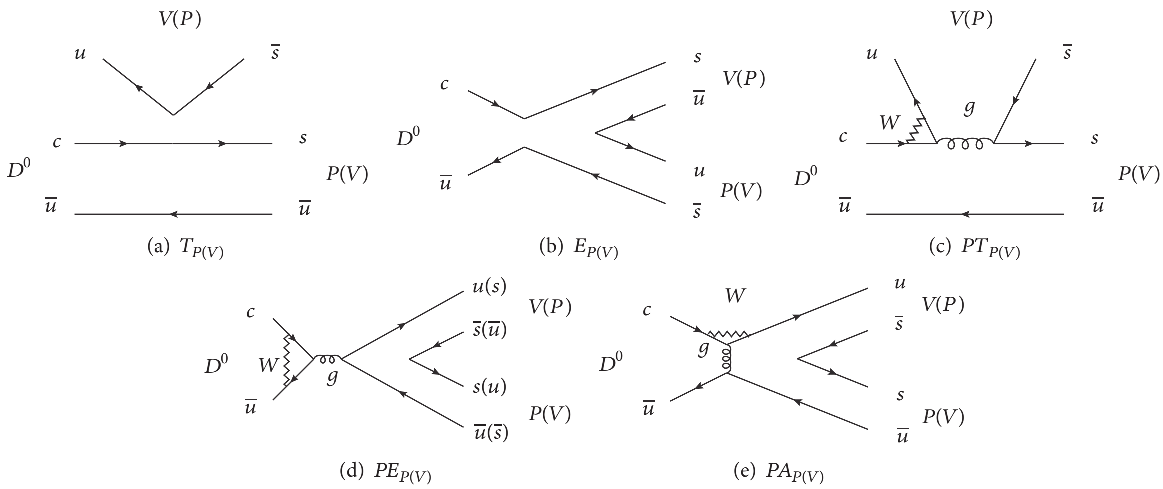


Figure 1: The relevant topological diagrams for $D \longrightarrow PV$ with (a) the color-favored tree amplitude $T_{P(V)}$, (b) the W-exchange amplitude $E_{P(V)}$, (c) the color-favored penguin amplitude $PT_{P(V)}$, (d) the gluon-annihilation penguin amplitude $PE_{P(V)}$, and (e) the gluon-exchange penguin amplitude $PA_{P(V)}$.

the aforementioned FAT approach [48, 49]. The relevant topological tree and penguin diagrams for $D \longrightarrow PV$ are displayed in Figure 1, where P and V denote a light pseudoscalar and vector meson (representing $K^{\pm}$ and $K^{*\pm}$ in this paper), respectively.

The two tree diagrams in first line of Figure 1 represent the color-favored tree diagram for $D \longrightarrow P(V)$ transition and the W-exchange diagram with the pseudoscalar (vector) meson containing the antiquark from the weak vertex, respectively. The amplitudes of these two diagrams will be, respectively, denoted as $T_{P(V)}$ and $E_{P(V)}$.

According to these topological structures, the amplitudes of the color-favored tree diagrams $T_{P(V)}$, which are dominated by the factorizable contributions, can be parameterized as

$$T_P = \frac{G_F}{\sqrt{2}} \lambda_s a_2(\mu) f_V m_V F_1^{D \longrightarrow P}(m_V^2)\, 2(\varepsilon^* \cdot p_D), \quad (9)$$

and

$$T_V = \frac{G_F}{\sqrt{2}} \lambda_s a_2(\mu) f_P m_V A_0^{D \longrightarrow V}(m_P^2)\, 2(\varepsilon^* \cdot p_D), \quad (10)$$

respectively, where G_F is the Fermi constant, $\lambda_s = V_{us}V_{cs}^*$, with V_{us} and V_{cs} being the CKM matrix elements, $a_2(\mu) = c_2(\mu) + c_1(\mu)/N_c$, with $c_1(\mu)$ and $c_2(\mu)$ being the scale-dependent Wilson coefficients, and the number of color $N_c = 3$, $f_{V(P)}$ and $m_{V(P)}$ are the decay constant and mass of the vector (pseudoscalar) meson, respectively, $F_1^{D \longrightarrow P}$ and $A_0^{D \longrightarrow V}$ are the form factors for the transitions $D \longrightarrow P$ and $D \longrightarrow V$, respectively, ε is the polarization vector of the vector meson, and p_D is the momentum of D meson. The scale μ of Wilson coefficients is set to energy release in individual decay channels [52, 53], which depends on masses of initial and final states and is defined as [48, 49]

$$\mu = \sqrt{\Lambda m_D (1 - r_P^2)(1 - r_V^2)}, \quad (11)$$

with the mass ratios $r_{V(P)} = m_{V(P)}/m_D$, where Λ represents the soft degrees of freedom in the D meson, which is a free parameter.

For the W-exchange amplitudes, since the factorizable contributions to these amplitudes are helicity-suppressed, only the nonfactorizable contributions need to be considered. Therefore, the W-exchange amplitudes are parameterized as

$$E_{P,V}^q = \frac{G_F}{\sqrt{2}} \lambda_s c_2(\mu) \chi_q^E e^{i\phi_q^E} f_D m_D \frac{f_P f_V}{f_\pi f_\rho} (\varepsilon^* \cdot p_D), \quad (12)$$

where m_D is the mass of D meson, f_D, f_π, and f_ρ are the decay constants of the D, π, and ρ mesons, respectively, and χ_q^E and ϕ_q^E characterize the strengths and the strong phases of the corresponding amplitudes, with $q = u, d, s$ representing the strongly produced q quark pair. The ratio of $f_P f_V$ over $f_\pi f_\rho$ indicates that the flavor $SU(3)$ breaking effects have been taken into account from the decay constants.

