



Corrigendum

Corrigendum to “Measurement of the $e^+e^- \rightarrow \pi^+\pi^-$ cross section between 600 and 900 MeV using initial state radiation” [Phys. Lett. B 753 (2016) 629–638]

BESIII Collaboration

M. Ablikim^a, M.N. Achasov^{j,3}, P. Adlarson^{bv}, S. Ahmed^o, M. Albrecht^d, R. Aliberti^{ad}, A. Amoroso^{bu,z}, Q. An^{br,ay,az}, X.H. Bai^{bk}, Y. Bai^{ax}, O. Bakina^{ae}, R. Baldini Ferroli^w, I. Balossino^x, Y. Ban^{an,11}, K. Begzsuren^{ab}, N. Berger^{ad}, M. Bertani^w, D. Bettoni^x, F. Bianchi^{bu,z}, J. Biernat^{bv}, J. Bloms^{bn}, A. Bortone^{bu,z}, I. Boyko^{ae}, R.A. Briere^e, H. Cai^{bw}, X. Cai^{a,ay,az}, A. Calcaterra^w, G.F. Cao^{a,bf}, N. Cao^{a,bf}, S.A. Cetin^{be}, J.F. Chang^{a,ay,az}, W.L. Chang^{a,bf}, G. Chelkov^{ae,2}, D.Y. Chen^f, G. Chen^a, H.S. Chen^{a,bf}, M.L. Chen^{a,ay,az}, S.J. Chen^{ak}, X.R. Chen^{aa}, Y.B. Chen^{a,ay,az}, Z.J. Chen^{t,12}, W.S. Cheng^z, G. Cibinetto^x, F. Cossio^z, X.F. Cui^{al}, H.L. Dai^{a,ay,az}, X.C. Dai^{a,bf}, A. Dbeyssi^o, R.E. de Boer^d, D. Dedovich^{ae}, Z.Y. Deng^a, A. Denig^{ad}, I. Denysenko^{ae}, M. Destefanis^{bu,z}, F. De Mori^{bu,z}, Y. Ding^{ai}, C. Dong^{al}, J. Dong^{a,ay,az}, L.Y. Dong^{a,bf}, M.Y. Dong^{a,ay,az,bf}, X. Dong^{bw}, S.X. Du^{bz}, J. Fang^{a,ay,az}, S.S. Fang^{a,bf}, Y. Fang^a, R. Farinelli^x, L. Fava^{bg,z}, F. Feldbauer^d, G. Felici^w, C.Q. Feng^{br,ay,az}, M. Fritsch^d, C.D. Fu^a, Y. Gao^{bs}, Y. Gao^{br,ay,az}, Y. Gao^{an,11}, Y.G. Gao^f, I. Garzia^{x,bh}, E.M. Gersabeck^{bl}, A. Gilman^{bm}, K. Goetzen^k, L. Gong^{ai}, W.X. Gong^{a,ay,az}, W. Gradl^{ad}, M. Greco^{bu,z}, L.M. Gu^{ak}, M.H. Gu^{a,ay,az}, S. Gu^b, Y.T. Gu^m, C.Y. Guan^{a,bf}, A.Q. Guo^v, L.B. Guo^{aj}, R.P. Guo^{ap}, Y.P. Guo^{i,8}, A. Guskov^{ae}, T.T. Han^{aq}, X.Q. Hao^p, F.A. Harris^{bj}, K.L. He^{a,bf}, F.H. Heinsius^d, C.H. Heinz^{ad}, T. Held^d, Y.K. Heng^{a,ay,az,bf}, C. Herold^{bb}, M. Himmelreich^{k,6}, T. Holtmann^d, Y.R. Hou^{bf}, Z.L. Hou^a, H.M. Hu^{a,bf}, J.F. Hu^{aw,13}, T. Hu^{a,ay,az,bf}, Y. Hu^a, G.S. Huang^{br,ay,az}, L.Q. Huang^{bs}, X.T. Huang^{aq}, Y.P. Huang^a, Z. Huang^{an,11}, N. Huesken^{bn}, T. Hussain^{bt}, W. Ikegami Andersson^{bv}, W. Imoehl^v, M. Irshad^{br,ay,az}, S. Jaeger^d, S. Janchiv^{ab,10}, Q. Ji^a, Q.P. Ji^p, X.B. Ji^{a,bf}, X.L. Ji^{a,ay,az}, H.B. Jiang^{aq}, X.S. Jiang^{a,ay,az,bf}, J.B. Jiao^{aq}, Z. Jiao^r, S. Jin^{ak}, Y. Jin^{bk},

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E-mail address: redmer@uni-mainz.de (C.F. Redmer).

¹ Also at Bogazici University, 34342 Istanbul, Turkey.

² Also at the Moscow Institute of Physics and Technology, Moscow 141700, Russia.

³ Also at the Novosibirsk State University, Novosibirsk, 630090, Russia.

⁴ Also at the NRC “Kurchatov Institute”, PNPI, 188300, Gatchina, Russia.

⁵ Also at Istanbul Arel University, 34295 Istanbul, Turkey.

⁶ Also at Goethe University Frankfurt, 60323 Frankfurt am Main, Germany.

⁷ Also at Key Laboratory for Particle Physics, Astrophysics and Cosmology, Ministry of Education; Shanghai Key Laboratory for Particle Physics and Cosmology; Institute of Nuclear and Particle Physics, Shanghai 200240, People's Republic of China.

⁸ Also at Key Laboratory of Nuclear Physics and Ion-beam Application (MOE) and Institute of Modern Physics, Fudan University, Shanghai 200443, People's Republic of China.

⁹ Also at Harvard University, Department of Physics, Cambridge, MA, 02138, USA.

¹⁰ Currently at: Institute of Physics and Technology, Peace Ave.54B, Ulaanbaatar 13330, Mongolia.

¹¹ Also at State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, People's Republic of China.

¹² School of Physics and Electronics, Hunan University, Changsha 410082, China.

