

The gravitational wave damping timescales of f -modes in neutron stars

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Abstract. Gravitational astroseismology studies applied to neutron stars aim to relate stellar properties to characteristics of the gravitational wave signal. In the case of non-rotating neutron stars, gravitational waves produced by linear oscillations are characterized by their pulsation frequency and damping timescale. We focus on the main emitter, the quadrupole fundamental mode, and review a higher order empirical relation for the damping timescale, which was initially presented in [1]. The relation is accurate throughout a wide range of compactness, covering the expected values for neutron stars, while involving only the mass and radius and no other quantity, such as the moment of inertia.

1. Introduction

Neutron stars (NSs) are the densest known stars in the Universe, admitting core densities which can be a few times the nuclear saturation density. The equation of state (EoS) at these very high densities remains still partially unknown, as they are not yet easily accessible by terrestrial experiments [2]. Different EoSs lead to different NS macroscopic characteristics, such as their mass and radius. Constraining such characteristics through observations can thus provide crucial information on the properties and composition of nuclear matter at those densities.

The detection of gravitational waves (GWs) from merging NSs in 2017 [3] highlights that GWs can be used as an observational channel for such systems. Other scenarios involving NSs emitting GWs, including pulsating isolated NSs, could also be prominent for observation in the near future [4]. Within the context of gravitational-wave astroseismology, characteristics such as the mass and radius of NSs have been found to be closely related to both the real frequencies, as well as the damping timescales of NS modes. In the case of isolated NSs various relations have been suggested, relating the mass, radius and an (effective) compactness derived from the moment of inertia of NSs to the gravitational wave frequency and damping timescale [5, 6, 7, 8, 9, 10, 11]. Similar relations involving the radius of non-rotating models and the frequency of the fundamental mode (f -mode) of the merger have been proposed for the case of binary neutron star systems [12, 13, 14, 15]. Overall such relations can be used alongside observations to constrain such macroscopic characteristics, thus helping to constrain the EoS. They are particularly interesting as they can provide us with a better insight of such systems,



especially from the dynamical point of view. Furthermore, a successful detection of GWs from those systems relies on accurate modeling of the expected waveforms [16, 17, 18, 19, 20].

Here we focus on the f -mode quadrupole oscillations. We focus our attention on the damping timescale of the f -mode and discuss a universal relation proposed in [1] involving the compactness, which is a generalization of the one presented in [6]. This relation is accurate throughout the whole compactness range and highlights that, in the case of NSs, no additional macroscopic quantities are required to describe the damping timescale.

In sections 2 and 3 we briefly present the formulation of the problem and the EoSs used in this study, respectively. Section 4 is devoted to the universal relation. Finally, section 5 concludes this work. In the following, unless otherwise stated, we employ gravitational units $c = G = 1$. Finally, the term “mass” refers to the gravitational mass of the isolated NS.

2. Formulation

Our implementation follows [1] (see also references therein, as well as [7, 21] for an extensive review). We focus on the $l = 2$ polar mode. We consider a nonrotating star in equilibrium for the background, while the matter is assumed to be a perfect fluid with stress-energy tensor

$$T_{\alpha\beta} = (\epsilon + p)u_\alpha u_\beta + pg_{\alpha\beta}, \quad (1)$$

where $g_{\alpha\beta}$ is the metric tensor, ϵ the energy density, p the pressure and u_α the 4-velocity of the fluid.

The symmetries of the background allow for the perturbations to be analyzed in terms of quasi-normal modes, which describe damped harmonic pulsations. The time dependence of these modes is of the form $e^{i\omega t}$, where

$$\omega = 2\pi f + \frac{i}{\tau_{GW}}, \quad (2)$$

f being the real part of the pulsation frequency and τ_{GW} the damping timescale.

