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## PAPER

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## Ferromagnetic gyroscopes for tests of fundamental physics

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### Abstract

A ferromagnetic gyroscope (FG) is a ferromagnet whose angular momentum is dominated by electron spin polarization and that will precess under the action of an external torque, such as that due to a magnetic field. Here we model and analyze FG dynamics and sensitivity, focusing on practical schemes for experimental realization. In the case of a freely floating FG, we model the transition from dynamics dominated by libration in relatively high externally applied magnetic fields, to those dominated by precession at relatively low applied fields. Measurement of the libration frequency enables *in situ* determination of the magnetic field and a technique to reduce the field below the threshold for which precession dominates the FG dynamics. We note that evidence of gyroscopic behavior is present even at magnetic fields much larger than the threshold field below which precession dominates. We also model the dynamics of an FG levitated above a type-I superconductor via the Meissner effect, and find that for FGs with dimensions larger than about 100 nm the observed precession frequency is reduced compared to that of a freely floating FG. This is due to an effect akin to negative feedback that arises from the distortion of the field from the FG by the superconductor. Finally we assess the sensitivity of an FG levitated above a type-I superconductor to exotic spin-dependent interactions under practical experimental conditions, demonstrating the potential of FGs for tests of fundamental physics.

## 1. Introduction

Gyroscopes are valuable tools for metrology and navigation due to their sensitivity to rotations. For example, the Gravity Probe B space mission contained several spinning spheres made of fused quartz and coated with a layer of niobium [1]. Changes in the direction of angular momentum and rate of rotation of these spheres were detected by a superconducting quantum interference device (SQUID). In a different technique, ring laser interferometers (optical gyroscopes based on the Sagnac effect) have been used for continuous measurement of the Earth rotation and tilt [2]. Yet another approach observes gyroscopic motion due to precession of molecules, atoms and nuclei [3]. In the present work, we investigate how the intrinsic spin of electrons can play the role of a gyroscope.

Atoms, molecules, and nuclei, that can possess angular momentum due to their rotational motion as well as due to intrinsic spin, can act as gyroscopes [4–8]. Atomic, molecular, and nuclear gyroscopes have proven to be particularly useful for precision tests of fundamental physics [9], including tests of Lorentz

symmetry [10–12], searches for exotic spin-dependent interactions [13–16], dark matter experiments [17–20], and measurements of electric dipole moments [21–23] and gravitational dipole moments [24–26]. It has recently been proposed that a ferromagnet can act as a new type of gyroscope that may be particularly useful for precision tests of fundamental physics [27]. However, in order to realize the potential sensitivity of a ferromagnetic gyroscope (FG), it is essential to decouple the ferromagnet from the environment, e.g., by requiring either microgravity or some method of frictionless suspension. A promising platform for FG-based fundamental physics experiments involves levitating an FG above a superconducting surface by taking advantage of the Meissner effect [28–32]. In the present work we model the dynamics of a freely floating FG and the dynamics of an FG levitated above a perfect type-I superconductor (SC). We find that the response of an FG to external torques is considerably modified in the case of an FG levitated above an SC: the precession frequency can be reduced by orders of magnitude as compared to that for a freely floating FG. Taking this effect into account, we analyze the sensitivity of an FG levitated above an SC to torques from exotic fields [9].

Under conditions where the angular momentum of a ferromagnet is dominated by the intrinsic spin of the polarized electrons, an applied torque is predicted to cause gyroscopic precession of the ferromagnet [27]. If such a FG can be sufficiently isolated from the environment, a measurement of the precession can yield sensitivity to torques far beyond that of other systems (such as atomic magnetometers [33] and conventional gyroscopes [34]). The high sensitivity of an FG is a result of the rapid averaging of quantum noise [27, 35]. A key enabling technology for practical realization of an FG is a method of near frictionless suspension. One approach is to levitate a ferromagnet above an SC [28]. Recently, there has been considerable interest and progress in development of sensors based on ferromagnets levitated above SCs [28–32, 36].

Magnetization dynamics of ferromagnets, including precession and nutation motions, have been observed in thin films using ferromagnetic resonance [37, 38]. Such dynamics occur on characteristic time scales of a picosecond, related to the time it takes for the electron spins to relax to their equilibrium state. The FG concept concerns dynamics on time scales much longer than the aforementioned relaxation time, involving macroscopic motion of the whole ferromagnet.

In the present work, we propose a strategy for a proof-of-principle experiment aimed at observing FG precession, and analyze a concrete example of such an experiment involving a levitating sphere above a type-I SC. We model the behavior of an FG levitated above an SC and compare to the behavior of a freely floating FG. Qualitative and quantitative differences are observed in the precession dynamics of the FG in the two cases. In relation to tests of fundamental physics, FGs have recently been proposed as tools to measure general-relativistic precession [39]; here we extend this discussion to show how FGs can be used in other searches for new physics.

As discussed in reference [27], in order for a ferromagnet to exhibit spin precession in an applied magnetic field, it should be in the regime where the intrinsic spin  $S$  due to the magnetization exceeds the classical rotational angular momentum  $L$  associated with the physical rotation of the ferromagnet,  $S \gg L$ . In the opposite case, where the orbital angular momentum associated with precession exceeds that of the spin along the axis, the ferromagnet ‘tips over’ or, in the undamped case, oscillates or librates about its equilibrium orientation along the applied magnetic field. These two regimes can be identified as the precessing regime and the tipping regime. Ferromagnetic compass needles operate in the tipping regime—they tip along the direction of the external magnetic field. Atomic and nuclear spins are in the precessing regime—they precess around the direction of the external magnetic field.

