

Fast Apparent Oscillations of Fundamental Constants

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Precision spectroscopy of atoms and molecules allows one to search for and to put stringent limits on the variation of fundamental constants. These experiments are typically interpreted in terms of variations of the fine structure constant α and the electron-to-proton mass ratio $\mu = m_e/m_p$. Atomic spectroscopy is usually less sensitive to other fundamental constants, unless the hyperfine structure of atomic levels is studied. However, the number of possible dimensionless constants increases when allowed for fast variations of the constants, where “fast” is determined by the time scale of the response of the studied species or experimental apparatus used. In this case, the relevant dimensionless quantity is, for example, the ratio $m_e/\langle m_e \rangle$ and $\langle m_e \rangle$ is the time average. In this sense, one may say that the experimental signal depends on the variation of dimensionful constants (m_e in this example).

We provide a general recipe of how to deal with apparent variation of constants in a situation where the time scale of the variation is faster than the response time of a part of the experimental system. This is important because some of the basic rules that were established for the case of slow variations need to be revisited and modified. We also discuss the often contentious question of whether only dimensionless constants may be allowed to vary in the case of the constants having fast variations on a time scale relevant to a measurement such as in the atomic experiments,^[12,13] or the experiment with resonant-mass antennae.^[14]

1. Introduction

Variations of “constants” have been extensively discussed in the literature, see, for example, a review,^[1] and references therein as well.^[2–9] However, in the previous literature, it was usually assumed that variations occur at time scales much longer than that of an individual measurement, so the “constants” could be safely assumed to be, in fact, constant during a given experimental run. The goal of the present article, is to clarify, following the earlier discussions by others,^[10,11] the beyond-the-standard-model context in which apparent variations of “constants” may arise.

The present work provides a full motivation for, and significantly expands on the analysis presented in ref. [13]. The new analysis is done using a fully relativistic Lagrangian for the description of the relevant physics. The presented formalism enables description of searches for other time-varying constants of nature which are not confined to the fine structure constant or the electron mass, but also changes of the quark masses and any other constants of nature.

The masses of the particles in the standard model (SM) are generated by the interaction with the scalar Higgs field, which forms vacuum condensate. In some models, dark matter (DM) is associated with ultralight scalar fields (see, e.g., refs. [1–7, 9, 11]).

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These fields do not necessarily form vacuum condensate, but exist as classical fields filling the space. The galactic DM field is known to be non-relativistic. This means that kinetic energy is small compared to the rest energy mc^2 and the field is oscillating at the frequency close to $\nu = mc^2/h$, where m is the mass of the scalar particle, c is the speed of light, and h is Planck's constant. Interaction of such a field with fermions leads to a term in their Lagrangian, which looks like an oscillating mass term.

In this scenario of oscillating DM field linearly coupled to fermions, the particles acquire apparent modifications to their masses,^[2,5–7,9,11] which oscillates at the frequency ν . The amplitude of these oscillations depends on the local density of the scalar field. If, for example, the apparent mass of the electron m_e is modified in such a way, this must affect the spectra of atoms and molecules. As long as this additional mass-like term appears from the interaction with the cosmological field, it does not violate the conservation of energy, though the energy of atoms is changing.^[15] If the scalar field is also coupled to the electromagnetic field, this generally leads to the variation in the strength of the electromagnetic coupling characterized by the fine structure constant $\alpha = \frac{e^2}{\hbar c}$.^[2–6,9,11] As a result, the fine structure constant also acquires oscillating components. For an example of a model with oscillating m_e and α , see for example, refs. [5, 7, 9]. Below we discuss how such effects can be observed in precision spectroscopic experiments.

In the non-relativistic approximation, the energy of any electronic level in an atom is proportional to the atomic unit of energy, Hartree:

$$E_H = \frac{m_e e^4}{\hbar^2} \approx 27 \text{ eV} \quad (1)$$

where e is elementary charge, and we write the analytical expression in Gaussian units. In this approximation, all atomic transition frequencies are also proportional to E_H and their ratios do not depend on fundamental constants.^[16,17] When relativistic corrections are taken into account, the energies acquire a dependence on the fine structure constant:

$$E_{\text{at}} = E_H [C_0 + C_1(\alpha Z)^2 + \dots] \quad (2)$$

where Z is the number of protons in the nucleus. For neutral atoms, the coefficients C_i are of the order of unity and depend on the quantum numbers of the level. For light atoms, $\alpha Z \ll 1$, and the dependence of the energies on α is weak; however, it becomes significant for heavy elements with $Z \approx 100$.