¹³ Also at Guangdong Provincial Key Laboratory of Nuclear Science, Institute of Quantum Matter, South China Normal University, Guangzhou 510006, China.

T. Johansson^{bv}, N. Kalantar-Nayestanaki^{bi}, X.S. Kang^{ai}, R. Kappert^{bi}, M. Kavatsyuk^{bi}, B.C. Ke^{as,a}, I.K. Keshk^d, A. Khoukaz^{bn}, P. Kiese^{ad}, R. Kiuchi^a, R. Kliemt^k, L. Koch^{af}, O.B. Kolcu^{be,5}, B. Kopf^d, M. Kuemmel^d, M. Kuessner^d, A. Kupsc^{bv}, M.G. Kurth^{a,bf}, W. Kühn^{af}, J.J. Lane^{bl}, J.S. Lange^{af}, P. Larin^o, A. Lavanina^u, L. Lavezzi^{bu,z}, Z.H. Lei^{br,ay,az}, H. Leithoff^{ad}, M. Lellmann^{ad}, T. Lenz^{ad}, C. Li^{ao}, C.H. Li^{ah}, Cheng Li^{br,ay,az}, D.M. Li^{bz}, F. Li^{a,ay,az}, G. Li^a, H. Li^{as}, H. Li^{br,ay,az}, H.B. Li^{a,bf}, H.J. Li^{i,8}, J.L. Li^{aq}, J.Q. Li^d, Ke Li^a, L.K. Li^a, Lei Li^c, P.L. Li^{br,ay,az}, P.R. Li^{ag}, S.Y. Li^{bc}, W.D. Li^{a,bf}, W.G. Li^a, X.H. Li^{br,ay,az}, X.L. Li^{aq}, Z.Y. Li^{ba}, H. Liang^{br,ay,az}, H. Liang^{a,bf}, H. Liang^{ac}, Y.F. Liang^{au}, Y.T. Liang^{aa}, G.R. Liao^l, L.Z. Liao^{a,bf}, J. Libby^u, C.X. Lin^{ba}, B.J. Liu^a, C.X. Liu^a, D. Liu^{br,ay,az}, F.H. Liu^{at}, Fang Liu^a, Feng Liu^f, H.B. Liu^m, H.M. Liu^{a,bf}, Huanhuan Liu^a, Huihui Liu^q, J.B. Liu^{br,ay,az}, J.Y. Liu^{a,bf}, K. Liu^a, K.Y. Liu^{ai}, Ke Liu^f, L. Liu^{br,ay,az}, M.H. Liu^{i,8}, Q. Liu^{bf}, S.B. Liu^{br,ay,az}, Shuai Liu^{av}, T. Liu^{a,bf}, W.M. Liu^{br,ay,az}, X. Liu^{ag}, Y.B. Liu^{al}, Z.A. Liu^{a,ay,az,bf}, Z.Q. Liu^{aq}, X.C. Lou^{a,ay,az,bf}, F.X. Lu^p, H.J. Lu^r, J.D. Lu^{a,bf}, J.G. Lu^{a,ay,az}, X.L. Lu^a, Y. Lu^a, Y.P. Lu^{a,ay,az}, C.L. Luo^{aj}, M.X. Luo^{by}, P.W. Luo^{ba}, T. Luo^{i,8}, X.L. Luo^{a,ay,az}, S. Lusso^z, X.R. Lyu^{bf}, F.C. Ma^{ai}, H.L. Ma^a, L.L. Ma^{aq}, M.M. Ma^{a,bf}, Q.M. Ma^a, R.Q. Ma^{a,bf}, R.T. Ma^{bf}, X.X. Ma^{a,bf}, X.Y. Ma^{a,ay,az}, F.E. Maas^o, M. Maggiora^{bu,z}, S. Maldaner^d, S. Malde^{bp}, Q.A. Malik^{bt}, A. Mangoni^y, Y.J. Mao^{an,11}, Z.P. Mao^a, S. Marcello^{bu,z}, Z.X. Meng^{bk}, J.G. Messchendorp^{bi}, G. Mezzadri^x, T.J. Min^{ak}, R.E. Mitchell^v, X.H. Mo^{a,ay,az,bf}, Y.J. Mo^f, N.Yu. Muchnoi^{j,3}, H. Muramatsu^{bm}, S. Nakhoul^{k,6}, Y. Nefedov^{ae}, F. Nerling^{k,6}, I.B. Nikolaev^{j,3}, Z. Ning^{a,ay,az}, S. Nisar^{h,9}, S.L. Olsen^{bf}, Q. Ouyang^{a,ay,az,bf}, S. Pacetti^{y,bo}, X. Pan^{i,8}, Y. Pan^{bl}, A. Pathak^a, P. Patteri^w, M. Pelizaeus^d, H.P. Peng^{br,ay,az}, K. Peters^{k,6}, J. Pettersson^{bv}, J.L. Ping^{aj}, R.G. Ping^{a,bf}, A. Pitka^d, R. Poling^{bm}, V. Prasad^{br,ay,az}, H. Qi^{br,ay,az}, H.R. Qi^{bc}, K.H. Qi^{aa}, M. Qi^{ak}, T.Y. Qiⁱ, T.Y. Qi^b, S. Qian^{a,ay,az}, W.