The penguin diagrams shown in the second line of Figure 1 represent the color-favored, the gluon-annihilation, and the gluon-exchange penguin diagrams, respectively, whose amplitudes will be denoted as $PT_{P(V)}$, $PE_{P(V)}$, and $PA_{P(V)}$, respectively.

Since a vector meson cannot be generated from the scalar or pseudoscalar operator, the amplitude PT_P does not include contributions from the penguin operator O_5 or O_6. Consequently, the color-favored penguin amplitudes PT_P and PT_V can be expressed as

$$PT_P = -\frac{G_F}{\sqrt{2}} \lambda_b a_4(\mu) f_V m_V F_1^{D \longrightarrow P}(m_V^2)\, 2(\varepsilon^* \cdot p_D), \quad (13)$$

and

$$PT_V = -\frac{G_F}{\sqrt{2}} \lambda_b \left[a_4(\mu) - r_\chi a_6(\mu)\right] f_P m_V A_0^{D \longrightarrow V}(m_P^2) \cdot 2(\varepsilon^* \cdot p_D), \quad (14)$$

respectively, where $\lambda_b = V_{ub}V_{cb}^*$ with V_{ub} and V_{cb}^* being the CKM matrix elements, $a_{4,6}(\mu) = c_{4,6}(\mu) + c_{3,5}(\mu)/N_c$, with $c_{3,4,5,6}$ being the Wilson coefficients, and r_χ is a chiral factor, which takes the form

$$r_\chi = \frac{2m_P^2}{(m_u + m_q)(m_q + m_c)}, \tag{15}$$

with $m_{u(c,q)}$ being the masses of $u(c,q)$ quark. Note that the quark-loop corrections and the chromomagnetic-penguin contribution are also absorbed into $c_{3,4,5,6}$ as shown in [49].

Similar to the amplitudes $E_{P,V}$, the amplitudes PE only include the nonfactorizable contributions as well. Therefore, the amplitudes $PE_{P,V}$, which are dominated by O_4 and O_6 [48], can be parameterized as

$$PE_{P,V}^q = -\frac{G_F}{\sqrt{2}}\lambda_b\left[c_4(\mu) - c_6(\mu)\right]\chi_q^E e^{i\phi_q^E} f_D m_D \cdot \frac{f_P f_V}{f_\pi f_\rho}(\varepsilon^* \cdot p_D). \tag{16}$$

For the amplitudes PA_P and PA_V, the helicity suppression does not apply to the matrix elements of $O_{5,6}$, so the factorizable contributions exist. In the pole resonance model [54], after applying the Fierz transformation and the factorization hypothesis, the amplitudes PA_P and PA_V can be expressed as

$$PA_P^q = -\frac{G_F}{\sqrt{2}}\lambda_b\left[(-2)a_6(\mu)(2g_S) \cdot \frac{1}{m_D^2 - m_{P^*}^2}\left(f_{P^*}m_{P^*}^0\right)\left(f_D\frac{m_D^2}{m_c}\right) + c_3(\mu) \cdot \chi_q^A e^{i\phi_q^A} f_D m_D \frac{f_P f_V}{f_\pi f_\rho}\right](\varepsilon^* \cdot p_D), \tag{17}$$

and

$$PA_V^q = -\frac{G_F}{\sqrt{2}}\lambda_b\left[(-2)a_6(\mu)(-2g_S) \cdot \frac{1}{m_D^2 - m_{P^*}^2}\left(f_{P^*}m_{P^*}^0\right)\left(f_D\frac{m_D^2}{m_c}\right) + c_3(\mu) \cdot \chi_q^A e^{i\phi_q^A} f_D m_D \frac{f_P f_V}{f_\pi f_\rho}\right](\varepsilon^* \cdot p_D), \tag{18}$$

respectively, where g_S is an effective strong coupling constant obtained from strong decays, e.g., $\rho \longrightarrow \pi\pi$, $K^* \longrightarrow K\pi$, and $\phi \longrightarrow KK$, and is set as $g_S = 4.5$ [54] in this work, m_{P^*} and f_{P^*} are the mass and decay constant of the pole resonant pseudoscalar meson P^*, respectively, and χ_q^A and ϕ_q^A are the strengths and the strong phases of the corresponding amplitudes.

From Figure 1, the decay amplitudes of $D^0 \longrightarrow K^+K^*(892)^-$ and $D^0 \longrightarrow K^-K^*(892)^+$ in the FAT approach can be easily written down

$$\mathcal{M}^\lambda_{D^0\longrightarrow K^+K^{*-}} = T_{K^{*-}} + E^u_{K^+} + PT_{K^{*-}} + PE^s_{K^{*-}} + PE^u_{K^+} + PA^s_{K^{*-}}, \tag{19}$$

and

$$\mathcal{M}^\lambda_{D^0\longrightarrow K^-K^{*+}} = T_{K^-} + E^u_{K^{*+}} + PT_{K^-} + PE^s_{K^-} + PE^u_{K^{*+}} + PA^s_{K^-}, \tag{20}$$

respectively, where λ is the helicity of the polarization vector $\varepsilon(p,\lambda)$. In the FAT approach, the fitted nonperturbative parameters, $\chi^E_{q,s}$, $\phi^E_{q,s}$, $\chi^A_{q,s}$, $\phi^A_{q,s}$, are assumed to be universal and can be determined by the data [49].