After adding polar perturbations, the full spacetime metric in the Regge-Wheeler gauge [22] is given by [23]

$$ds^2 = -e^{2\nu} \left(1 + r^l H_0^{lm} Y_{lm} e^{i\omega t} \right) dt^2 - 2i\omega r^{l+1} H_1^{lm} Y_{lm} e^{i\omega t} dt dr \\ + e^{2\lambda} \left(1 - r^l H_0^{lm} Y_{lm} e^{i\omega t} \right) dr^2 + r^2 \left(1 - r^l K^{lm} Y_{lm} e^{i\omega t} \right) (d\theta^2 + \sin^2 \theta d\phi^2), \quad (3)$$

where $\nu(r)$ and $\lambda(r)$ are the background metric functions, while $H_0(r)$, $H_1(r)$ and $K(r)$ are functions used to describe polar perturbations of the spacetime. Y_{lm} denotes spherical harmonics.

At the same time, the covariant components of the Lagrangian displacement ξ^μ of fluid elements become

$$\xi_r = e^{\lambda} r^{l-1} W^{lm} Y_{lm} e^{i\omega t}, \quad (4a)$$

$$\xi_\theta = -r^l V^{lm} \partial_\theta Y_{lm} e^{i\omega t}, \quad (4b)$$

$$\xi_\phi = -r^l V^{lm} \partial_\phi Y_{lm} e^{i\omega t}, \quad (4c)$$

where $W^{lm}(r)$ and $V^{lm}(r)$ can be related to the fluid perturbations.

A new variable X^{lm} can be defined [24] as

$$X^{lm} = \omega^2 (\epsilon + p) e^{-\nu} V^{lm} - \frac{1}{r} \frac{dp}{dr} e^{\nu-\lambda} W^{lm} + \frac{e^\nu}{2} (\epsilon + p) H_0^{lm}. \quad (5)$$

Employing this new variable X^{lm} , a system of 4 1st-order differential equations for H_1^{lm} , K^{lm} , W^{lm} and X^{lm} is formed, which describes the interior of the star.

The exterior of the star is accounted for by following the approach described in [25]. Since there is no fluid present outside the star, only the perturbations of the spacetime survive. Namely, only H_1^{lm} and K^{lm} have to be determined.

The final solution is obtained by matching the solutions for the interior and the exterior of the star at its surface, by requiring continuity of the solution and the first derivatives. Demanding purely outgoing modes at infinity leads to a discrete set of eigenfrequencies for the oscillations of the star. These frequencies are the quasi-normal modes of the system, as they describe the oscillations of the star, which require no external stimulation.

3. Equations of State

We consider a set of 9 cold tabulated EoSs (MDI[26], L[27], O[28], SKI4[29, 30], WFF1[31], WFF2[31], N[32], SKA[29, 30], HHJ[33, 34]). Figure 1 shows the mass-radius diagram for each one of them. Evidently, we investigate soft, stiff, as well as intermediate EoSs, with radii ranging between ~ 10 km and ~ 15 km, and maximum masses between $2M_\odot$ and $2.7M_\odot$.

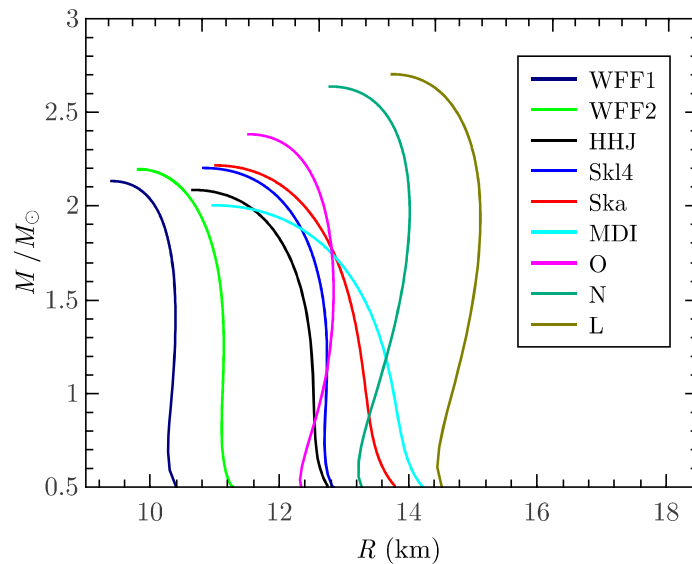


Figure 1. Mass-radius relations for all EoSs used in this study. Figure from [1].