-B. Qian^{bf}, Z. Qian^{ba}, C.F. Qiao^{bf}, L.Q. Qin^l, X.S. Qin^d, Z.H. Qin^{a,ay,az}, J.F. Qiu^a, S.Q. Qu^{al}, K. Ravindran^u, C.F. Redmer^{ad}, A. Rivetti^z, V. Rodin^{bi}, M. Rolo^z, G. Rong^{a,bf}, Ch. Rosner^o, M. Rump^{bn}, H.S. Sang^{br}, A. Sarantsev^{ae,4}, Y. Schelhaas^{ad}, C. Schnier^d, K. Schoenning^{bv}, M. Scodeggio^{x,bh}, D.C. Shan^{av}, W. Shan^s, X.Y. Shan^{br,ay,az}, M. Shao^{br,ay,az}, C.P. Shenⁱ, P.X. Shen^{al}, X.Y. Shen^{a,bf}, H.C. Shi^{br,ay,az}, R.S. Shi^{a,bf}, X. Shi^{a,ay,az}, X.D. Shi^{br,ay,az}, J.J. Song^{aq}, W.M. Song^{ac,a}, Y.X. Song^{an,11}, S. Sosio^{bu,z}, S. Spataro^{bu,z}, K.X. Su^{bw}, F.F. Sui^{aq}, G.X. Sun^a, H.K. Sun^a, J.F. Sun^p, L. Sun^{bw}, S.S. Sun^{a,bf}, T. Sun^{a,bf}, W.Y. Sun^{aj}, X. Sun^{t,12}, Y.J. Sun^{br,ay,az}, Y.K. Sun^{br,ay,az}, Y.Z. Sun^a, Z.T. Sun^a, Y.H. Tan^{bw}, Y.X. Tan^{br,ay,az}, C.J. Tang^{au}, G.Y. Tang^a, J. Tang^{ba}, J.X. Teng^{br,ay,az}, V. Thoren^{bv}, I. Uman^{be}, B. Wang^a, C.W. Wang^{ak}, D.Y. Wang^{an,11}, H.P. Wang^{a,bf}, K. Wang^{a,ay,az}, L.L. Wang^a, M. Wang^{aq}, M.Z. Wang^{an,11}, Meng Wang^{a,bf}, W.H. Wang^{bw}, W.P. Wang^{br,ay,az}, X. Wang^{an,11}, X.F. Wang^{ag}, X.L. Wang^{i,8}, Y. Wang^{ba}, Y. Wang^{br,ay,az}, Y.D. Wang^{am}, Y.F. Wang^{a,ay,az,bf}, Y.Q. Wang^a, Z. Wang^{a,ay,az}, Z.Y. Wang^a, Ziyi Wang^{bf}, Zongyuan Wang^{a,bf}, D.H. Wei^l, P. Weidenkaff^{ad}, F. Weidner^{bn}, S.P. Wen^a, D.J. White^{bl}, U. Wiedner^d, G. Wilkinson^{bp}, M. Wolke^{bv}, L. Wollenberg^d, J.F. Wu^{a,bf}, L.H. Wu^a, L.J. Wu^{a,bf}, X. Wu^{i,8}, Z. Wu^{a,ay,az}, L. Xia^{br,ay,az}, H. Xiao^{i,8}, S.Y. Xiao^a, Z.J. Xiao^{aj}, X.H. Xie^{an,11}, Y.G. Xie^{a,ay,az}, Y.H. Xie^f, T.Y. Xing^{a,bf}, G.F. Xu^a, J.J. Xu^{ak}, Q.J. Xuⁿ, W. Xu^{a,bf}, X.P. Xu^{av}, Y.C. Xu^{bf}, F. Yan^{i,8}, L. Yan^{i,8}, L. Yan^{bu,z,8}, W.B. Yan^{br,ay,az}, W.C. Yan^{bz}, Xu Yan^{av}, H.J. Yang^{ar,7}, H.X. Yang^a, L. Yang^{as}, S.L. Yang^{bf}, Y.H. Yang^{ak}, Y.X. Yang^l, Yifan Yang^{a,bf}, Zhi Yang^{aa}, M. Ye^{a,ay,az}, M.H. Ye^g, J.H. Yin^a, Z.Y. You^{ba}, B.X. Yu^{a,ay,az,bf}, C.X. Yu^{al}, G. Yu^{a,bf}, J.S. Yu^{t,12}, T. Yu^{bs}, C.Z. Yuan^{a,bf}, L. Yuan^b, W. Yuan^{bu,z}, X.Q. Yuan^{an,11}, Y. Yuan^a, Z.Y. Yuan^{ba}, C.X. Yue^{ah}, A. Yuncu^{be,1}, A.A. Zafar^{bt}, Y. Zeng^{t,12}, B.X. Zhang^a, Guangyi Zhang^p, H. Zhang^{br}, H.H. Zhang^{ba}, H.H. Zhang^{ac}, H.Y. Zhang^{a,ay,az}, J.J. Zhang^{as}, J.L. Zhang^{bx}, J.Q. Zhang^{aj}, J.W. Zhang^{a,ay,az,bf}, J.Y. Zhang^a, J.Z. Zhang^{a,bf}, Jianyu Zhang^{a,bf}, Jiawei Zhang^{a,bf}, Lei Zhang^{ak}, S. Zhang^{ba}, S.F. Zhang^{ak}, Shulei Zhang^{t,12}, X.D. Zhang^{am}, X.Y. Zhang^{aq}, Y. Zhang^{bp}, Y.H. Zhang^{a,ay,az}, Y.T. Zhang^{br,ay,az}, Yan Zhang^{br,ay,az}, Yao Zhang^a, Yi Zhang^{i,8}, Z.H. Zhang^f, Z.Y. Zhang^{bw}, G. Zhao^a, J. Zhao^{ah}, J.Y. Zhao^{a,bf}, J.Z. Zhao^{a,ay,az}, Lei Zhao^{br,ay,az}, Ling Zhao^a, M.G. Zhao^{al}, Q. Zhao^a, S.J. Zhao^{bz}, Y.B. Zhao^{a,ay,az}, Y.X. Zhao^{aa}, Z.G. Zhao^{br,ay,az}, A. Zhemchugov^{ae,2}, B. Zheng^{bs}, J.P. Zheng^{a,ay,az}, Y. Zheng^{an,11}, Y.H. Zheng^{bf}, B. Zhong^{aj}, C. Zhong^{bs}, L.P. Zhou^{a,bf}, Q. Zhou^{a,bf}, X. Zhou^{bw}, X.K. Zhou^{bf}, X.R. Zhou^{br,ay,az}, A.N. Zhu^{a,bf}, J. Zhu^{al}, K. Zhu^a,