In Table 1, we list the magnitude of each topological amplitude for $D^0 \longrightarrow K^+K^*(892)^-$ and $D^0 \longrightarrow K^-K^*(892)^+$ by using the global fitted parameters for $D \longrightarrow PV$ in [49]. One can see from Table 1 that the penguin contributions are greatly suppressed. PT is dominant in the penguin contributions of $D^0 \longrightarrow K^-K^*(892)^+$, while PT is small in $D^0 \longrightarrow K^+K^*(892)^-$, which is even smaller than the amplitude PA. This difference is because of the chirally enhanced factor contained in (14) while not in (13). The very small PE do not receive the contributions from the quark-loop and chromomagnetic penguins, since these two contributions to c_4 and c_6 are canceled with each other in (16). Besides, the relations $PE^s_V = PE^s_P$, $PE^u_V = PE^u_P$, and $PE^s_V \neq PE^u_V$ can be read from Table 1; this is because that the isospin symmetry and the flavor $SU(3)$ breaking effect have been considered.

Since the form factors are inevitably model-dependent, we list in Table 2 the branching ratios of $D^0 \longrightarrow K^+K^*(892)^-$ and $D^0 \longrightarrow K^-K^*(892)^+$ predicted by the FAT approach, by various form factor models. The pole, dipole, and covariant light-front (CLF) models are adopted. The uncertainties in Table 2 mainly come from decay constants. The CLF model agrees well with the data for both decay channels, and other models are also consistent with the data. However, the model-dependence of form factor leads to large uncertainty of the branching fraction, as large as 20%. Because of the smallness of the Wilson coefficients and the CKM-suppression of the penguin amplitudes, the branching ratios are dominated by the tree amplitudes. Therefore, there is no much difference for the branching ratios whether we consider the penguin amplitudes or not.

3. *CP* Asymmetries for $D^0 \longrightarrow K^\pm K^*(892)^\mp$ and $D^0 \longrightarrow K^+K^-\pi^0$

The direct CP asymmetry for the two-body decay $D \longrightarrow PV$ is defined as

$$A_{CP}^{D\longrightarrow PV} = \frac{\left|\mathcal{M}_{D\longrightarrow PV}\right|^2 - \left|\mathcal{M}_{\overline{D}\longrightarrow\overline{PV}}\right|^2}{\left|\mathcal{M}_{D\longrightarrow PV}\right|^2 + \left|\mathcal{M}_{\overline{D}\longrightarrow\overline{PV}}\right|^2}, \tag{21}$$

where $\mathcal{M}_{\overline{D}\longrightarrow\overline{PV}}$ represents the decay amplitude of the CP conjugate process $\overline{D} \longrightarrow \overline{PV}$, such as $\overline{D}^0 \longrightarrow K^+K^*(892)^-$ or $\overline{D}^0 \longrightarrow K^-K^*(892)^+$. In the framework of FAT approach,

Table 1: The magnitude of tree and penguin contributions (in unit of 10^{-3}) corresponding to the topological amplitudes in (19) and (20). The factors "$(G_F/\sqrt{2})\lambda_s(\varepsilon^* \cdot p_D)$" and "$-(G_F/\sqrt{2})\lambda_b(\varepsilon^* \cdot p_D)$" are omitted in this table.

Decay modes	$T_{K^{*-}}$	$E^u_{K^+}$	$PT_{K^{*-}}$	$PE^s_{K^{*-}}$	$PE^u_{K^+}$	$PA^s_{K^{*-}}$
$D^0 \longrightarrow K^+K^*(892)^-$	0.23	−0.02 + 0.15i	3.83 + 4.32i	0.96 − 0.03i	0.13 − 0.81i	6.73 + 8.22i
	T_{K^-}	$E^u_{K^{*+}}$	PT_{K^-}	$PE^s_{K^-}$	$PE^u_{K^{*+}}$	$PA^s_{K^-}$
$D^0 \longrightarrow K^-K^*(892)^+$	0.44	−0.02 + 0.15i	−23.3 − 19.3i	0.96 − 0.03i	0.13 − 0.81i	−8.53 − 5.53i

Table 2: Branching ratios (in unit of 10^{-3}) of singly-Cabibbo-suppressed decays $D^0 \longrightarrow K^+K^*(892)^-$ and $D^0 \longrightarrow K^-K^*(892)^+$. Both experimental data [55–57] and theoretical predictions of FAT approach of the branching ratios are listed.

Form factors	$\mathrm{Br}(D^0 \longrightarrow K^+K^*(892)^-)$	$\mathrm{Br}(D^0 \longrightarrow K^-K^*(892)^+)$
Pole	1.57 ± 0.04	3.73 ± 0.17
Dipole	1.69 ± 0.04	4.02 ± 0.19
CLF	1.45 ± 0.04	4.44 ± 0.20
Exp.	1.56 ± 0.12	4.38 ± 0.21

we predict very small direct CP asymmetries of $D^0 \longrightarrow K^+K^*(892)^-$ and $D^0 \longrightarrow K^-K^*(892)^+$ presented in Table 3. The uncertainties induced by the model-dependence of form factor to the CP asymmetries of $D^0 \longrightarrow K^+K^*(892)^-$ and $D^0 \longrightarrow K^-K^*(892)^+$ are about 30% and 10%, respectively.