Electronic energy of light molecules is also proportional to E_H , but now there are also vibrational and rotational energies E_{vib} and E_{rot} , which depend on the electron-to-proton mass ratio $\mu = m_e/m_p$.^[18]

$$E_{\text{vib}} = C_v E_H \mu^{1/2}, \quad E_{\text{rot}} = C_r E_H \mu \quad (3)$$

Because of these vibrational and rotational energies, molecular spectra are sensitive to the mass ratio μ . Relativistic corrections again introduce an α dependence: $C_v = C_{v,0} + C_{v,1}(\alpha Z)^2 + \dots$ and similarly for C_r . This dependence comes about because the molecular potential and the inter-nuclear distance (that

enters the moment of inertia and thus the rotational energy) depend on the electronic wavefunctions and thus on αZ .

For completeness, we need to mention that the hyperfine structure of atomic and molecular levels is sensitive to the nuclear magnetic and quadrupole moments, which depend on other fundamental constants. With this exception, all the ratios of atomic and molecular transition frequencies are sensitive only to the values of two fundamental constants, namely, α and μ . Strictly speaking, the ratios of atomic frequencies depend on all fundamental constants. However, their sensitivity to other fundamental constants is orders of magnitude smaller. For example, the finite nuclear size leads to the “volume shifts” of atomic levels, typically on the scale $10^{-5} E_H$. The size of the nucleus depends on the strong coupling constant. Thus, the sensitivity of atomic energy levels to the variations of the strong coupling constant is suppressed by roughly five orders of magnitude. The advent of laser spectroscopy of a low-energy *nuclear* transition in ^{229}Th is expected to be a game-changer with greatly enhanced sensitivity to nuclear parameters.^[19,20]

2. Experimental Consequences

Let us first assume slow variation of the “constants” on all time scales relative to a measurement. Many spectroscopic experiments use optical resonators (cavities). The latest state-of-the-art optical resonators use crystalline material, instead of amorphous low-expansion glasses, for cavity spacers.^[21,22] The length L of such a cavity depends on the lattice constant of the material its spacer is made of. The latter, in turn, is proportional to the Bohr radius

$$r_0 = \frac{\hbar^2}{m_e e^2} \quad (4)$$

The resonant frequency of such cavity is proportional to c/r_0 :

$$\nu_{\text{cav}} = C_c \frac{c}{r_0} = C_c \frac{m_e e^2 c}{\hbar^2} = C_c \frac{E_H}{\hbar \alpha} \quad (5)$$

where $C_c = C_{c,0} + C_{c,1}(\alpha Z)^2 + \dots$. We see that the ratio of atomic transition frequency ν_{at} to ν_{cav} to a first approximation is proportional to α :

$$\frac{\delta\left(\frac{\nu_{\text{at}}}{\nu_{\text{cav}}}\right)}{\left(\frac{\nu_{\text{at}}}{\nu_{\text{cav}}}\right)} = \frac{\delta\alpha}{\alpha} [1 + \mathcal{O}(\alpha Z)] \quad (6)$$

If the constants are rapidly oscillating, the spectra we study will depend on some average values of the constants and the corresponding averaging time depends on the response time of the atoms/molecules and the apparatus we use. For an atom, the response time depends on the lifetime of the level τ_{at} and the width of the transition Γ . For a resonator with a finesse $\mathcal{F}$ the response time is $\tau_{\text{cav},1} \propto \mathcal{F}L/c$.

For a resonator there is also another relevant time. This is the time $\tau_{\text{cav},2}$ during which the length L may adjust to the changing value of the atomic length scale r_0 . We can estimate $\tau_{\text{cav},2}$ in terms of the speed of sound in the material v_s ,^[23] $\tau_{\text{cav},2} \approx L/v_s$. If the

fineness is $\mathcal{F} < c/v_s$, then $\tau_{\text{cav}} = \tau_{\text{cav},2} > \tau_{\text{cav},1}$. A more accurate analysis has to account for other vibrational modes of the cavity,^[14] but for the estimates one can still use $\tau_{\text{cav}} \approx L/v_s$.