K.J. Zhu ^{a,ay,az,bf}, S.H. Zhu ^{bq}, T.J. Zhu ^{bx}, W.J. Zhu ^{al}, Y.C. Zhu ^{br,ay,az}, Z.A. Zhu ^{a,bf}, B.S. Zou ^a, J.H. Zou ^a

^a Institute of High Energy Physics, Beijing 100049, People's Republic of China

^b Beihang University, Beijing 100191, People's Republic of China

^c Beijing Institute of Petrochemical Technology, Beijing 102617, People's Republic of China

^d Bochum Ruhr-University, D-44780 Bochum, Germany

^e Carnegie Mellon University, Pittsburgh, PA 15213, USA

^f Central China Normal University, Wuhan 430079, People's Republic of China

^g China Center of Advanced Science and Technology, Beijing 100190, People's Republic of China

^h COMSATS University Islamabad, Lahore Campus, Defence Road, Off Raiwind Road, 54000 Lahore, Pakistan

ⁱ Fudan University, Shanghai 200443, People's Republic of China

^j G.I. Budker Institute of Nuclear Physics SB RAS (BINP), Novosibirsk 630090, Russia

^k GSI Helmholtzcentre for Heavy Ion Research GmbH, D-64291 Darmstadt, Germany

^l Guangxi Normal University, Guilin 541004, People's Republic of China

^m Guangxi University, Nanning 530004, People's Republic of China

ⁿ Hangzhou Normal University, Hangzhou 310036, People's Republic of China

^o Helmholtz Institute Mainz, Johann-Joachim-Becher-Weg 45, D-55099 Mainz, Germany

^p Henan Normal University, Xinxiang 453007, People's Republic of China

^q Henan University of Science and Technology, Luoyang 471003, People's Republic of China

^r Huangshan College, Huangshan 245000, People's Republic of China

^s Hunan Normal University, Changsha 410081, People's Republic of China

^t Hunan University, Changsha 410082, People's Republic of China

^u Indian Institute of Technology Madras, Chennai 600036, India

^v Indiana University, Bloomington, IN 47405, USA

^w INFN Laboratori Nazionali di Frascati, I-00044, Frascati, Italy

^x INFN Sezione di Ferrara, I-44122, Ferrara, Italy

^y INFN Sezione di Perugia, I-06100, Perugia, Italy

^z INFN Sezione di Torino, I-10125, Turin, Italy

^{aa} Institute of Modern Physics, Lanzhou 730000, People's Republic of China

^{ab} Institute of Physics and Technology, Peace Ave. 54B, Ulaanbaatar 13330, Mongolia

^{ac} Jilin University, Changchun 130012, People's Republic of China

^{ad} Johannes Gutenberg University of Mainz, Johann-Joachim-Becher-Weg 45, D-55099 Mainz, Germany

^{ae} Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia

^{af} Justus-Liebig-Universitaet Giessen, II. Physikalisches Institut, Heinrich-Buff-Ring 16, D-35392 Giessen, Germany

^{ag} Lanzhou University, Lanzhou 730000, People's Republic of China

^{ah} Liaoning Normal University, Dalian 116029, People's Republic of China

^{ai} Liaoning University, Shenyang 110036, People's Republic of China

^{aj} Nanjing Normal University, Nanjing 210023, People's Republic of China

^{ak} Nanjing University, Nanjing 210093, People's Republic of China

^{al} Nankai University, Tianjin 300071, People's Republic of China

^{am} North China Electric Power University, Beijing 102206, People's Republic of China

^{an} Peking University, Beijing 100871, People's Republic of China

^{ao} Qufu Normal University, Qufu 273165, People's Republic of China

^{ap} Shandong Normal University, Jinan 250014, People's Republic of China

^{aq} Shandong University, Jinan 250100, People's Republic of China

^{ar} Shanghai Jiao Tong University, Shanghai 200240, People's Republic of China

^{as} Shanxi Normal University, Linfen 041004, People's Republic of China

^{at} Shanxi University, Taiyuan 030006, People's Republic of China

^{au} Sichuan University, Chengdu 610064, People's Republic of China

^{av} Soochow University, Suzhou 215006, People's Republic of China

^{aw} South China Normal University, Guangzhou 510006, People's Republic of China

^{ax} Southeast University, Nanjing 211100, People's Republic of China

^{ay} State Key Laboratory of Particle Detection and Electronics, Beijing 100049, People's Republic of China

^{az} State Key Laboratory of Particle Detection and Electronics, Hefei 230026, People's Republic of China

^{ba} Sun Yat-Sen University, Guangzhou 510275, People's Republic of China

^{bb} Suranaree University of Technology, University Avenue 111, Nakhon Ratchasima 30000, Thailand

^{bc} Tsinghua University, Beijing 100084, People's Republic of China

^{bd} Turkish Accelerator Center Particle Factory Group, Istanbul Bilgi University, 34060 Eyup, Istanbul, Turkey

^{be} Turkish Accelerator Center Particle Factory Group, Near East University, Nicosia, North Cyprus, Mersin 10, Turkey

^{bf} University of Chinese Academy of Sciences, Beijing 100049, People's Republic of China

^{bg} University of Eastern Piedmont, I-15121, Alessandria, Italy

^{bh} University of Ferrara, I-44122, Ferrara, Italy

^{bi} University of Groningen, NL-9747 AA Groningen, the Netherlands

^{bj} University of Hawaii, Honolulu, HI 96822, USA

^{bk} University of Jinan, Jinan 250022, People's Republic of China

^{bl} University of Manchester, Oxford Road, Manchester, M13 9PL, United Kingdom

^{bm} University of Minnesota, Minneapolis, MN 55455, USA

^{bn} University of Muenster, Wilhelm-Klemm-Str. 9, 48149 Muenster, Germany

^{bo} University of Perugia, I-06100, Perugia, Italy

^{bp} University of Oxford, Keble Rd, Oxford, OX13RH, United Kingdom

^{bq} University of Science and Technology Liaoning, Anshan 114051, People's Republic of China

^{br} University of Science and Technology of China, Hefei 230026, People's Republic of China