4. Universal relation

The damping timescale τ_{GW} describes the time required for the amplitude of the oscillation to drop to $1/e$ of its initial value. It is therefore proportional to the energy stored in the oscillation over the rate at which this energy is being dissipated through GWs. In [6] Andersson and Kokkotas used the lowest order dependence of this ratio on the mass and radius to scale the damping time appropriately. The scaled damping timescale was found to relate tightly to the compactness of the NS for which it was computed. A different approach was followed in [8]. They argued that both the real and the imaginary part of the frequency, when normalized by the mass, satisfy a relation involving only powers of the compactness. Another accurate relation, involving an effective compactness computed using the moment of inertia, was shown to hold in [9] for both NSs and Quark stars.

In [1], we extended the relation proposed in [6] to higher order. We take into account EoSs which satisfy the maximum mass observational constraint of $\sim 2M_\odot$ [35, 36, 37, 38], while we

also cover a wider range of compactness, particularly at the higher end. Based on our data we suggested a relation of the form

$$\frac{M}{\tau_{GW}} = 0.112 \left(\frac{M}{R}\right)^4 - 0.53 \left(\frac{M}{R}\right)^5 + 0.628 \left(\frac{M}{R}\right)^6. \quad (6)$$

This relation was built using only configurations with a compactness greater than 0.1. Those models are more physical and thus expected to be more relevant in such a survey. The relation however describes models of lower compactness with high accuracy as well, as can be seen in Figure 2.

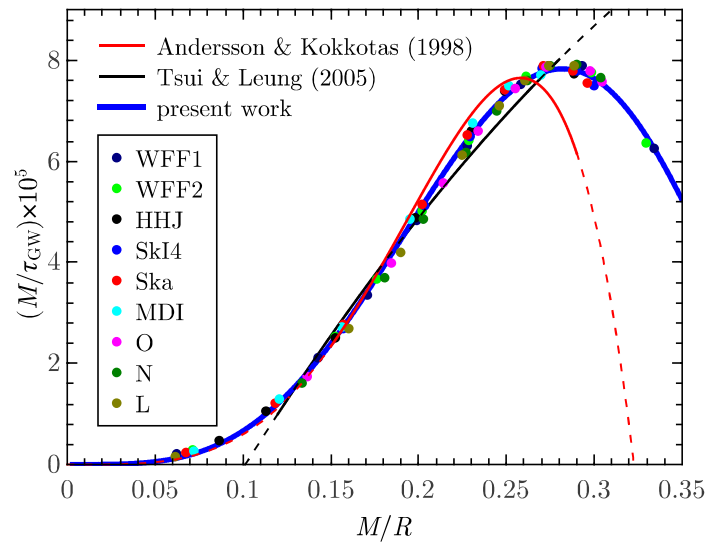


Figure 2. Gravitational-wave damping timescale scaled with the mass versus the compactness. The blue line represents Eq. (6). The previous relations by Andersson & Kokkotas [6] (red) and by Tsui & Leung [8] (black), are also shown with solid lines in their respective range of validity and with dashed lines outside of that range. Figure from [1].

As discussed in [1], this relation reproduces the correct Newtonian limit $1/\tau_{GW} \rightarrow 0$ as the compactness approaches 0. At the same time it extends the compactness validity range of previously suggested relations [6, 8], especially towards higher values of the compactness, up to 0.33. Its accuracy throughout the whole range of expected values for the compactness eliminates, in the case of NSs, the need for the use of another parameter e.g. the moment of inertia. The mass and radius are sufficient to accurately model the damping timescale. This result is important because the determination of the mass and radius of a neutron star should be easier than that of the moment of inertia in a potential observation.

5. Conclusions

Following [1] we discussed an EoS-independent relation between the GW damping timescale of the quadrupole f -mode of non-rotating NSs and macroscopic stellar parameters, in particular the mass and radius of the configuration. The relation is an extension of the one presented in [6] to higher order. It extends the compactness range of validity of previously suggested relations. It manages to do so without the use of additional parameters, such as the moment of inertia, and thus proves that, in the case of NSs, no additional parameter to the mass and radius is required

to model the damping timescale. Use of the relation should provide highly reliable data, which can also be used to model the expected waveforms from such systems, thus aiding in the effort to observe them.

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