As an example, consider the experiment^[13] where the frequency of the $6s \rightarrow 6p_{3/2}$ transition in Cs is compared to the frequency of an optical resonator with an invar spacer^[24] of length $L = 12$ cm. The lifetime of the atomic upper state here is $\tau_{\text{at}} = 30.5 \times 10^{-9}$ s. The speed of sound for steel is $v_s \approx 6 \cdot 10^5$ cm/s, and $\tau_{\text{cav}} = 2 \times 10^{-5}$ s. If we assume that all fundamental constants oscillate at some common frequency f_a , then the experiment^[13] is sensitive to different combinations of constants depending on the frequency f_a . If $f_a \ll \tau_{\text{cav}}^{-1}$ then Equation (6) holds. If $\tau_{\text{cav}}^{-1} \ll f_a \ll \tau_{\text{at}}^{-1}$, then the cavity is sensitive only to the averaged values of E_H and α , while the atoms maintain sensitivity to the variation. As a result,

$$\frac{\delta\left(\frac{v_{\text{at}}}{v_{\text{cav}}}\right)}{\left(\frac{v_{\text{at}}}{v_{\text{cav}}}\right)} = \frac{\delta E_H}{E_H} [1 + \mathcal{O}(\alpha Z)] \quad (7)$$

where $\delta E_H = E_H - \langle E_H \rangle$ is the deviation from the time averaged value.

Equation (7) shows that for intermediate frequencies f_a the ratio $\frac{v_{\text{at}}}{v_{\text{cav}}}$ depends on the variation of the dimensionful parameter E_H . At this point we need to specify what kind of models we are interested in.

3. Discussion of Models

First, we assume that at short distances our system is described by a local perturbative Lorentz invariant quantum field theory (QFT), which implies no charge, parity and time-reversal (CPT) violation. For this case we have fairly good understanding of how to proceed. Without loss of generality, we are allowed to use natural units $\hbar = c = 1$ (see, e.g., ref. [25–27]). We also have examples of working models (e.g., dilation, relaxion, and SUSY theories). For the gauge field we can use a normalization where the coupling constant α is absorbed into the field $(\alpha A_\mu \rightarrow A_\mu)$.^[28] Then the kinetic term for the gauge field has the form: $\mathcal{L}_{\text{kin}} = -\frac{1}{4\alpha} F_{\mu\nu} F^{\mu\nu}$.

Using the above conventions we can now consider a model with relevant fields (omitting for simplicity the weak and strong gauge fields): A_μ , the photon field (with $F_{\mu\nu}$ stands for the corresponding field strength), a lepton doublet, $L_e^T = (v_e, e_L)$, with v_e electron–neutrino and $e_{L,R}$ left-handed and right-handed electron fields, the Higgs field written in unitary gauge as $H^T = (0, h + v)/\sqrt{2}$, with h being the celebrated Higgs boson, and $v \simeq 246$, GeV being the Higgs vacuum expectation value (VEV). In addition, we have a new scalar field φ , the singlet of the SM gauge interactions. The relevant part of the Lagrangian is (for more detail see, e.g., ref. [29, 30]):

$$\begin{aligned} \mathcal{L}_{\text{free}} = & -\frac{1}{4\alpha} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (\partial_\mu \varphi \partial^\mu \varphi - m^2 \varphi^2) \\ & + \mathcal{L}_{\text{kin}}^{\text{SM}} - \sqrt{2} \frac{m_e}{v} H \bar{L}_e e_R + h.c. \\ & - \mu^2 H^\dagger H + \lambda (H^\dagger H)^2 + \mu_{\phi h} \phi H^\dagger H \end{aligned} \quad (8)$$

where $\mathcal{L}_{\text{kin}}^{\text{SM}}$ stands for the SM matter field's kinetic terms, m_e being the electron mass, μ^2 (λ) being the Higgs quadratic (quartic) coupling and m ($\mu_{\phi h}$) are the singlet mass (cubic coupling) and higher order terms being suppressed. The electromagnetic interactions for the electron field, relevant for low energy physics discussed below are

$$\mathcal{L}_{\text{gauge}} = \bar{e} A_\mu \gamma^\mu e \quad (9)$$

The coupling $\mu_{\phi h}$ in Equation (8) induces mixing between φ and h with the mixing angle usually designated as θ (see, e.g., ref. [31] for a recent review). Then we find that the Yukawa interaction in (8) between the electron and the field H leads to a similar term between the electron and the scalar field φ :

$$\sin \theta \sqrt{2} m_e \left(\frac{\varphi}{v} \right) \bar{e}_L e_R \quad (10)$$

At the one-loop level, a coupling between the scalar φ and the photon is induced (see, e.g., ref. [32]), approximately given by

$$\sin \theta \frac{\alpha}{4\pi} \left(\frac{\varphi}{v} \right) F_{\mu\nu} F^{\mu\nu} \quad (11)$$

Of course, there will be similar induced terms for other particles and interactions of the standard model, which we omitted here for simplicity. We note that couplings of similar form are also expected for a simple dilaton model which couples to the gauge fields via the anomalous contribution to the trace of the energy–momentum tensor (see, e.g., refs. [5, 33]).