^{bs} University of South China, Hengyang 421001, People's Republic of China

^{bt} University of the Punjab, Lahore-54590, Pakistan

^{bu} University of Turin, INFN, I-10125, Turin, Italy

^{bv} Uppsala University, Box 516, SE-75120 Uppsala, Sweden

^{bw} Wuhan University, Wuhan 430072, People's Republic of China

^{bx} Xinyang Normal University, Xinyang 464000, People's Republic of China

^{by} Zhejiang University, Hangzhou 310027, People's Republic of China
^{bz} Zhengzhou University, Zhengzhou 450001, People's Republic of China

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ABSTRACT

In Ref. [1] the BESIII collaboration published a cross section measurement of the process $e^+e^- \rightarrow \pi^+\pi^-$ in the energy range between 600 and 900 MeV. In this corrigendum, we report a corrected evaluation of the statistical errors in terms of a fully propagated covariance matrix. The correction also yields a reduced statistical uncertainty for the hadronic vacuum polarization contribution to the anomalous magnetic moment of the muon, which now reads as $a_\mu^{\pi\pi,LO}(600-900 \text{ MeV}) = (368.2 \pm 1.5_{\text{stat}} \pm 3.3_{\text{syst}}) \times 10^{-10}$. The central values of the cross section measurement and of $a_\mu^{\pi\pi,LO}$, as well as the systematic uncertainties remain unchanged.

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1. Introduction

Previously, we reported [1] a measurement of the cross section $\sigma^{\text{bare}}(e^+e^- \rightarrow \pi^+\pi^-)$ and the pion form factor $|F_\pi|^2$ in the energy range between 600 MeV and 900 MeV. As pointed out in Refs. [2] and [3], there exists a difference between the statistical uncertainties of the tabulated cross section of Ref. [1] and the covariance matrix, which is documented as a supplemental material to the publication. Furthermore, when including the covariance matrix, it is not possible to reproduce the fit of the form factor presented in Ref. [1].

In scrutinizing the published analysis, we realized that the covariance matrix had been provided at the level of the event yield, not, as claimed, at the level of the cross section. The same matrix is erroneously used in the calculation of $a_\mu^{\pi\pi,LO}$. At the same time, the statistical errors in Tab. 4 of Ref. [1] are taken from the event yield prior to unfolding and are propagated to the level of the cross section and form factor, respectively, producing two different statistical uncertainties for the results.

In this work, the statistical uncertainties are reevaluated and an updated value of the uncertainty of the two-pion contribution to the hadronic vacuum polarization contribution of the anomalous magnetic moment of the muon, $a_\mu^{\pi\pi,LO}$, is calculated.

2. Reevaluation of the statistical covariance matrix

The covariance matrix results from the unfolding procedure, which is applied at the level of the event yield to compensate for mass resolution effects of the detector. The underlying algorithm of the procedure is based on singular value decomposition [4]. The covariance matrix needs to be propagated according to generalized Gaussian error propagation to correctly reflect the statistical correlations of the cross section and the form factor, respectively.

In an initial state radiation (ISR) measurement, the dressed cross section $\sigma^{\text{dressed}}(e^+e^- \rightarrow \pi^+\pi^-)$ is calculated from the unfolded event yield N_{unf} of $\pi^+\pi^- \gamma_{\text{ISR}}$ events according to

$$\sigma^{\text{dressed}}(e^+e^- \rightarrow \pi^+\pi^-) = \frac{N_{\text{unf}}}{\varepsilon_{\pi\pi}^{\text{global}} \cdot \mathcal{L}_{\text{int}} \cdot H(s, s') \cdot (1 + \delta_{\text{FSR}}^{\pi\pi\gamma})}, \quad (1)$$

where $\varepsilon_{\pi\pi}^{\text{global}}$ is the reconstruction efficiency, $\mathcal{L}_{\text{int}}$ is the integrated luminosity, and $H(s, s')$ is the radiator function, where the implementation of Ref. [5] is considered. The correction $(1 + \delta_{\text{FSR}}^{\pi\pi\gamma})$

denotes the final state radiation (FSR) corrections on the level of radiative $\pi^+\pi^-\gamma$ events.¹⁴

The bare cross section is obtained from the dressed cross section by applying mass-dependent corrections for vacuum polarization δ_{VP} [6] and by adding back effects of FSR on the level of the non-radiative $\pi^+\pi^-$ cross sections as parametrized within scalar QED in the Schwinger term $1 + \eta(s') \frac{\alpha}{\pi}$ [7]. The final formula for the bare cross section reads as:

$$\sigma^{\text{bare}}(e^+e^- \rightarrow \pi^+\pi^- (\gamma_{\text{FSR}})) = \sigma^{\text{dressed}}(e^+e^- \rightarrow \pi^+\pi^-) \frac{1 + \eta(s') \frac{\alpha}{\pi}}{\delta_{\text{VP}}(s')}, \quad (2)$$

where s' denotes the two-pion invariant mass squared.

Since all the above mentioned values remain unchanged compared to the original work [1], the central value of the cross section does not change.

The covariance matrix obtained from the unfolding procedure is propagated taking into account Eqs. (1) and (2) to the level of the bare cross section. It is, assuming no correlations between the contributing quantities, thus, given by

$$C^{\sigma^{\text{bare}}} = \sum_{k \in \{N, \varepsilon, \mathcal{L}_{\text{int}}, H, (1 + \delta_{\text{FSR}}^{\pi\pi\gamma})\}} (J^T)^k C^k J^k, \quad (3)$$

with $J_{ij}^k = \frac{\partial \sigma_i^{\text{bare}}}{\partial k_j}$ being the Jacobian matrix of the bare cross section with respect to the contribution k to the statistical uncertainty, according to generalized Gaussian error propagation.

Since the time integrated luminosity is a single scalar value, its covariance matrix is simply given by the squared statistical uncertainty of the time integrated luminosity: $C^{\mathcal{L}_{\text{int}}} = (\Delta \mathcal{L}_{\text{int}})^2$.

