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Time-evolution of fluctuations as signal of the phase transition dynamics in a QCD-assisted transport approach

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Abstract

For the understanding of fluctuation measurements in heavy-ion collisions it is crucial to develop quantitatively reliable dynamical descriptions which take the non-perturbative nature of QCD near the phase transition into account. We discuss a novel QCD-assisted transport approach based on non-equilibrium chiral fluid dynamics and the effective action of low energy QCD. In this framework, we study the time-evolution of fluctuation measures of the critical mode, notably the kurtosis, for a non-expanding system. From this, we can estimate the equilibration times of critical mode fluctuations in the QCD phase diagram. These allow us to identify both the phase boundary and the critical region near the QCD critical point.

Keywords: QCD-assisted transport, time-evolution of fluctuations, equilibration dynamics, critical slowing down

1. Introduction

In the last decades tremendous progress has been made in understanding the phase structure of strongly interacting matter. This has been driven by heavy-ion collisions on the experimental side and by Lattice QCD, functional approaches to QCD, perturbation theory and effective theories on the theoretical side. Most notably the existence of a deconfined phase, i.e. the Quark-Gluon plasma, and its phase transition at vanishing and small net-baryon density are by now well established from both theory and experiment. Despite these efforts, the situation at larger densities is less clear. In order to verify the existence of a possible critical endpoint in the phase diagram of QCD, a more detailed understanding of the connection between the equilibrium phase structure and the highly dynamical non-equilibrium situation created in heavy-ion collisions needs to be established. Only then firm conclusions can unambiguously be drawn from fluctuation measurements.

First attempts in this direction were based on non-equilibrium chiral fluid dynamics studies connected with an effective mean-field model for QCD [1, 2, 3]. A recent approach [4] studies a fully interacting stochastic description of the non-equilibrium evolution of fluctuation observables. In the present work, we explore a novel method which is capable of connecting the equilibrium physics of QCD, obtained beyond mean field within the Functional Renormalization Group (FRG) approach to QCD, with the non-equilibrium evolution around an equilibrium state in a systematic manner. In a first study, we apply this method to the

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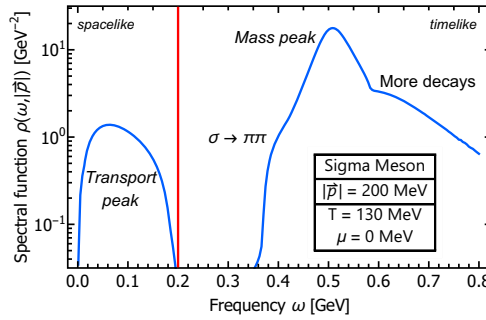


Fig. 1: Spectral function of the sigma meson at $T = 130$ MeV, $\mu = 0$ MeV in the phase diagram. The transport peak and the mass peak are associated with the diffusion in the transport equation. A detailed discussion of the seen structures can be found in e.g. [8].

time-evolution of the critical mode around the various equilibrium states in the phase diagram of a 2+1 flavour Quark-Meson model. This is achieved by solving a transport equation with the corresponding linear response functions of a given equilibrium state as input.

In section 2, we describe the calculation of the necessary equilibrium input, i.e. the effective potential and the spectral functions of the sigma meson. In section 3, the dynamical evolution of the critical mode around this equilibrium result is described.

2. Equilibrium linear response functions

The calculation of equilibrium correlation functions needed as input for the transport evolution utilizes the Functional Renormalization Group. The FRG is a versatile, first principle tool that has been applied successfully to QCD, see e.g. [5], and low-energy effective versions thereof, see e.g. [6, 7]. The advantage of the FRG in the present context is that it allows for the computation of the phase structure, i.e. the effective potential, and momentum dependent correlation functions within a unified framework.

The equilibrium part of our work, i.e. the equation of state and the equilibrium correlation functions, is based on a 2+1 flavour study of a low-energy effective description of QCD, where the dynamics of constituent quarks as well as the lowest scalar- and pseudoscalar meson nonets are taken into account [7]. It captures, by design, the relevant physical effects at small chemical potential μ and temperatures $T \lesssim T_c$. Additionally, it features a critical endpoint which is in the same static universality class as the one potentially present in QCD. Therefore this model provides a well-suited base for studying how dynamical non-equilibrium effects manifest themselves in observables.