Now we can introduce a time and a space dependent classical field φ and see what the implications are. This leads to a theory with a space and time dependent effective Higgs-VEV. The terms in Equations (10) and (11) modify the kinetic and mass terms in the Lagrangian of Equation (8). Implications of these modifications are the same as those of varying coupling constant $\alpha(\varphi)$ and mass $m_e(\varphi)$:

$$\alpha(\varphi) = \alpha \left(1 + \sin \theta \frac{1}{\pi} \frac{\varphi}{v} \right) \quad (12a)$$

$$m_e(\varphi) = m_e \left(1 - \sin \theta \frac{\varphi}{v} \right) \quad (12b)$$

We see that in this model an effective variation of the fine-structure constant α and mass m_e appears, which is linear in the field φ .

Now we need to find out how this affects atomic unit E_H in Equation (7). With the chosen units ($\hbar = c = 1$), any variation of the atomic unit E_H is induced by the variations of α and m_e . If we rewrite (1) as

$$E_H = m_e c^2 \alpha^2 \quad (13)$$

we see that

$$\frac{\delta E_H}{E_H} = \frac{\delta m_e}{m_e} + 2 \frac{\delta \alpha}{\alpha} \quad (14)$$

Substituting (12) into (14), we find a (unit independent) result:

$$\frac{\delta E_H}{E_H} = -\sin \theta \frac{\varphi}{v} \left(1 - \frac{2}{\pi}\right) \quad (15)$$

which is connected to the experimental observables via Equation (7).

The above mechanism can be actually realized in dilaton-DM theory^[5] and in cases where φ is an axion-like DM field that is subject to (spontaneous) charge and parity (CP) violation, as in the case of relaxion dark matter models.^[7,34]

4. Summary

The presence of oscillating background fields in a broad class of QFT models, may be interpreted as temporal variations of fundamental constants. In the case of variation of a constant q , it is possible to find setups where q is calibrated by its own average value $\langle q \rangle$, resulting in a comparison of a dimensionless ratio $\frac{q - \langle q \rangle}{\langle q \rangle}$. In this sense, for the case of rapid variations, it is possible to test variations of dimensionful constants as well as that of dimensionless ones. To be sensitive to such variations requires two systems, one of which has a faster response (such as an atom) and another is more inertial (such as a cavity). Then the faster-response system tracks instantaneous values of the constants, while the inertial one depends only on their average values.

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Conflict of Interest

The authors declare no conflict of interest.

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- [1] A. M. Safronova, D. Budker, D. DeMille, D. F. J. Kimball, A. Derevianko, C. W. Clark, *Rev. Mod. Phys.* **2018**, 90, 025008.
[2] T. Damour, A. M. Polyakov, *Nucl. Phys. B* **1994**, 423, 532.