The differential CP asymmetry of the three-body decay $D^0 \longrightarrow K^+K^-\pi^0$, which is a function of the invariant mass of $s_{\pi^0K^+}$ and $s_{\pi^0K^-}$, is defined as

$$A_{CP}^{D^0\longrightarrow K^+K^-\pi^0}\left(s_{\pi^0K^+}, s_{\pi^0K^-}\right) = \frac{\left|\mathcal{M}_{D^0\longrightarrow K^+K^-\pi^0}\right|^2 - \left|\mathcal{M}_{\overline{D}^0\longrightarrow K^-K^+\pi^0}\right|^2}{\left|\mathcal{M}_{D^0\longrightarrow K^+K^-\pi^0}\right|^2 + \left|\mathcal{M}_{\overline{D}^0\longrightarrow K^-K^+\pi^0}\right|^2}, \quad (22)$$

where the invariant mass $s_{\pi^0K^\pm} = (p_{\pi^0} + p_{K^\pm})^2$. As can be seen from (4), the differential CP asymmetry $A_{CP}^{D^0\longrightarrow K^+K^-\pi^0}$ depends on the relative strong phase δ, which is impossible to be calculated theoretically because of its nonperturbative origin. Despite this, we can still acquire some information of this relative strong phase δ from data. By using a Dalitz plot technique [55, 58, 59], the phase difference $\delta^{\exp}$ between D^0 decays to $K^+K^*(892)^-$ and $K^-K^*(892)^+$ can be extracted from data. One should notice that $\delta^{\exp}$ is not the same as the strong phase δ defined in (4). The strong phase δ is the relative phase between the decay amplitudes of $D^0 \longrightarrow K^+K^*(892)^-$ and $D^0 \longrightarrow K^-K^*(892)^+$. On the other hand, the phase $\delta^{\exp}$ is defined through

$$\mathcal{M}_{D^0\longrightarrow K^+K^-\pi^0} = \left(\left|\mathcal{M}_{K^{*+}}\right| + e^{i\delta^{\exp}}\left|\mathcal{M}_{K^{*-}}\right|\right)e^{i\delta_{K^{*+}}} \quad (23)$$

in the overlapped region of the phase space, where $\delta_{K^{*\pm}}$ is the phase of the amplitude $\mathcal{M}_{K^{*\pm}}$:

$$\mathcal{M}_{K^{*\pm}} = \left|\mathcal{M}_{K^{*\pm}}\right|e^{i\delta_{K^{*\pm}}}. \quad (24)$$

Therefore, neglecting the CKM suppressed penguin amplitudes, $\delta^{\exp}$ and δ can be related by

$$\delta^{\exp} - \delta \approx \delta^{K^{*-}K^+} - \delta^{K^{*+}K^-}, \quad (25)$$

where $\delta^{K^{*\mp}K^\pm} = \arg(T_{K^{*\mp}} + E^u_{K^\pm})$ are the phases in tree-level amplitudes of $D^0 \longrightarrow K^\pm K^*(892)^\mp$ and are equivalent to $\delta_{K^{*\mp}}$ if the penguin amplitudes are neglected. With the relation of (25), and $\delta^{\exp} = -35.5^\circ \pm 4.1^\circ$ measured by the BABAR Collaboration [56], we have $\delta \approx -51.85^\circ \pm 4.1^\circ$.

In Figure 2, we present the differential CP asymmetry of $D^0 \longrightarrow K^+K^-\pi^0$ in the overlapped region of $K^*(892)^-$ and $K^*(892)^+$ in the phase space, with $\delta = -51.85^\circ$. Namely, we will focus on the region $m_{K^*} - 2\Gamma_{K^*} < \sqrt{s_{\pi^0K^-}}, \sqrt{s_{\pi^0K^+}} < m_{K^*} + 2\Gamma_{K^*}$ of the phase space. One can see from Figure 2 that the differential CP asymmetry of $D^0 \longrightarrow K^+K^-\pi^0$ can reach 3.0×10^{-4} in the overlapped region, which is about 10 times larger than the CP asymmetries of the corresponding two-body decay channels shown in Table 3.

The behavior of the differential CP asymmetry of $D^0 \longrightarrow K^+K^-\pi^0$ in Figure 2 motivates us to separate this region into four areas, area A ($m_{K^*} < \sqrt{s_{\pi^0K^-}} < m_{K^*} + 2\Gamma_{K^*}, m_{K^*} - 2\Gamma_{K^*} < \sqrt{s_{\pi^0K^+}} < m_{K^*}$), area B ($m_{K^*} < \sqrt{s_{\pi^0K^-}} < m_{K^*} + 2\Gamma_{K^*}, m_{K^*} < \sqrt{s_{\pi^0K^+}} < m_{K^*} + 2\Gamma_{K^*}$), area C ($m_{K^*} - 2\Gamma_{K^*} < \sqrt{s_{\pi^0K^-}} < m_{K^*}, m_{K^*} - 2\Gamma_{K^*} < \sqrt{s_{\pi^0K^+}} < m_{K^*}$), and area D ($m_{K^*} - 2\Gamma_{K^*} < \sqrt{s_{\pi^0K^-}} < m_{K^*}, m_{K^*} < \sqrt{s_{\pi^0K^+}} < m_{K^*} + 2\Gamma_{K^*}$). We further consider the observable of regional CP asymmetry in areas A, B, C, and D displayed in Table 4, which is defined by

$$A_{CP}^{\Omega} = \frac{\int_\Omega \left(\left|\mathcal{M}_{\mathrm{tot}}\right|^2 - \left|\overline{\mathcal{M}}_{\mathrm{tot}}\right|^2\right) \mathrm{d}s_{\pi^0K^-}s_{\pi^0K^+}}{\int_\Omega \left(\left|\mathcal{M}_{\mathrm{tot}}\right|^2 + \left|\overline{\mathcal{M}}_{\mathrm{tot}}\right|^2\right) \mathrm{d}s_{\pi^0K^-}s_{\pi^0K^+}}, \quad (26)$$

where Ω represents a certain region of the phase space.