In general, spectral functions can be obtained either via analytically continuing numerical data, see e.g. [9] or via a direct computation from analytically continued equations, see e.g. [10, 11]. If possible, the latter is preferred and also the option utilized in this work. The spectral functions of the sigma meson are calculated similarly to [12, 8] with suitable modifications in order to take non-trivial wave-function renormalizations into account. As a result we have access to the two-point correlator $\Gamma_{\sigma\sigma}^{(2)}(\omega, |\vec{p}|)$, depending on an external frequency ω and an external momentum \vec{p} , as well as momentum independent vertices $\Gamma_{\sigma^n}^{(n)}$ which are extracted from the full effective potential computed in [7]. An exemplary spectral function is shown in Figure 1. The two main features that influence the behaviour of the dynamical evolution are the transport peak and the mass peak. The transport peak, if present at small frequencies $\omega < |\vec{p}|$, dominates the long range behaviour of the sigma field. The mass peak, instead, becomes the driving force for the evolution dynamics when the transport peak is absent, e.g. in the vacuum.

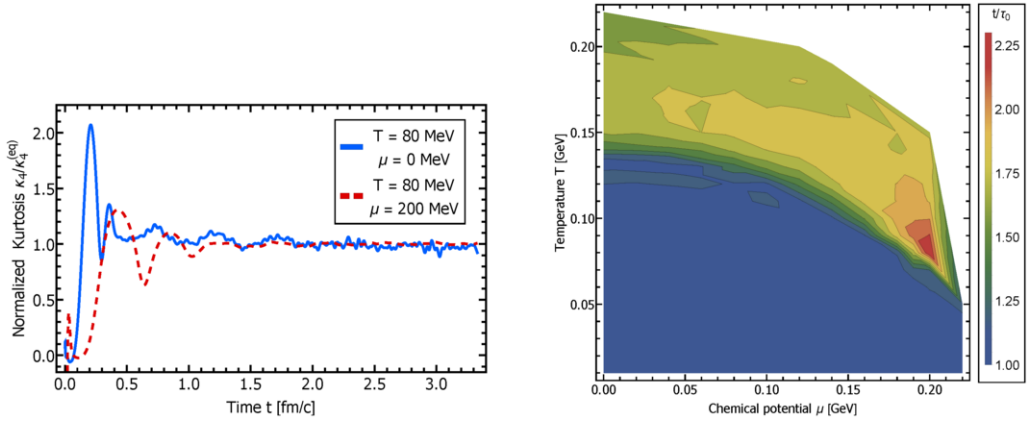


Fig. 2: Left: Scaled kurtosis as a function of time for a quench from high T to two different points in the phase diagram. Within statistical deviations, the equilibration time is found to be significantly increased near the critical endpoint (red, dashed curve) compared to a quench far away from it (blue, solid curve). Right: Equilibration time t in units of $\tau_0 \approx 0.4$ fm/c in the QCD phase diagram based on the analysis of the scaled kurtosis in the quench scenario (see left panel).

3. Time-evolution of fluctuation measures

We are now in the position to study the time-evolution of the critical mode and its event-by-event fluctuations. For this purpose, we solve the Langevin-type transport equation

$$\frac{d\Gamma}{d\sigma} = \xi. \quad (1)$$

In (1), the equation of motion contains a kinetic term related to the real part of $\Gamma_{\sigma\sigma}^{(2)}$, a diffusion term sensitive to the imaginary part of $\Gamma_{\sigma\sigma}^{(2)}$, and the effective potential, discussed in section 2, while ξ represents the noise field chosen such that the fluctuation-dissipation balance is guaranteed.

For the numerical results presented in the following we consider the critical mode to be spatially isotropic, i.e. $\sigma(\vec{x}, t) = \sigma(r, t)$, where we split $\sigma = \sigma_0 + \delta\sigma$. We study the time-evolution of the critical fluctuations for a system subject to a sudden quench from high temperatures to a specific point in the QCD phase diagram. Accordingly, the system is initialized such that $\sigma(r, t = 0) = 0$ and $\partial_r \sigma(r, t = 0) = 0$ which implies that the initial fluctuations $\delta\sigma(t = 0)$ are of the magnitude of the equilibrium value σ_0 after the quench. Moreover, we consider spatially constant Gaussian white noise, with zero mean and a variance given as [1]

$$\langle \xi(t) \xi(t') \rangle = \frac{1}{V} \delta(t - t') m_\sigma \eta \coth\left(\frac{m_\sigma}{2T}\right), \quad (2)$$

where the diffusion coefficient η is extracted from the imaginary part of $\Gamma_{\sigma\sigma}^{(2)}$, such that the fluctuation-dissipation theorem is recovered.

In Figure 2 (left panel), we show, as an example, the time-evolution of the kurtosis scaled by its late-time equilibrium limit for the quench to two different points in the phase diagram. Far away from the critical endpoint the scaled kurtosis exhibits a rather quick equilibration while close to it the corresponding time scale is clearly increased. For the quench through the phase boundary we furthermore observe that the equilibrium value is approached from above as the equilibrium kurtosis is larger near the phase boundary than in the low-temperature phase.