Let us reformulate the criterion  $L \ll S$  for a ferromagnet to be in the precessing regime in the following way: the product of the moment of inertia  $I$  and the precession frequency  $\Omega$  (that represents the classical rotational angular momentum of the system) should be smaller than the spin content of the ferromagnet

$$I\Omega \ll N\frac{\hbar}{2}, \quad (1)$$

where  $N$  is the number of polarized spins and  $\hbar$  is Planck’s constant, such that each electron has an intrinsic spin of  $\hbar/2$ . Rephrasing (1) as a bound on frequency, we have

$$\Omega \ll \Omega^* = \frac{N\hbar}{2I}, \quad (2)$$

or as a bound on the external magnetic field  $B$  applied on the ferromagnet,

$$|B| \ll B^* = \left| \frac{\hbar\Omega^*}{g\mu_B} \right|. \quad (3)$$

Here  $g$  is the Landé  $g$ -factor and  $\mu_B$  is the Bohr magneton. If the applied magnetic field  $B$  is smaller than  $B^*$ , we expect the ferromagnet to be in the precessing regime.

One of the key features of an FG is the fact that a torque on the electron spins generates macroscopic rotation of the ferromagnet. This behavior of an FG is closely related to the Barnett [40] and Einstein–de Haas [41, 42] effects. In contrast to nuclear spins, whose precession is largely decoupled from the crystal lattice as observed in solid-state nuclear magnetic resonance experiments, in a ferromagnet there is strong coupling between electron spins and the crystal lattice via the exchange interaction. These internal dynamics governing an FG are well-described by the Landau–Lifshitz–Gilbert model [43, 44]. Thus, when the electron spins within the ferromagnet are made to precess, the entire ferromagnet rotates.

Constructing an FG by creating suitable conditions for a magnet to precess instead of tipping opens the possibility of a sensitive measurement device. For instance, by bringing an SQUID near the FG and measuring the change in magnetic flux as the FG precesses, the torques acting on the FG can be precisely measured [27].

## 2. Model of a freely floating ferromagnetic gyroscope

To better understand the dynamics of an FG, we model a freely floating FG in space subjected to a constant magnetic field  $\mathbf{B}$ , similar to the modelling in reference [39]. A weak magnetic field causes precession of the FG with Larmor frequency

$$\omega_L = \frac{g_e \mu_B}{\hbar} B = \gamma B, \quad (4)$$

where  $g_e$  is the electron  $g$ -factor, and  $\gamma$  is the gyromagnetic ratio. We consider a spherical FG with radius of  $30 \mu\text{m}$  and  $7 \times 10^{15}$  electron spins, identical to the microsphere used in the experiment described in reference [30]. Note that the threshold precession frequency  $\Omega^*$  described in equation (2) is equal to the Einstein–de Haas frequency

$$\omega_I = \frac{S}{I} = \frac{N\hbar}{2I}, \quad (5)$$

where  $I = 2mr^2/5$  is the moment of inertia for a sphere of mass  $m$  and radius  $r$ . The frequency  $\omega_I$  plays the role of the nutation frequency in the zero magnetic field limit for the FG dynamics.

The equations of motion for the ferromagnet are

$$\frac{\partial \mathbf{j}}{\partial t} = \omega_L (\mathbf{n} \times \hat{\mathbf{B}}), \quad (6)$$

$$\frac{\partial \mathbf{n}}{\partial t} = \omega_I (\mathbf{j} \times \mathbf{n}), \quad (7)$$

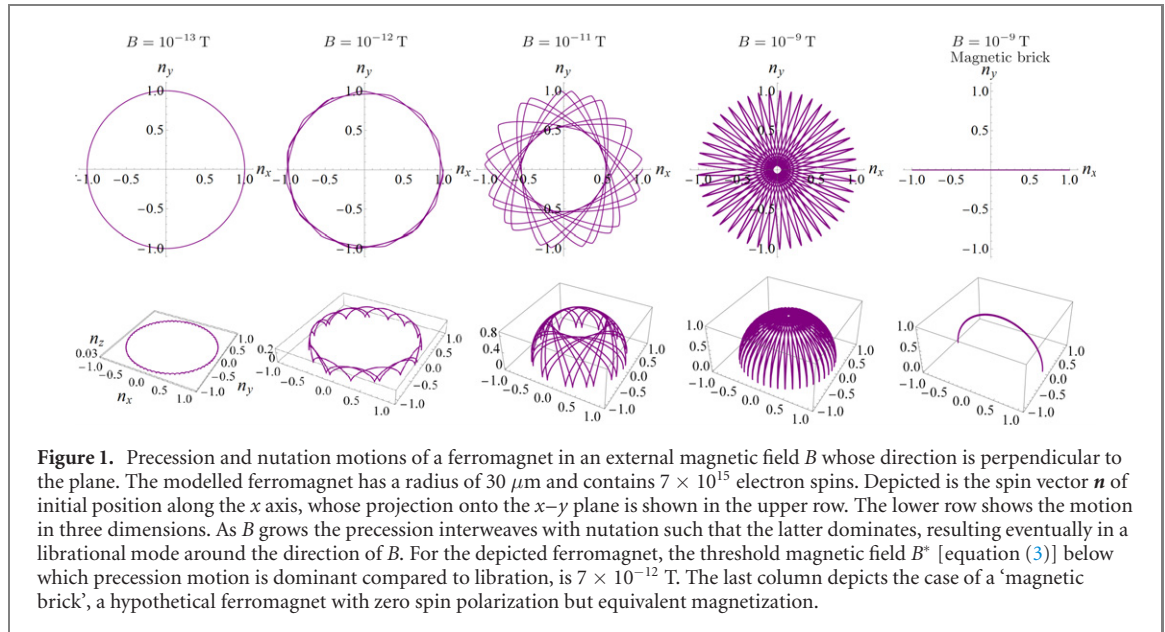
where we defined the following dimensionless vectors: the unit spin  $\mathbf{n} \equiv \mathbf{S}/S$ , the rotational angular momentum  $\boldsymbol{\ell} \equiv \mathbf{L}/S$ , and the total angular momentum  $\mathbf{j} = \mathbf{n} + \boldsymbol{\ell}$ . Equations (6) and (7) are derived from Landau–Lifshitz–Gilbert equations under the assumption that the spin vector is locked to the easy axis, as was done in the modelling of the FG in [35, 39].