Comparing with the CP asymmetries of two-body decays, the regional CP asymmetries, from Table 4, are less sensitive to the models we have used. We would like to use only the CLF model for the following discussion. The uncertainties in Table 4 come from decay constants as well as the relative phase $\delta^{\exp}$. In addition, if we focus on the right part of area A, that is, $m_{K^*} < \sqrt{s_{\pi^0K^-}} < m_{K^*} + 2\Gamma_{K^*}, m_{K^*} - \Gamma_{K^*} < \sqrt{s_{\pi^0K^+}} < m_{K^*}$, the regional CP violation will be $(1.09 \pm 0.16) \times 10^{-4}$.

The energy dependence of the propagator of the intermediate resonances can lead to a small correction to CP asymmetry. For example, if we replace the Breit-Wigner

Table 3: CP asymmetries (in unit of 10^{-5}) of $D^0 \longrightarrow K^+K^*(892)^-$ and $D^0 \longrightarrow K^-K^*(892)^+$ predicted by the FAT approach with pole, dipole, and CLF models adopted. The uncertainties in this table are mainly from decay constants.

Form factors	$A_{CP}(D^0 \longrightarrow K^+K^*(892)^-)$	$A_{CP}(D^0 \longrightarrow K^-K^*(892)^+)$
Pole	-1.45 ± 0.25	3.60 ± 0.23
Dipole	-1.63 ± 0.26	3.70 ± 0.24
CLF	-1.27 ± 0.25	3.86 ± 0.26

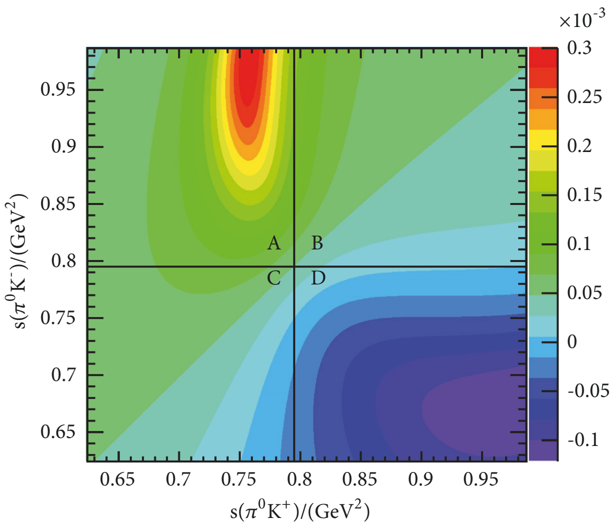


Figure 2: The differential CP asymmetry distribution of $D^0 \longrightarrow K^+K^-\pi^0$ in the overlapped region of $K^*(892)^-$ and $K^*(892)^+$ in the phase space.

propagator by the Flatté Parametrization [60], the correction to the regional CP asymmetry will be about 1%.

Since the CP asymmetry of $D^0 \longrightarrow K^+K^-\pi^0$ is extremely suppressed, it should be more sensitive to the NP. For example, some NPs have considerable impacts on the chromomagnetic dipole operator O_{8g} [34, 61–66]. Consequently, the CP violation in SCS decays may be further enhanced. In practice, the NP contributions can be absorbed into the corresponding effective Wilson coefficient c_{8g}^{eff} [67, 68]. For comparison, we first consider a relative small value of c_{8g}^{eff} (as in [48, 64]) lying within the range $(0, 1)$ and the global CP asymmetry of $D^0 \longrightarrow K^*(892)^{\pm}K^{\mp}$ are no larger than 5×10^{-5}. Moreover, if we follow [49] taking $c_{8g}^{\mathrm{eff}} \approx 10$ (while $c_{8g}^{\mathrm{eff}} = 10$, which is extracted from ΔA_{CP} measured by LHCb [69], is a quite large quantity even for the coefficients corresponding tree-level operators, however, such large contribution can be realized if some NPs effects are pulled in. For example, the up squark-gluino loops in supersymmetry (SUSY) can arise significant contributions to c_{8g}. More details about the squark-gluino loops and other models in SUSY can be found in [34, 62, 70–72]), the global CP asymmetries of $D^0 \longrightarrow K^+K^*(892)^-$ and $D^0 \longrightarrow K^-K^*(892)^+$ are then $(0.56\pm0.08)\times 10^{-3}$ and $(-0.50 \pm 0.04) \times 10^{-3}$, respectively.