It is assumed that the reconstruction efficiency, the time integrated luminosity, the radiator function, as well as the final state radiation correction term are completely uncorrelated. The respective diagonal elements of the covariance matrices are given by the square of the uncertainties. The contribution of the Schwinger correction term is neglected, since as a QED calculation, it is assumed to be exact. In the original work, the uncertainty of the vacuum polarization effect is considered to be purely systematic. Hence, it

¹⁴ In Eq. (1) of Ref. [1], the factor $(1 + \delta_{\text{FSR}}^{\pi\pi\gamma})$ should be read as $\left[\frac{1 + \eta(s) \frac{\alpha}{\pi}}{1 + \delta_{\text{FSR}}^{\pi\pi\gamma}} \right]$, contrary to the description in Section 6.3 therein.

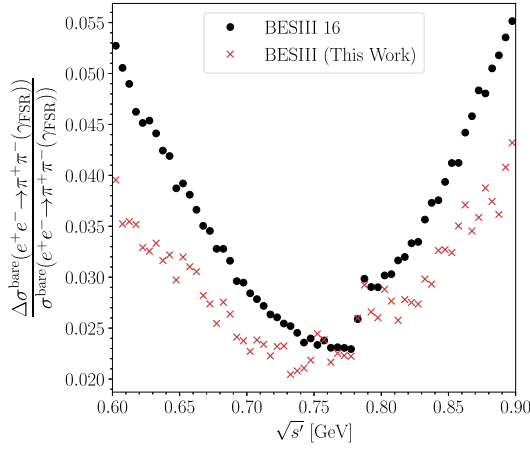


Fig. 1. Relative uncertainty of the bare cross section $\sigma^{\text{bare}}(e^+e^- \rightarrow \pi^+\pi^-(\gamma_{\text{FSR}}))$ of this work (red crosses) compared to the results of Ref. [1] (black circles). The uncertainties of the cross section of this work are the square roots of the diagonal elements of the matrix.

is also neglected in the calculation of the statistical covariance matrix.

In the original publication, the error propagation of the covariance matrix had not been carried out properly. As a result, the statistical uncertainties of the published cross section do not reflect the information of the unfolding. Fig. 1 shows a comparison of the relative statistical errors of the bare cross sections calculated as the diagonal uncertainties of this work (red crosses) and the uncertainties published in Ref. [1] (black circles). The values of the diagonal errors are listed in Table 1.

It must be stressed that only the statistical uncertainties of the measurements of $\sigma^{\text{bare}}(e^+e^- \rightarrow \pi^+\pi^-(\gamma_{\text{FSR}}))$ and of $|F_\pi|^2$ have been reevaluated. Thus, the systematic uncertainty of 0.9% evaluated in Ref. [1] is unchanged.

The BESIII collaboration has approved new data taking at 3.773 GeV in 2021-2022, aiming at a total data set of 20 fb⁻¹ [8]. In addition to a significant reduction of the statistical uncertainty, the new data will also allow for the alternative normalization scheme for $\sigma^{\text{bare}}(e^+e^- \rightarrow \pi^+\pi^-(\gamma_{\text{FSR}}))$, discussed in Eq. (3) of Ref. [1], in which the dominating systematic uncertainties cancel. A total uncertainty of 0.6% can be expected.

3. Gounaris-Sakurai fit of the pion form factor

The pion form factor $|F_\pi|^2$ is defined as

$$|F_\pi|^2 = \frac{3s'}{\pi\alpha\beta_\pi^3(s')} \cdot \sigma^{\text{dressed}}(e^+e^- \rightarrow \pi^+\pi^-), \quad (4)$$

where $\beta_\pi = \sqrt{1 - 4m_\pi^2/s'}$ denotes the pion velocity. The factor $\frac{3s'}{\pi\alpha\beta_\pi^3(s')}$ from pure QED calculations is considered to be exact. Thus, the statistical error-covariance matrix of the pion form factor is constructed analogously to Eq. (3) from the covariance matrix of the event yield, which is propagated according to Eqs. (1) and (4) to the level of the form factor. The diagonal elements of the matrix are presented as updated statistical uncertainties of the pion form factor in Table 1.

In the original work, a fit of the Gounaris-Sakurai parametrization [9] to the pion form factor is used to compare the BESIII measurement to previous publications. In the fit, the statistical covariance matrix is not considered. Instead, the uncertainties before having applied the unfolding procedure are considered. These are assumed to implicitly take into account all correlations. A good fit quality is achieved, but cannot be reproduced using the originally published covariance matrix in Ref. [2].

In this corrigendum, we repeat the fit of the form factor as a cross check of the newly derived covariance matrix. In order to evaluate the effects of the different treatment of the statistical errors, the width of the ω meson Γ_ω is fixed to the PDG

Table 1

Results for the bare cross section $\sigma^{\text{bare}}_{\pi^+\pi^-}$ and the pion form factor together with their statistical uncertainties. The systematical uncertainties are given by 0.9% [1].