Solving for  $\mathbf{j}(t)$  and  $\mathbf{n}(t)$ , figure 1 shows the different kinds of motions of a freely floating FG in an external magnetic field. For magnetic fields below the threshold in equation (3) the precession motion is prominent. In the intermediate regime  $B \approx B^*$  both precession and nutation manifest. At fields much larger than  $B^*$  the amplitude of the nutation grows so large that it manifests as oscillation, i.e. libration, of the ferromagnet about the direction of the applied magnetic field. Note that even in the case where libration is the dominant motion, precession of the plane of libration can still be observed. The frequencies observed in the periodic FG dynamics in each regime can be obtained by analytical approximate solutions of the equations of motion [45].

## 3. Experimental strategy

To observe precession of a FG, we propose to work at an external magnetic field weaker than the threshold, below which precession dominates (at sufficiently low magnetic fields, the amplitude of nutation becomes relatively small so that the dynamics of the FG are dominated by the precession). Generally, in experiments, this will require both shielding and careful control of the external magnetic field. Fortunately, the ferromagnet itself can be used as a magnetometer even for fields larger than the threshold field for precession by measuring the libration frequency  $\omega_l$ . Oscillation of a ferromagnet at the libration frequency  $\omega_l$  was observed in soft ferromagnetic levitating particles [32] (denoted there as  $\omega_\phi$ ) and with ferromagnets levitating above type-I SC [30]. For a hard ferromagnet,  $\omega_l$  is the geometrical average of the Larmor frequency  $\omega_L$  and the Einstein–de Haas frequency  $\omega_I$  [29]

$$\omega_l^2 = \omega_L \omega_I. \quad (8)$$



Since the libration frequency  $\omega_l$  depends on the magnetic field, it can be used to measure and reduce the magnetic field until precession dominates the dynamics. For a freely floating FG, as one reduces the magnetic field below the threshold field defined in equation (3), the frequencies will split on the logarithmic scale (figure 2) such that they can be resolved in the magnetic flux spectrum measured by an SQUID pick-up loop.

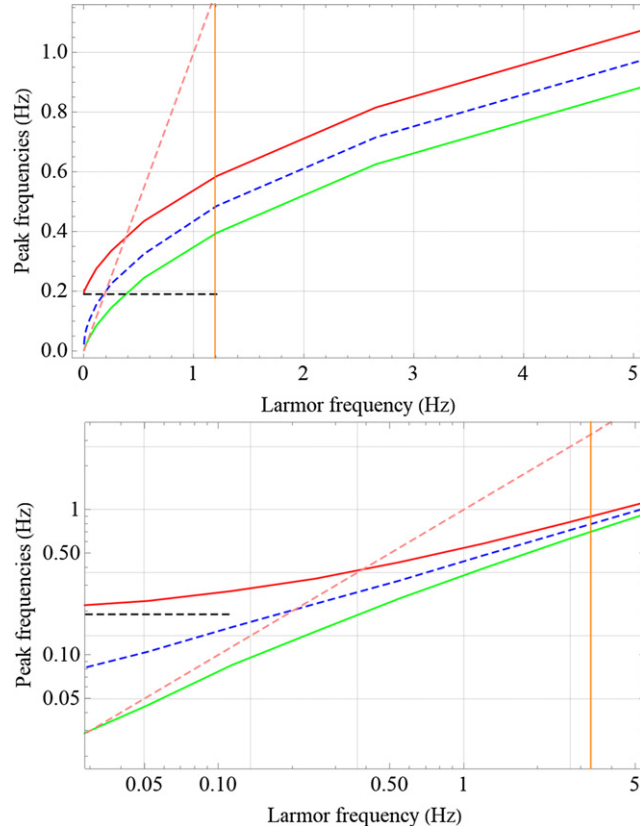
Quantitatively, solving equations (6) and (7) for  $\mathbf{j}(t)$  and  $\mathbf{n}(t)$  for various  $\omega_l$ , the model of the dynamics of a freely floating FG shows signals at two distinct frequencies in the magnetic flux observed along the  $x$ -direction (perpendicular to the magnetic field applied along  $z$ ). These frequencies are fractionally split in low magnetic fields, that is, the difference between the frequencies, normalized by their geometric average, becomes bigger at lower frequencies, as shown in figure 2, where the fractional behavior is emphasized on the logarithmic scale. The apparent splitting of the nutation and precession curves on the logarithmic scale in figure 2 (the red and green curves) points to the transition from the librational behavior above the threshold (the vertical line) into the precession and nutation motion below the threshold. The two frequencies can be viewed as a modulation of a central frequency (dashed blue line in figure 2) which appears in the case of a ‘magnetic brick’, a hypothetical ferromagnet with zero spin polarization ( $\mathbf{j} = 0\ \mathbf{n} + \ell$ ) but equivalent magnetization. The concept of a magnetic brick is introduced to separate, in the model, effects due to magnetic torques from effects related to the gyroscopic nature of the ferromagnet.

As for the precession frequency, green curve in figure 2, it deviates from the Larmor frequency in equation (4), as expected from the interplay between nutation and precession motions [46]. The Larmor frequency is a dashed pink line with a unit slope on the linear scale of figure 2. The parameters used in the model match those for the experimental setup discussed in the next section, which result in  $\omega_l = 1.193\ \text{rad s}^{-1}$  [30]. This  $\omega_l$  is plotted as a dashed black horizontal line in figure 2.