We further display the CP asymmetry of $D^0 \longrightarrow K^+K^-\pi^0$ in the overlapped region of $K^*(892)^-$ and $K^*(892)^+$ in Figures 3(a) and 3(b) for $c_{8g}^{\mathrm{eff}} = 1$ and $c_{8g}^{\mathrm{eff}} = 10$, respectively. After taking the interference effect into account, the differential CP asymmetry of $D^0 \longrightarrow K^+K^-\pi^0$ can be increased as large as 5.5×10^{-4} and 2.8×10^{-3} for $c_{8g}^{\mathrm{eff}} = 1$ and $c_{8g}^{\mathrm{eff}} = 10$, respectively. The regional ones (in phase space of $\sqrt{0.74}$ GeV $< \sqrt{s_{\pi^0K^-}} < \sqrt{0.81}$ GeV, $\sqrt{0.84} < \sqrt{s_{\pi^0K^+}} < m_{K^*} + 2\Gamma_{K^*}$) can reach $(2.7\pm0.5)\times10^{-4}$ and $(1.3\pm0.3)\times10^{-3}$ for $c_{8g}^{\mathrm{eff}} = 1$ and $c_{8g}^{\mathrm{eff}} = 10$, respectively.

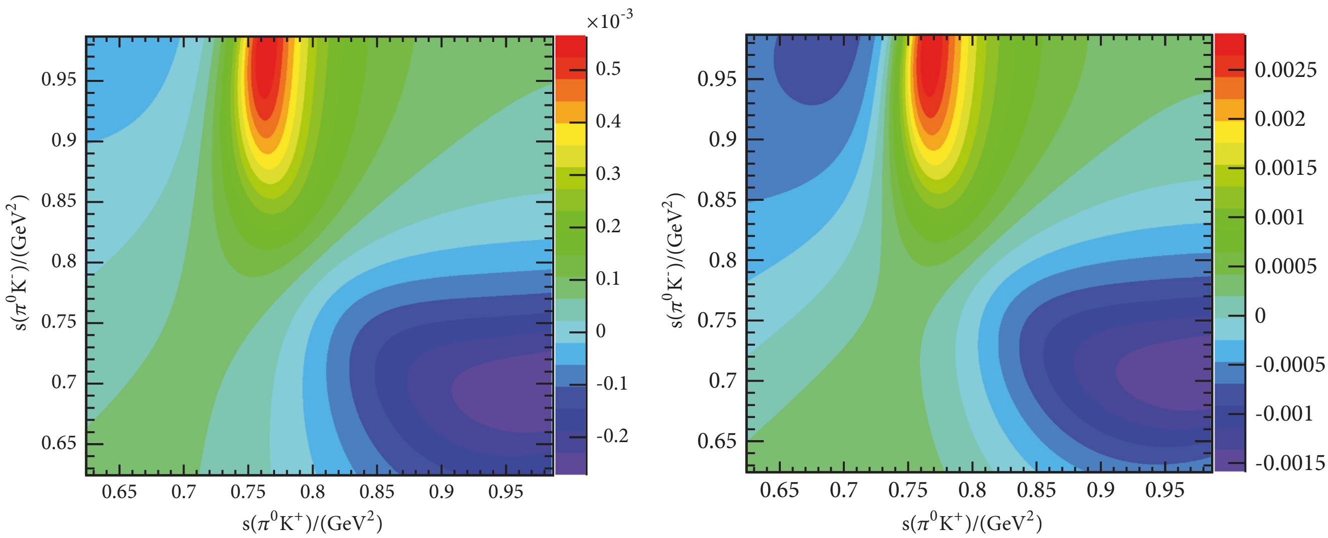
4. Discussion and Conclusion

In this work, we studied CP violations in $D^0 \longrightarrow K^*(892)^{\pm}K^{\mp} \longrightarrow K^+K^-\pi^0$ via the FAT approach. The CP violations in two-body decay processes $D^0 \longrightarrow K^+K^*(892)^-$ and $D^0 \longrightarrow K^-K^*(892)^+$ are very small, which are $(-1.27 \pm 0.25) \times 10^{-5}$ and $(3.86 \pm 0.26) \times 10^{-5}$, respectively. Our discussion shows that the CP violation can be enhanced by the interference effect in three-body decay $D^0 \longrightarrow K^+K^-\pi^0$. The differential CP asymmetry can reach 3.0×10^{-4} when the interference effect is taken into account, while the regional one can be as large as $(1.09 \pm 0.16) \times 10^{-4}$.

Besides, since the chromomagnetic dipole operator O_{8g} is sensitive to some NPs, the inclusion of this kind of NPs will lead to a much larger global CP asymmetries of $D^0 \longrightarrow K^+K^*(892)^-$ and $D^0 \longrightarrow K^-K^*(892)^+$, which are $(0.56 \pm 0.08) \times 10^{-3}$ and $(-0.50 \pm 0.04) \times 10^{-3}$, respectively, while the regional CP asymmetry of $D^0 \longrightarrow K^+K^-\pi^0$ can be also increased to $(1.3 \pm 0.3) \times 10^{-3}$ when considering the interference effect in the phase space. Since the $\mathcal{O}(10^{-3})$ of

Table 4: Three from factor models: the pole, dipole, and CLF models are used for the regional *CP* asymmetries (in unit of 10^{-4}) in the four areas, A, B, C, and D, of the phase space.

Form factors	A_{CP}^{A}	A_{CP}^{B}	A_{CP}^{C}	A_{CP}^{D}	A_{CP}^{All}
Pole	0.87 ± 0.11	0.42 ± 0.08	0.39 ± 0.07	-0.30 ± 0.08	0.33 ± 0.05
Dipole	0.87 ± 0.11	0.41 ± 0.08	0.38 ± 0.07	-0.30 ± 0.08	0.32 ± 0.05
CLF	0.84 ± 0.10	0.45 ± 0.08	0.42 ± 0.07	-0.25 ± 0.08	0.36 ± 0.06



(a)

(b)

Figure 3: The differential *CP* asymmetry distribution of $D^0 \longrightarrow K^+K^-\pi^0$ for (a) $c_{8g}^{\text{eff}} = 1$ and (b) $c_{8g}^{\text{eff}} = 10$, in the overlapped region of $K^*(892)^-$ and $K^*(892)^+$ in the phase space.