$\sqrt{s'}$ [MeV]	$\sigma^{\text{bare}}_{\pi^+\pi^-}(\gamma_{\text{FSR}})$ [nb]	$ F_\pi ^2$	$\sqrt{s'}$ [MeV]	$\sigma^{\text{bare}}_{\pi^+\pi^-}(\gamma_{\text{FSR}})$	$ F_\pi ^2$
602.5	288.3 ± 11.4	6.9 ± 0.3	752.5	1276.1 ± 31.2	41.8 ± 1.0
607.5	306.6 ± 10.8	7.4 ± 0.3	757.5	1315.9 ± 31.4	43.6 ± 1.0
612.5	332.8 ± 11.8	8.2 ± 0.3	762.5	1339.3 ± 29.0	44.8 ± 1.0
617.5	352.5 ± 12.4	8.7 ± 0.3	767.5	1331.9 ± 30.0	45.0 ± 1.0
622.5	367.7 ± 12.1	9.2 ± 0.3	772.5	1327.0 ± 29.6	45.2 ± 1.0
627.5	390.1 ± 12.7	9.8 ± 0.3	777.5	1272.7 ± 28.3	43.7 ± 1.0
632.5	408.0 ± 13.6	10.4 ± 0.3	782.5	1031.5 ± 26.8	37.1 ± 1.0
637.5	426.6 ± 13.5	11.0 ± 0.3	787.5	810.7 ± 23.7	30.3 ± 0.9
642.5	453.5 ± 14.6	11.8 ± 0.4	792.5	819.7 ± 21.8	30.6 ± 0.8
647.5	477.7 ± 14.2	12.5 ± 0.4	797.5	803.1 ± 20.9	30.1 ± 0.8
652.5	497.4 ± 15.9	13.2 ± 0.4	802.5	732.4 ± 21.1	27.7 ± 0.8
657.5	509.2 ± 15.8	13.6 ± 0.4	807.5	679.9 ± 18.8	25.9 ± 0.7
662.5	543.4 ± 16.6	14.7 ± 0.4	812.5	663.6 ± 17.1	25.5 ± 0.7
667.5	585.0 ± 16.5	16.0 ± 0.4	817.5	622.2 ± 17.3	24.1 ± 0.7
672.5	642.7 ± 17.6	17.7 ± 0.5	822.5	585.0 ± 16.1	22.9 ± 0.6
677.5	640.5 ± 16.3	17.8 ± 0.5	827.5	540.8 ± 14.8	21.4 ± 0.6
682.5	668.0 ± 18.4	18.8 ± 0.5	832.5	496.4 ± 14.8	19.8 ± 0.6
687.5	724.4 ± 19.1	20.6 ± 0.5	837.5	450.4 ± 13.2	18.1 ± 0.5
692.5	783.5 ± 18.9	22.5 ± 0.5	842.5	404.7 ± 13.2	16.4 ± 0.5
697.5	858.6 ± 20.4	24.9 ± 0.6	847.5	391.3 ± 12.8	16.0 ± 0.5
702.5	893.8 ± 20.3	26.2 ± 0.6	852.5	364.0 ± 11.8	15.0 ± 0.5
707.5	897.8 ± 21.4	26.6 ± 0.6	857.5	339.6 ± 11.9	14.2 ± 0.5
712.5	978.6 ± 22.9	29.3 ± 0.7	862.5	310.0 ± 11.5	13.0 ± 0.5
717.5	1059.1 ± 23.6	32.0 ± 0.7	867.5	283.8 ± 9.8	12.1 ± 0.4
722.5	1086.0 ± 25.2	33.2 ± 0.8	872.5	256.5 ± 9.2	11.0 ± 0.4
727.5	1088.4 ± 25.3	33.6 ± 0.8	877.5	237.3 ± 9.2	10.3 ± 0.4
732.5	1158.8 ± 23.7	36.2 ± 0.7	882.5	229.7 ± 8.6	10.0 ± 0.4
737.5	1206.5 ± 25.1	38.2 ± 0.8	887.5	224.0 ± 8.1	9.9 ± 0.4
742.5	1229.9 ± 25.9	39.3 ± 0.8	892.5	196.1 ± 8.0	8.7 ± 0.4
747.5	1263.3 ± 27.6	40.9 ± 0.9	897.5	175.9 ± 7.6	7.9 ± 0.3

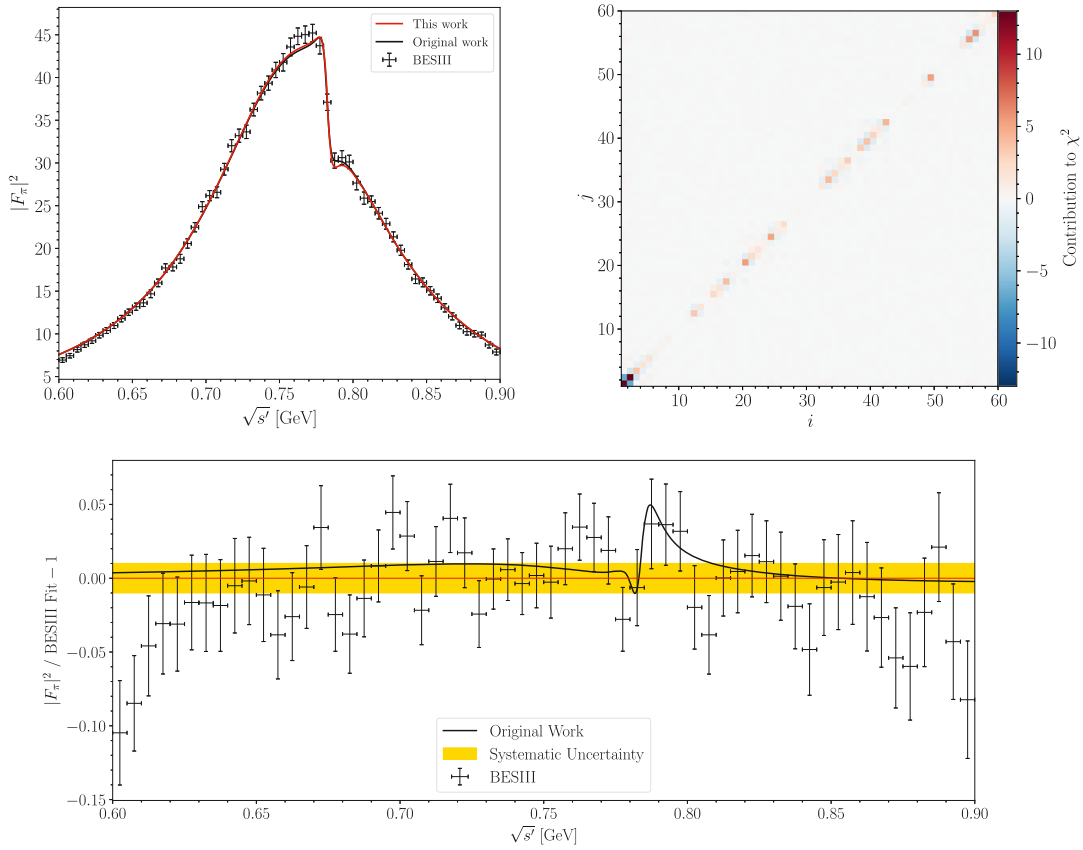


Fig. 2. **Top Left:** Results of the Gounaris-Sakurai fit of the original work [1] (black) and this work (red); **Top Right:** Contribution to the χ^2 value of the individual bins of the covariance matrix; **Bottom:** Deviations between the fit result of this work (red line) and the data as well as the old fit [1] (black line).