Based on our preliminary results for the scaled kurtosis in the quench scenario, we may estimate the equilibration time of the critical fluctuations within the QCD phase diagram. This is shown in Figure 2 (right panel). One can clearly identify both the phase boundary and the region near the critical endpoint and observe the expected increase of the equilibration time in that region. Nevertheless, we find this increase

to be rather moderate suggesting that phenomena associated with critical slowing down are only moderately pronounced. This hints towards equilibrium dominated measurements and, thus, to the feasibility of studying the QCD phase diagram by means of heavy-ion collisions.

4. Conclusion

In this work we developed a novel method to study dynamical non-equilibrium effects in the phase diagram of the 2+1 flavour Quark-Meson model. This method is valid not only in the scaling region but across the entire region of the phase transition in QCD because it contains critical and non-critical contributions to fluctuation observables in a single framework. It builds upon the dynamics of a quantum field around an equilibrium state with an appropriate Langevin-type transport equation. We study the time-evolution of the critical mode from a quench at high temperatures to different points within the phase diagram. The time-evolution of different cumulants of the sigma field was calculated, and an equilibration time was extracted from the kurtosis. We find a moderate increase of the equilibration time near the phase boundary and when approaching the critical point. A region of critical slowing down is clearly identifiable, but even close to the critical point only an enhancement of roughly a factor of two is found.

In order to connect even closer to the full dynamics of a heavy-ion collision, the scenario of a temperature quench will be subsequently improved towards the full dynamics of the underlying quantum field theory.

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References

- [1] M. Nahrgang, S. Leupold, C. Herold, M. Bleicher, Nonequilibrium chiral fluid dynamics including dissipation and noise, *Phys. Rev. C* 84 (2011) 024912. [arXiv:1105.0622](#), doi:10.1103/PhysRevC.84.024912.
- [2] M. Nahrgang, S. Leupold, M. Bleicher, Equilibration and relaxation times at the chiral phase transition including reheating, *Phys. Lett. B* 711 (2012) 109–116. [arXiv:1105.1396](#), doi:10.1016/j.physletb.2012.03.059.
- [3] C. Herold, M. Nahrgang, Y. Yan, C. Kobdaj, Dynamical net-proton fluctuations near a QCD critical point, *Phys. Rev. C* 93 (2) (2016) 021902. [arXiv:1601.04839](#), doi:10.1103/PhysRevC.93.021902.
- [4] M. Nahrgang, M. Bluhm, T. Schäfer, S. A. Bass, Diffusive dynamics of critical fluctuations near the QCD critical point [arXiv:1804.05728](#).
- [5] A. K. Cyrol, M. Mitter, J. M. Pawłowski, N. Strodthoff, Nonperturbative quark, gluon, and meson correlators of unquenched QCD, *Phys. Rev. D* 97 (5) (2018) 054006. [arXiv:1706.06326](#), doi:10.1103/PhysRevD.97.054006.
- [6] T. K. Herbst, J. M. Pawłowski, B.-J. Schaefer, The phase structure of the Polyakov–quark–meson model beyond mean field, *Phys. Lett. B* 696 (2011) 58–67. [arXiv:1008.0081](#), doi:10.1016/j.physletb.2010.12.003.
- [7] F. Rennecke, B.-J. Schaefer, Fluctuation-induced modifications of the phase structure in (2+1)-flavor QCD, *Phys. Rev. D* 96 (1) (2017) 016009. [arXiv:1610.08748](#), doi:10.1103/PhysRevD.96.016009.
- [8] J. M. Pawłowski, N. Strodthoff, N. Wink, Finite temperature spectral functions in the $O(N)$ -model [arXiv:1711.07444](#).
- [9] A. K. Cyrol, J. M. Pawłowski, A. Rothkopf, N. Wink, Reconstructing the gluon [arXiv:1804.00945](#).
- [10] S. Floerchinger, Analytic Continuation of Functional Renormalization Group Equations, *JHEP* 05 (2012) 021. [arXiv:1112.4374](#), doi:10.1007/JHEP05(2012)021.
- [11] K. Kamikado, N. Strodthoff, L. von Smekal, J. Wambach, Real-time correlation functions in the $O(N)$ model from the functional renormalization group, *Eur. Phys. J. C* 74 (3) (2014) 2806. [arXiv:1302.6199](#), doi:10.1140/epjc/s10052-014-2806-6.
- [12] R.-A. Tripolt, N. Strodthoff, L. von Smekal, J. Wambach, Spectral Functions for the Quark-Meson Model Phase Diagram from the Functional Renormalization Group, *Phys. Rev. D* 89 (3) (2014) 034010. [arXiv:1311.0630](#), doi:10.1103/PhysRevD.89.034010.