CP asymmetry is attributed to the large c_{8g}^{eff}, which is almost impossible for the SM to generate such large contribution, it will indicate NP if such *CP* violation is observed. Here, we roughly estimate the number of $D^0\overline{D}^0$ needed for testing such kind of asymmetries, which is about $(1/Br)(1/A_{CP}^2) \sim 10^9$. This could be observed in the future experiments at Belle II [73, 74], while the current largest $D^0\overline{D}^0$ yields are about 10^8 at BABAR and Belle [75, 76] and 10^7 at BESIII [77].

Appendix

Some Useful Formulas and Input Parameters

(1) Effective Hamiltonian and Wilson Coefficients. The weak effective Hamiltonian for SCS D meson decays, based on the Operator Product Expansion (OPE) and Heavy Quark Effective Theory (HQET), can be expressed as [78]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}}\left[\sum_{q=d,s}\lambda_q\left(c_1 O_1^q + c_2 O_2^q\right) - \lambda_b\left(\sum_{i=3}^{6} c_i O_i + c_{8g}O_{8g}\right)\right] + h.c., \tag{A.1}$$

where G_F is the Fermi constant, $\lambda_q = V_{uq}V_{cq}^*$, c_i $(i = 1, \ldots, 6)$ is the Wilson coefficient, and O_1^q, O_2^q, O_i $(i = 1, \ldots, 6)$, and O_{8g} are four-fermion operators which are constructed from different combinations of quark fields. The four-fermion operators take the following form:

$$\begin{aligned}
O_1^q &= \bar{u}_\alpha\gamma_\mu\left(1-\gamma_5\right)q_\beta\bar{q}_\beta\gamma^\mu\left(1-\gamma_5\right)c_\alpha, \\
O_2^q &= \bar{u}\gamma_\mu\left(1-\gamma_5\right)q\bar{q}\gamma^\mu\left(1-\gamma_5\right)c, \\
O_3 &= \bar{u}\gamma_\mu\left(1-\gamma_5\right)c\sum_{q'}\bar{q}'\gamma^\mu\left(1-\gamma_5\right)q', \\
O_4 &= \bar{u}_\alpha\gamma_\mu\left(1-\gamma_5\right)c_\beta\sum_{q'}\bar{q}'_\beta\gamma^\mu\left(1-\gamma_5\right)q'_\alpha, \\
O_5 &= \bar{u}\gamma_\mu\left(1-\gamma_5\right)c\sum_{q'}\bar{q}'\gamma^\mu\left(1+\gamma_5\right)q', \\
O_6 &= \bar{u}_\alpha\gamma_\mu\left(1-\gamma_5\right)c_\beta\sum_{q'}\bar{q}'_\beta\gamma^\mu\left(1+\gamma_5\right)q'_\alpha, \\
O_{8g} &= -\frac{g_s}{8\pi^2}m_c\bar{u}\sigma_{\mu\nu}\left(1+\gamma_5\right)G^{\mu\nu}c,
\end{aligned} \tag{A.2}$$

where α and β are color indices and $q' = u, d, s$. Among all these operators, O_1^q and O_2^q are tree operators, $O_3 - O_6$ are QCD penguin operators, and O_{8g} is chromomagnetic dipole operator. The electroweak penguin operators are neglected in practice. One should notice that SCS decays receive contributions from all aforementioned operators while only tree operators can contribute to CF decays and DCS decays.

The Wilson coefficients used in this paper are evaluated at $\mu = 1\text{GeV}$, which can be found in [48].

(2) CKM Matrix. We use the Wolfenstein parameterization for the CKM matrix elements, which up to order $\mathcal{O}(\lambda^8)$ read [79, 80]

$$\begin{aligned} V_{us} &= \lambda - \frac{1}{2}A^2\lambda^7\left(\rho^2+\eta^2\right), \\ V_{cs} &= 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4\left(1+4A^2\right) \\ &\quad - \frac{1}{16}\lambda^6\left(1-4A^2+16A^2\left(\rho+i\eta\right)\right) \\ &\quad - \frac{1}{128}\lambda^8\left(5-8A^2+16A^4\right), \\ V_{ub} &= A\lambda^3\left(\rho - i\eta\right), \\ V_{cb} &= A\lambda^2 - \frac{1}{2}A^3\lambda^8\left(\rho^2+\eta^2\right), \end{aligned} \tag{A.3}$$

where $A, \rho, \eta,$ and λ are the Wolfenstein parameters, which satisfy following relation:

$$\rho + i\eta = \frac{\sqrt{1-A^2\lambda^4}\left(\overline{\rho}+i\overline{\eta}\right)}{\sqrt{1-\lambda^2}\left[1-A^2\lambda^4\left(\overline{\rho}+i\overline{\eta}\right)\right]}. \tag{A.4}$$

Numerical values of Wolfenstein parameters which have been used in this work are as follows:

$$\begin{aligned} \lambda &= 0.22548^{+0.00068}_{-0.00034}, \\ A &= 0.810^{+0.018}_{-0.024}, \\ \overline{\rho} &= 0.145^{+0.013}_{-0.007}, \\ \overline{\eta} &= 0.343^{+0.011}_{-0.012}. \end{aligned} \tag{A.5}$$