Table 2

Fit results together with the statistical uncertainties from this work (BESIII), the original work (BESIII 16 [1]), the BaBar measurement [11], and the PDG values [10].

Parameter	BESIII	BESIII 16	BaBar	PDG
m_ρ [MeV]	776.58 ± 0.42	776.0 ± 0.4	775.02 ± 0.31	775.26 ± 0.25
Γ_ρ [MeV]	152.05 ± 0.65	151.7 ± 0.7	149.59 ± 0.67	147.8 ± 0.9
m_ω [MeV]	782.69 ± 0.34	782.2 ± 0.6	781.91 ± 0.18	782.65 ± 0.12
$ c_\omega [10^{-3}]$	1.92 ± 0.16	1.7 ± 0.2	1.644 ± 0.061	–
ϕ_ω [rad]	0.15 ± 0.11	0.04 ± 0.13	-0.011 ± 0.037	–
$\chi^2/\text{n.d.f.}$	70.70/56	49.1/56	–	–

value [10], and the masses and widths of the higher ρ states $\rho(1450)$, $\rho(1700)$, and $\rho(2150)$ are fixed to the values obtained by the BaBar collaboration [11], as done in the original work. The updated fit result is illustrated with a red line in the top left panel of Fig. 2 and compared to the original fit result. The updated fit yields a reduced χ^2 value of $\chi^2/\text{n.d.f.} = 70.70/56$. The bottom panel of Fig. 2 illustrates the deviations of the updated fit result and the old fit result from Ref. [1]. A clear deviation is found at the $\rho - \omega$ interference, where using the covariance matrix in the fit has worsened the agreement with data. The effect is related to fixing Γ_ω in the fit. However, floating the width does not only result in a better agreement between data and fit function, but also in a value of Γ_ω more than two standard deviations larger than the PDG value. The top right panel of Fig. 2 shows the individual contributions of the bins of the covariance matrix to the total χ^2 value. Large fluctuations, as reported by Colangelo et al. [2] are not observed. The largest contribution to χ^2 stems from the mass region between 600 and 615 MeV, where there is a systematic difference between the data and the Gounaris-Sakurai parametrization. The fit results are summarized in Table 2.

By comparing the resulting parameters one finds a significant improvement of the uncertainty of the ω mass. The results obtained for other parameters agree well with the original work. Thus, the systematic differences found between the BESIII result and previous measurements using the Gounaris-Sakurai fit in Ref. [1] can be considered unchanged. The deviations between the fit results of BESIII and BaBar are on the level of 3σ or less, which might be well covered by systematic effects that are neglected at this point. It should also be noted that the BaBar results do not consider the covariance matrix in the fit due to expected biases [11]. The precise determination of resonance parameters is not the purpose of this corrigendum.

4. Reevaluation of $a_\mu^{\pi\pi, \text{LO}}$ (600 – 900 MeV)

The hadronic vacuum polarization (HVP) contribution to the muon anomalous magnetic moment a_μ can be connected to the cross section $\sigma(e^+e^- \rightarrow \text{hadrons})$ using the optical theorem [12]. The contribution of $e^+e^- \rightarrow \pi^+\pi^-$ to a_μ in the mass range of the $\rho - \omega$ interference is given by

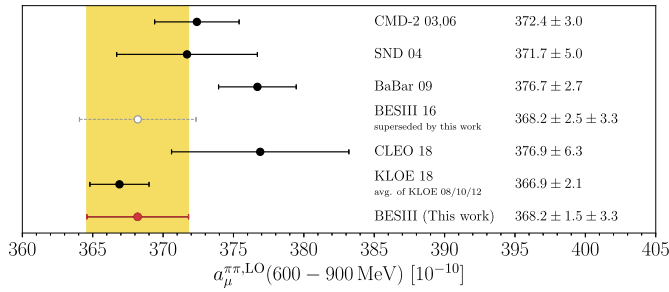


Fig. 3. Comparison of the updated calculation of the leading-order (LO) hadronic vacuum polarization contribution to $(g-2)_\mu$ due to $\pi^+\pi^-$ in the energy range 600–900 MeV from BESIII and the corresponding results from CMD-2 [13,14], SND [15], BaBar [11], BESIII 16 [1], CLEO [16], and KLOE [17]. The respective values are taken from the white paper of the Muon $g-2$ Theory Initiative [2,3,18–22]. The yellow band indicates the 1σ range of the updated BESIII result.

$$a_\mu^{\pi\pi, \text{LO}}(600-900 \text{ MeV}) = \frac{1}{4\pi^3} \int_{(600 \text{ MeV})^2}^{(900 \text{ MeV})^2} ds' K(s') \sigma^{\text{bare}}(e^+e^- \rightarrow \pi^+\pi^-(\gamma_{\text{FSR}})), \quad (5)$$

where $K(s')$ is a kernel function.

With the systematical uncertainty remaining at 0.9% [1], the BESIII result on the hadronic vacuum polarization now reads as $a_\mu^{\pi\pi, \text{LO}}(600-900 \text{ MeV}) = (368.2 \pm 1.5_{\text{stat}} \pm 3.3_{\text{syst}}) \times 10^{-10}$.

Fig. 3 shows the results of the calculation compared to previous measurements. The statistical uncertainty is reduced by 40% compared to the original work. The result lines up well with the KLOE results, while the 1.7σ discrepancy between the BESIII and BaBar results remains.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary material

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.physletb.2020.135982>.

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