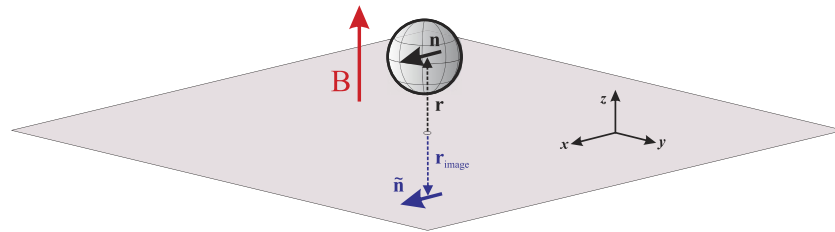
Above the threshold, the librational-mode frequency  $\omega_l$  can be observed and used to measure and control the magnetic field. Using this technique the magnetic field can be tuned below the threshold field where the FG dynamics clearly display precession and nutation, demonstrating the gyroscopic behavior of a ferromagnet and confirming experimentally the prediction of precession. In the next section we examine this experimental strategy in the context of a ferromagnetic microsphere levitating above a type-I SC.

#### 4. Ferromagnetic gyroscope levitated above a type-I superconductor

A promising avenue for experimental realization of an FG are optomechanical and magnetomechanical systems, for instance: a ferromagnet levitated by magnetic or electric fields. In particular, the motion, dynamics and stability of a magnetically levitated ferromagnet have been studied [45] and are in agreement with expectations regarding the precessing and tipping regimes. Here we consider a ferromagnetic microsphere levitating above a type-I SC (figure 3). In this case, the expulsion of the magnetic field from the SC by the Meissner effect creates a field in the region above the SC mathematically equivalent to that



**Figure 2.** Modelling the dynamics of an FG by equations (6) and (7). In linear and logarithmic scales, presented are the frequencies of the maxima in the spectrum of FG dynamics as can be measured with an SQUID pick-up loop, as a function of the Larmor frequency  $\omega_L$ . The external magnetic field direction is perpendicular to the precession plane, as in figure 1. The SQUID pick-up loop measures the flux from the FG in the horizontal direction  $x$ . The middle line (dashed blue) is the sole frequency appearing in the spectrum of a ‘magnetic brick’ (hypothetical ferromagnet with zero spin polarization but equivalent magnetization, see main text) with a radius of  $30\ \mu\text{m}$  in an external magnetic field. The red and green curves are the frequencies of a fully spin-polarized ferromagnet with the same radius, corresponding respectively to nutation and precession frequencies. As might be expected from equation (8), the blue line is the geometric average of the red and green lines, above the threshold frequency. The orange vertical line is the threshold frequency  $\Omega^*$ . The dashed pink line is the Larmor frequency of equation (4). The dashed black line,  $\omega_J$ , corresponds to the nutation frequency in the zero magnetic field limit.



**Figure 3.** Schematic setup for an FG levitated above a type-I SC. The sphere and arrow labeled  $\mathbf{n}$  represent the FG, the gray plane represents the surface of the SC, the blue arrow labeled  $\tilde{\mathbf{n}}$  represents the image dipole, the red arrow indicates the external magnetic field applied along the vertical direction,  $\mathbf{r}$  is the vector pointing to the location of the center of the sphere and  $\mathbf{r}_{\text{image}}$  is the vector pointing to the location of the image dipole.

from image dipole. The image-dipole magnetic field pushes the microsphere up while gravity pulls it down. To investigate the effect of the SC on the FG dynamics, we include the field from the image dipole in the modelling.

The image field  $\mathfrak{B}$  is a magnetic field emanating from the image dipole located at a vertical distance  $2z$  (center to center) below the levitating FG, where  $z$  is the height of the FG above the SC plane. In SI units,

$$\mathfrak{B} = -\frac{\mu_0 \mu}{4\pi \tilde{r}^5} \{3\tilde{\mathbf{r}} (\tilde{\mathbf{r}} \cdot \tilde{\mathbf{n}}) - \tilde{\mathbf{n}} \tilde{r}^2\}, \quad (9)$$



where  $\mu_0$  is the permeability of free space,  $\tilde{\mathbf{r}}$  is relative to the position of the image dipole

$$\begin{aligned}\tilde{\mathbf{r}} &= \mathbf{r} - \mathbf{r}_{\text{image}}, \\ \mathbf{r} &= (x, y, z), \\ \mathbf{r}_{\text{image}} &= (x, y, -z),\end{aligned}\tag{10}$$

$\mu\tilde{\mathbf{n}}$  is the magnetic moment of the image dipole and  $\mu\mathbf{n}$  is the magnetic moment of the levitating ferromagnet

$$\begin{aligned}\mathbf{n} &= (n_x, n_y, +n_z), \\ \tilde{\mathbf{n}} &= (n_x, n_y, -n_z).\end{aligned}\tag{11}$$

Here we take the origin of the coordinate system to be on the SC plane. The image dipole has the same horizontal component of the magnetic moment as the FG, and opposite vertical component.

To include  $\mathfrak{B}$  into the equations of motion in section 2, we derive the Larmor frequency associated with this image dipole field

$$\omega_{\mathfrak{B}} = \gamma\mathfrak{B},\tag{12}$$

so that equation (6) contains the term

$$\omega_{\mathfrak{B}} (\mathbf{n} \times \mathfrak{B}).\tag{13}$$

Moreover, we include the ferromagnet center-of-mass equations of motion

$$\frac{\partial \mathbf{p}}{\partial t} = \frac{\mu}{2} \nabla (\mathfrak{B} \cdot \mathbf{n}) + m\mathbf{g},\tag{14}$$

$$\frac{\partial \mathbf{r}}{\partial t} = \frac{\mathbf{p}}{m},\tag{15}$$

where  $\mathbf{p}$  is the FG center-of-mass momentum,  $\mathbf{g}$  is the gravitational acceleration, and the factor 1/2 is a consequence of the image dipole being not frozen in type-I SC, i.e., following the levitating dipole [47–49].