- [3] J. D. Barrow, *Phys. Rev. D* **1999**, 59, 043515.
[4] H. B. Sandvik, J. D. Barrow, J. Magueijo, *Phys. Rev. Lett.* **2002**, 88, 031302.
[5] A. Arvanitaki, J. Huang, K. Van Tilburg, *Phys. Rev. D* **2015**, 91, 015015.
[6] Y. V. Stadnik, V. V. Flambaum, *Phys. Rev. Lett.* **2015**, 115, 201301.
[7] A. Banerjee, H. Kim, G. Perez, *Phys. Rev. D* **2019**, 100, 115026.
[8] M. G. Kozlov, D. Budker, *Ann. Phys.* **2018**, 1800254.
[9] A. Hees, O. Minazzoli, E. Savalle, Y. V. Stadnik, P. Wolf, *Phys. Rev. D* **2018**, 98, 064051.
[10] A. Arvanitaki, P. W. Graham, J. M. Hogan, S. Rajendran, K. Van Tilburg, *Phys. Rev. D* **2018**, 97, 075020.
[11] E. Savalle, B. M. Roberts, F. Frank, P. E. Pottie, B. T. McAllister, C. Dailey, A. Derevianko, P. Wolf, arXiv:1902.07192 [gr-qc] **2019**.
[12] S. Aharony, N. Akerman, R. Ozeri, G. Perez, I. Savoray, R. Shaniv, arXiv:1902.02788 [hep-ph] **2019**.
[13] D. Antypas, O. Tretiak, A. Garcon, R. Ozeri, G. Perez, D. Budker, *Phys. Rev. Lett.* **2019**, 123, 141102.
[14] A. Arvanitaki, S. Dimopoulos, K. Van Tilburg, *Phys. Rev. Lett.* **2016**, 116, 031102.
[15] In general, all varying fundamental “constants” must be properly described as dynamic scalar fields. Indeed, a variation of “constants” leads to a change of energy of a system (e.g., an atom), which, assuming energy conservation, must be compensated by the energy taken up by the field.
[16] V. A. Dzuba, V. V. Flambaum, J. K. Webb, *Phys. Rev. Lett.* **1999**, 82, 888.
[17] M. P. Svedoff, *Nature* **1956**, 178, 688.
[18] More generally, the ratio of m_p to the nucleon mass or the strong interaction scale Λ_{QCD} may be considered.
[19] V. V. Flambaum, *Phys. Rev. Lett.* **2006**, 97, 092502.
[20] P. G. Thirolf, B. Seiferle, L. von der Wense, *J. Phys. B: At. Mol. Opt. Phys.* **2019**, 52, 203001.
[21] D. G. Matei, T. Legero, S. Häfner, C. Grebing, R. Weyrich, W. Zhang, L. Sonderhouse, J. M. Robinson, J. Ye, F. Riehle, U. Sterr, *Phys. Rev. Lett.* **2017**, 118, 263202.
[22] W. Zhang, M. Robinson, L. Sonderhouse, E. Oelker, C. Benko, J. L. Hall, T. Legero, D. G. Matei, F. Riehle, U. Sterr, J. Ye, *Phys. Rev. Lett.* **2017**, 119, 243601.
[23] A. A. Geraci, C. Bradley, D. Gao, J. Weinstein, A. Derevianko, *Phys. Rev. Lett.* **2019**, 123, 031304.
[24] The material of the cavity is generally important for precision measurements. The length of a crystalline cavity is conceptually connected to fundamental constants, which is different from a cavity based on amorphous glass. However, in the frequency range considered in this work for fast variations of fundamental constants, the relatively slow creeps of the glass material should not make a significant contribution.
[25] D. Tong, *Part III Cambridge University Mathematics Tripos*, Michaelmas **2006**.
[26] D. B. Kaplan, arXiv:nucl-th/0510023 **2005**.
[27] M. E. Peskin, D. V. Schroeder, *An Introduction To Quantum Field Theory*, CRC Press, Boca Raton, FL **1999**.
[28] F. Wilczek, in *More Things in Heaven and Earth* (Ed: B. Bederson), Springer, New York **1999**.
[29] M. Tanabashi, K. Hagiwara, K. Hikasa, K. Nakamura, Y. Sumino, F. Takahashi, J. Tanaka, K. Agashe, G. Aielli, C. Amsler, M. Antonelli, D. M. Asner, H. Baer, Sw. Banerjee, R. M. Barnett, T. Basaglia, C. W. Bauer, J. J. Beatty, V. I. Belousov, J. Beringer, S. Bethke, A. Bettini, H. Bichsel, O. Biebel, K. M. Black, E. Blucher, O. Buchmüller, V. Burkert, M. A. Bychkov, R. N. Cahn, et al. (Particle Data Group), *Phys. Rev. D* **2018**, 98, 030001.
[30] European Strategy for Particle Physics Preparatory Group, arXiv:1910.11775 [hep-ex] **2019**.
[31] J. Beacham, C. Burrage, D. Curtin, A. De Roeck, J. Evans, J. L. Feng, C. Gatto, S. Gninenko, A. Hartin, I. Irastorza, J. Jaeckel, K. Jungmann,

K. Kirch, F. Kling, S. Knapen, M. Lamont, G. Lanfranchi, C. Lazzeroni, A. Lindner, F. Martinez-Vidal, M. Moulson, N. Neri, M. Papucci, I. Pedraza, K. Petridis, M. Pospelov, A. Rozanov, G. Ruoso, P. Schuster, Y. Semertzidis, T. Spadaro, C. Vallee, G. Wilkinson, arXiv:1901.09966 [hep-ex] **2019**.

[32] A. Djouadi, *Phys. Rept.* **2008**, 457, 1.

[33] W. D. Goldberger, B. Grinstein, W. Skiba, *Phys. Rev. Lett.* **2008**, 100, 111802.

[34] T. Flacke, C. Frugiuele, E. Fuchs, R. S. Gupta, G. Perez, *J. High Energ. Phys* **2017**, 06, 50.