(3) Decay Constants and Form Factors. In (17) and (18), the pole resonance model was employed for the matrix element $\langle PV|\overline{q}_1q_2|0\rangle$ in the annihilation diagrams. By considering angular momentum conservation at weak vertex and all conservation laws are preserved at strong vertex, the matrix element $\langle PV|\overline{q}_1q_2|0\rangle$ is therefore dominated by a pseudoscalar resonance [54],

$$\begin{aligned} \langle PV\left|\overline{q}_1q_2\right|0\rangle &= \langle PV \mid P^*\rangle\langle P^*\left|\overline{q}_1q_2\right|0\rangle \\ &= g_{P^*PV}\frac{m_{P^*}}{m_D^2-m_{P^*}^2}f_{P^*}, \end{aligned} \tag{A.6}$$

where g_{P^*PV} is a strong coupling constant and m_{P^*} and f_{P^*} are the mass and decay constant of the pseudoscalar resonance P^*. Therefore, η and η' are the dominant resonances for the final states of $K^{*\pm}K^{\mp}$, which can be expressed as flavor mixing of η_q and η_s,

$$\begin{pmatrix}\eta \\ \eta'\end{pmatrix} = \begin{pmatrix}\cos\phi & -\sin\phi \\ \sin\phi & \cos\phi\end{pmatrix}\begin{pmatrix}\eta_q \\ \eta_s\end{pmatrix} \tag{A.7}$$

where ϕ is the mixing angle and η_q and η_s are defined by

$$\begin{aligned} \eta_q &= \frac{1}{\sqrt{2}}\left(u\overline{u}+d\overline{d}\right), \\ \eta_s &= s\overline{s}. \end{aligned} \tag{A.8}$$

The decay constants of η and η' are defined by

$$\begin{aligned} \left\langle 0\left|\overline{u}\gamma_\mu\gamma_5u\right|\eta(p)\right\rangle &= if_\eta^u p_\mu, \\ \left\langle 0\left|\overline{u}\gamma_\mu\gamma_5u\right|\eta'(p)\right\rangle &= if_{\eta'}^u p_\mu, \\ \left\langle 0\left|\overline{d}\gamma_\mu\gamma_5d\right|\eta(p)\right\rangle &= if_\eta^d p_\mu, \\ \left\langle 0\left|\overline{d}\gamma_\mu\gamma_5d\right|\eta'(p)\right\rangle &= if_{\eta'}^d p_\mu, \\ \left\langle 0\left|\overline{s}\gamma_\mu\gamma_5s\right|\eta(p)\right\rangle &= if_\eta^s p_\mu, \\ \left\langle 0\left|\overline{s}\gamma_\mu\gamma_5s\right|\eta'(p)\right\rangle &= if_{\eta'}^s p_\mu, \end{aligned} \tag{A.9}$$

where

$$\begin{aligned} f_\eta^u &= f_\eta^d = \frac{1}{\sqrt{2}}f_\eta^q, \\ f_{\eta'}^u &= f_{\eta'}^d = \frac{1}{\sqrt{2}}f_{\eta'}^q. \end{aligned} \tag{A.10}$$

According to [81, 82], the decay constants of η and η' can be expressed as

$$\begin{aligned} f_\eta^q &= f_q\cos\phi, \\ f_{\eta'}^q &= f_q\sin\phi, \\ f_\eta^s &= -f_s\sin\phi, \\ f_{\eta'}^s &= f_s\cos\phi, \end{aligned} \tag{A.11}$$

where $f_q = (1.07 \pm 0.02)f_\pi$ and $f_s = (1.34 \pm 0.02)f_\pi$ [81], and the mixing angle $\phi = (40.4 \pm 0.6)^\circ$ [83]. Other decay constants used in this paper are listed in Table 5.

The transition form factors $A_0^{D^0\longrightarrow K^{*-}}$ and $F_1^{D^0\longrightarrow K^-}$, based on the relativistic covariant light-front quark model [85], are expressed as a momentum-dependent, 3-parameter form (the parameters can be found in Table 6):

$$F\left(q^2\right) = \frac{F(0)}{1 - a\left(q^2/m_D^2\right) + b\left(q^2/m_D^2\right)^2}. \tag{A.12}$$

(4) Decay Rate. The decay width takes the form

$$\Gamma_{D\longrightarrow KK^*} = \frac{\left|\mathbf{p}_1\right|^3}{8\pi m_{K^*}^2}\left|\frac{\mathscr{M}_{D\longrightarrow KK^*}}{\varepsilon^*\cdot p_D}\right|^2, \tag{A.13}$$

TABLE 5: The meson decay constants used in this paper (MeV) [57, 84].

f_{K^*}	f_ρ	f_K	f_π	f_D
220(5)	216(3)	156(0.4)	130(1.7)	208(10)

TABLE 6: The parameters of $D \longrightarrow K^*, K$ transitions form factors in (A.12).

Form factor	$A_0^{D\longrightarrow K^*}$	$F_1^{D\longrightarrow K}$
$F(0)$	0.69	0.78
a	1.04	1.05
b	0.44	0.23

where $\mathbf{p}_1$ represents the center of mass (c.m.) 3-momentum of each meson in the final state and is given by

$$|\mathbf{p}_1| = \frac{\sqrt{\left[\left(m_D^2-(m_{K^*}+m_K)^2\right)\left(m_D^2-(m_{K^*}-m_K)^2\right)\right]}}{2m_D}. \quad \text{(A.14)}$$

$\mathscr{M}$ is the corresponding decay amplitude.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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