Modelling the levitating FG dynamics of the spin vector  $\mathbf{n}$  and the center-of-mass motion, we recover the frequencies  $\omega_z$  and  $\omega_\beta$  experimentally observed in reference [30], which describe oscillation of the center-of-mass in the vertical direction and libration of the magnetic moment about the vertical axis, respectively. Libration of the levitated ferromagnet at the frequency  $\omega_\beta$  is predominantly caused by the image dipole field. Before introducing an additional external magnetic field to observe the effects of Larmor precession, let us note the precession motion that exists even without the introduction of an external magnetic field. We observe in the modelling a precession in the horizontal plane, with a frequency of

$$\omega_{xy} = \omega_I n_{z0},\tag{16}$$

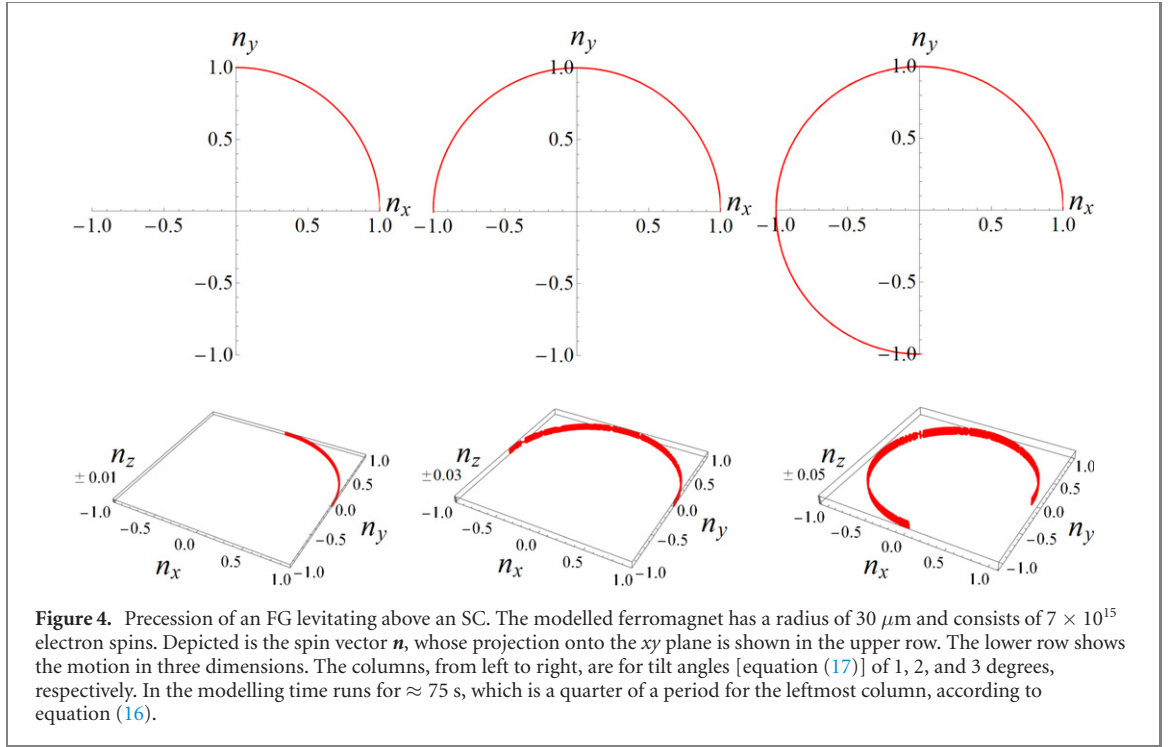
where  $n_{z0}$  is the initial vertical component of the FG magnetic moment, which is linked to the tilt angle  $\beta$

$$\sin \beta = \frac{n_{z0}}{n_0},\tag{17}$$

where  $n_0$  is the unit spin vector and  $n_z$  is the length of its vertical component, at the initial moment of the modelling. Such a vertical component of the magnetic moment and spin translates to a vertical component of the total angular momentum, since  $\mathbf{j} = \mathbf{n} + \ell$ . The librational mode  $\omega_\beta$  corresponds to an oscillation between  $n_z$  and  $\ell_z$ . The image field does not change  $\mathbf{j}_z = \mathbf{n}_z + \ell_z$ , thus as long as the mean value of  $\ell_z$  is not zero, precession occurs around the vertical axis, i.e. rotation of  $\mathbf{n}$  in the horizontal plane ensues. In figure 4 we present examples of such a precession in the modelling. We also observe in the modelling that setting the FG's initial angular momentum to  $\ell_z = -\sin \beta$  counteracts the effect of the tilt, as expected from conservation of angular momentum.

We can explain the appearance of  $\omega_{xy}$  in terms of the image dipole. The image dipole precesses with the FG so the component of the image field in the horizontal  $xy$  plane acting on the FG is constant in the rotating frame. On the other hand, the tilt of the FG with respect to the vertical axis changes the field acting on the FG due to the image dipole—the librational oscillation causes an oscillating field along  $z$  that induces FG precession. Since the librational oscillation frequency is fast compared to the precession frequency, effectively the FG is sensitive to the average field, such that bigger librational oscillation results in a bigger effective field and faster precession. Note that the vertical component of the field appears due to the initial tilt angle of the FG magnetization axis out of the horizontal plane.

Such a precession was not observed in previous experiments as the ferromagnetic microsphere was not free to rotate in the horizontal plane, either because of the SC's tilt out of the horizontal plane in reference



[30] or because of frozen flux in reference [28]. This tilt (or frozen flux) introduces a preferred direction for the magnetic moment of the levitating microsphere, so that it is situated in an energetic minimum; thus the ferromagnet oscillates around this direction (with frequency  $\omega_\alpha$  [30]) instead of precessing in the horizontal plane.

The above precession occurs due to the image field, while to use an FG to measure external torques we seek to observe the effect due to, for example, an external magnetic field. Therefore we introduce an external magnetic field  $\mathbf{B}_{\text{ext}}$ , consequently adding  $\mu \nabla (\mathbf{B}_{\text{ext}} \cdot \mathbf{n})$  to the right-hand side of equation (14); equation (6) is modified to read

$$\frac{\partial \mathbf{j}}{\partial t} = \omega_L (\mathbf{n} \times \hat{\mathbf{B}}_{\text{ext}}) + \omega_{\mathfrak{B}} (\mathbf{n} \times \hat{\mathfrak{B}}). \quad (18)$$

Since  $\mathbf{j}_z$  is a constant of motion, if the tilt angle  $\beta$  is initially zero (horizontal FG with respect to the SC surface), so that  $\mathbf{j}_z(t=0) = 0$ , then the angular momentum associated with precession  $\ell_z = I\Omega$  must be equal and opposite to  $\mathbf{s}_z = N\hbar \sin \beta$ . Thus we have

$$\sin \beta = \frac{I\Omega}{N\hbar}. \quad (19)$$

The magnitude of the image dipole field a distance  $z_0$  above the SC surface is

$$|\mathfrak{B}| = \frac{\mu\mu_0}{4\pi z_0^3}, \quad (20)$$

and its  $z$ -component is

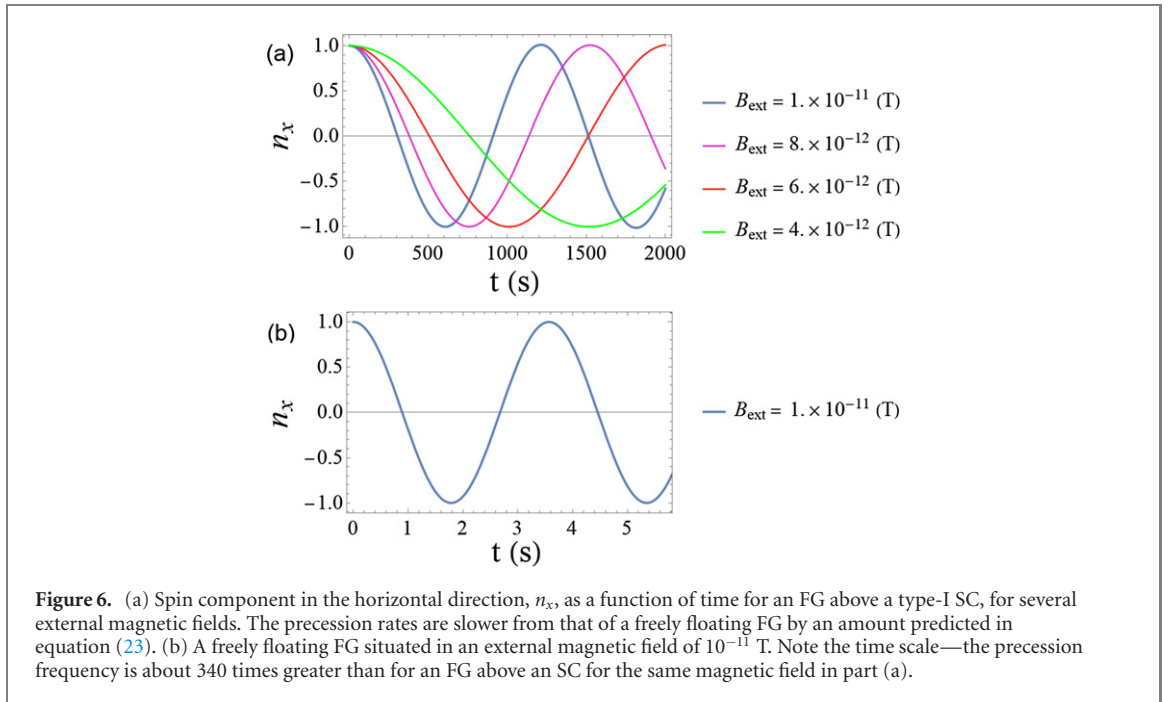
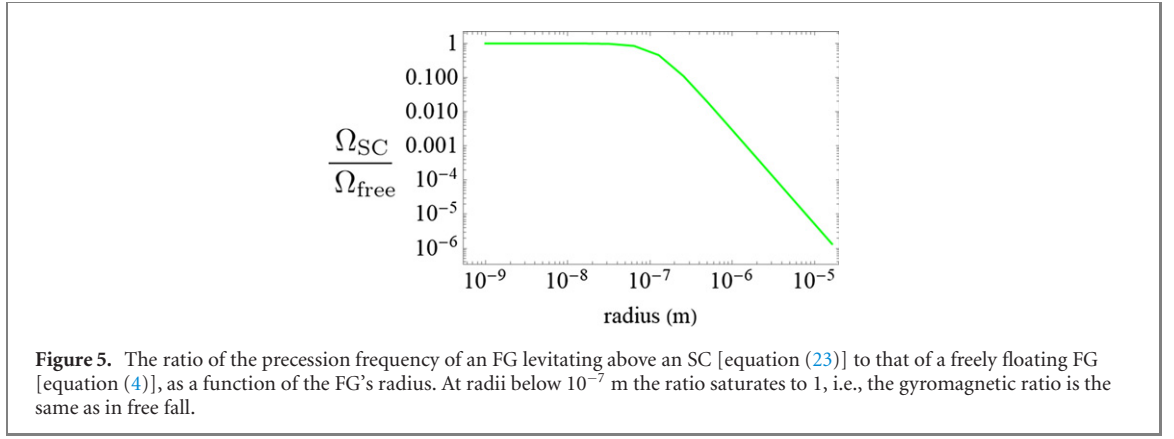
$$\mathfrak{B}_z = \mathfrak{B} \sin \beta. \quad (21)$$

The effective magnetic field that the FG experiences is the vector sum of the external magnetic field  $\mathbf{B}_{\text{ext}}$  (taken to be along the  $z$ -axis) and  $\mathfrak{B}_z$ , so the precession frequency  $\Omega$  is now given by

$$\Omega = \gamma (B_{\text{ext}} - \mathfrak{B}_z) = \gamma \left( B_{\text{ext}} - \mathfrak{B} \frac{I\Omega}{N\hbar} \right). \quad (22)$$

Solving for  $\Omega$  we find

$$\begin{aligned} \Omega &= \frac{\gamma B_{\text{ext}}}{1 + (\gamma \mathfrak{B} / \omega_I)}, \\ &= \frac{\gamma B_{\text{ext}}}{1 + \frac{\gamma \mu \mu_0 I}{2\pi z_0^3 N \hbar}}. \end{aligned} \quad (23)$$



Thus an FG levitated above an SC possesses an effectively reduced gyromagnetic ratio compared to the freely floating FG. The suppression of the effective gyromagnetic ratio due to the image field can be explained by a mechanism analogous to negative feedback [50]. Requiring  $\Omega \ll \Omega^*$  for the FG to be in the precession regime gives

$$B_{\text{ext}} \ll \mathfrak{B} + \frac{\omega_I}{\gamma} \approx \mathfrak{B}. \quad (24)$$

The image field is typically much larger than  $B^*$ ; hence precession can be observed in higher magnetic fields for an FG levitated over an SC compared to a freely floating FG. However, the corresponding precession frequency is smaller compared to a free FG. For a spherical FG with 30 micron radius, matching the experimental conditions of reference [30], the ratio of a free FG precession frequency to that for an FG levitated above an SC is  $\approx 4 \times 10^6$ . For 1 micron radius, this ratio is  $\approx 340$ , so the suppression of precession frequency is reduced in the case of a smaller-radius FG. Based on equation (23), in figure 5 we plot as a function of the FG radius the ratio of the precession rate for a levitating FG above an SC to that for a freely floating FG.

This constraint on  $B_{\text{ext}}$  can be viewed as an effective threshold field for a levitated FG above an SC due to the image field. Modelling such a system for the conditions of the levitated ferromagnet from [30], but with 1 micron radius instead of 30 microns, we observe gyroscopic behavior in the time domain, as shown in figure 6, consistently 340 times slower than that in the case of precession of a freely floating FG. As another check of the negative feedback explanation, we have varied the magnitude of the image field  $\mathfrak{B}$  (by varying the gravitational field magnitude) and observed in the modelling suppressed precession rates (compared to freely floating FG) matching the expected rates from equation (23). In figure 6 we decoupled



the motion of the center of mass from that of the spin vector  $\mathbf{n}$ , for clarity. In the presence of both  $B_{\text{ext}}$  and a finite initial tilt angle  $\beta$  [that of equation (17)], the resulting precession frequency is the difference between the precession frequency in the case of initial tilt angle with null  $B_{\text{ext}}$ , and that with  $B_{\text{ext}}$  and null initial tilt angle [in accordance with equation (22)].

To observe  $\omega_L$ , and then precession, an external magnetic field should be introduced, and several modifications should be made to the experimental apparatus used in [30]. One challenge is that external torques can effectively lock the ferromagnet's orientation and prevent precession. In a previous study of a ferromagnet levitated above a type-II SC, the ferromagnet's orientation was locked by the magnetic field due to trapped flux in the SC [28]. In type-I SC, however, flux trapping can be eliminated. Yet in a recent experiment with a levitating microsphere the ferromagnet was not free to rotate in the horizontal plane because of locking due to a relative tilt of the surface of the cylindrical 'bowl-shaped' trap in the SC [30]. In order to allow the microsphere to nutate and precess in the horizontal plane, a spherical 'bowl-shaped' trap, instead of a cylindrical one, could be used. Following an observation of the horizontal precession due to the image field, an external magnetic field  $\mathbf{B}_{\text{ext}}$  can be introduced in the  $z$  direction, much like was done in [28]. This field is expected to cause a librational motion of the FG around it, with a frequency  $\omega_L$  which can be detected with a sufficiently sensitive magnetic field sensor, such as an SQUID. Reducing  $B_{\text{ext}}$  below the threshold in equation (24), nutational motion will appear. Further reducing the magnetic field will reveal  $\omega_L$ . Note that the threshold in equation (24) is larger relative to equation (3), and thus is easier to control technically.

## 5. Sensitivity to new physics

An FG is a correlated system of  $N$  electron spins that acts as a gyroscope with total spin  $\sim N\hbar/2$ . Spin projections transverse to the FG's magnetization axis fluctuate rapidly due to interaction with the crystalline lattice while, unless acted upon by an external torque, the expectation value of the total spin vector  $\mathbf{S}$  remains fixed due to angular momentum conservation. This behavior enables rapid averaging of quantum uncertainty, opening the possibility of measuring torques on electron spins with a sensitivity many orders of magnitude beyond the present state-of-the-art [27, 39]. For this reason, FGs can be powerful tools to search for physics beyond the standard model [9].

Sensitivity estimates carried out in references [27, 39] assume a freely floating FG in ultrahigh cryogenic vacuum at temperatures  $\approx 0.1$  K (residual He vapor density  $\approx 10^3$  atoms/cm<sup>3</sup>). Here we carry out sensitivity estimates for an FG levitated above a type-I SC under the vacuum conditions achieved in the experiment of reference [30] (residual helium pressure  $\approx 10^{-5}$  mbar, corresponding to a He vapor density of  $n \approx 3 \times 10^{13}$  atoms/cm<sup>3</sup>) at a temperature of  $\approx 4$  K. We assume a spherical FG with radius  $\approx 1$   $\mu\text{m}$ . Therefore the conditions assumed in the following discussion are practically realizable with relatively minor modifications to existing experimental apparatuses.

In reference [30], the dominant source of noise comes from collisions of He atoms with the FG. These collisions transfer angular momentum to the FG and cause a random walk of precession angle  $\phi$  [27, 39]. For a spherical freely floating FG, the uncertainty in the precession frequency caused by gas collisions is given by [39]

$$\Delta\Omega_{\text{col}} \approx \frac{m_g r^2}{6N\hbar} \sqrt{\frac{nv_{\text{th}}^3}{\pi t}}, \quad (25)$$

where  $v_{\text{th}}$  is the mean thermal velocity of the residual gas atoms and  $m_g$  is their mass. However, in the case of an FG levitated above an SC, 'negative feedback' from the image dipole field  $\mathfrak{B}$  affects the FG's response to random torques caused by gas collisions in much the same way as it affects the Larmor precession frequency as described by equation (23). In general, the effect of any external torque  $\boldsymbol{\tau}_{\text{ext}}$  acting on the FG is modified by this 'negative feedback' mechanism. The equation, analogous to equation (18), describing the rate of change of total angular momentum  $\mathbf{J}$  is

$$\frac{d\mathbf{J}}{dt} = \boldsymbol{\tau}_{\text{ext}} + \omega_{\mathfrak{B}} (\mathbf{S} \times \hat{\mathfrak{B}}). \quad (26)$$

The external torques from gas collisions generate stochastic (random) variation in the precession frequency, and, because of nutation, a correlated stochastic variation in the tilt angle  $\beta$  and thus  $\mathfrak{B}$ . Just as equation (25) was derived using equation (6) as a starting point [27, 39], we can start from equation (26) and, following the same logic used to derive equation (23), show that the uncertainty in the precession frequency due to gas collisions for an FG levitated above an SC is given by

$$\Delta\Omega'_{\text{col}} \approx \frac{\Delta\Omega_{\text{col}}}{1 + (\gamma\mathfrak{B}/\omega_I)}. \quad (27)$$

Therefore, under the conditions considered here, the effects of gas collisions on FG dynamics are smaller (compared to a freely floating FG) by 340 times. This results in an uncertainty in the measured FG precession frequency of

$$\Delta\Omega'_{\text{col}} \sim \frac{10^{-5}}{\sqrt{t}} \text{ rad s}^{-1}. \quad (28)$$

Other potential noise sources, such as thermal currents and blackbody radiation, were considered in references [27, 39] and are also found to be negligible under the experimental conditions of reference [30]. Furthermore, the experimental results in reference [30] for a 30-micron-radius levitated ferromagnet showed that eddy current damping was negligible, and eddy current damping contributes even less for smaller FG radii: the eddy current power dissipation in a conducting sphere is  $\propto r^5$ . A one-micron radius ferromagnet can be single domain, in which case direct (hysteresis-based) magnetic losses should be largely suppressed as well.

The precession of the FG can be measured with an SQUID. For an SQUID with pick-up loop radius of  $\approx 1 \mu\text{m}$  situated about a micron from an FG (such that the flux capture is maximal), the amplitude of the time-varying magnetic flux is  $\Phi \approx 10^{-12} \text{ T} \cdot \text{m}^2$ . Low-temperature SQUIDS have a flux sensitivity of  $\delta\Phi \lesssim 10^{-21} \text{ T} \cdot \text{m}^2 (\sqrt{\text{Hz}})^{-1}$  [51–54], which yields a corresponding sensitivity to the precession angle of  $\delta\phi \approx \delta\Phi/\Phi \approx 10^{-9} \text{ rad } (\sqrt{\text{Hz}})^{-1}$ . Thus the detection-limited uncertainty in a measurement of the FG precession frequency  $\Omega = d\phi/dt$  integrating over a time  $t$  is  $\Delta\Omega_{\text{det}} \sim 10^{-9} t^{-3/2} \text{ rad s}^{-1}$ . Since the uncertainty in the measurement of precession due to gas collisions is far larger than  $\Delta\Omega_{\text{det}}$ , requirements on the pick-up loop geometry and SQUID sensitivity are correspondingly relaxed. For example, a pick-up loop radius of  $R \approx 1 \text{ mm}$  positioned  $\approx 1 \text{ mm}$  from the FG would achieve a detection-limited sensitivity in 1 s of integration time better than the gas collision limit,

$$\Delta\Omega'_{\text{det}} \sim 10^{-6} \frac{1}{t^{3/2}} \text{ rad s}^{-1}. \quad (29)$$

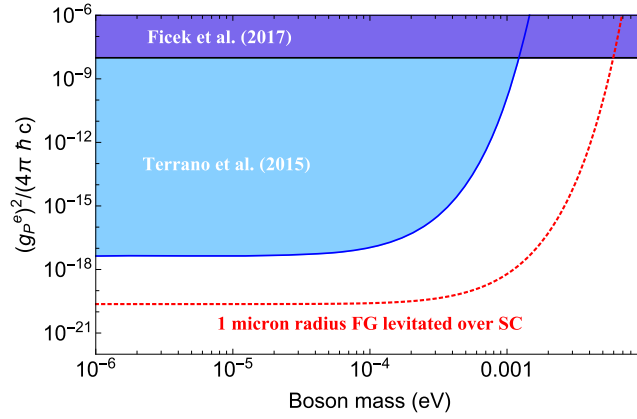
Vibrations were another important source of technical noise in the experiment described in reference [30]. Relative motion between the position of the FG and the SQUID pick-up loop lead to variations in the flux through the loop and consequently generate noise in the precession measurement. Commercial vibration isolation systems used, for example, in atomic force microscopy experiments can reduce vibration amplitudes to  $\delta x \lesssim 10^{-5} \text{ mm}$  at frequencies  $\lesssim 1 \text{ Hz}$  [55]. The fractional flux noise in the pick-up loop  $\delta\Phi/\Phi \sim \delta x/x$ , where  $x \approx R \approx 1 \text{ mm}$  is the distance between the FG and the pick-up loop. This corresponds to an uncertainty in the precession measurement similar to the gas-collision limit,

$$\Delta\Omega_{\text{vib}} \sim \frac{10^{-5}}{\sqrt{t}} \text{ rad s}^{-1}. \quad (30)$$

It is notable that  $\Delta\Omega_{\text{det}}$  appears to surpass the ‘standard quantum noise limit’ [27, 56]. While the energy resolution per bandwidth ( $E_R$ ) for existing magnetometers is at or above the quantum limit  $\hbar$ , an FG can in principle achieve  $E_R \ll \hbar$ , under conditions where external sources of error are controlled so that the FG sensitivity is limited by detector noise [33]. Such accuracy arises because the quantum uncertainty is rapidly averaged by the internal ferromagnetic spin-lattice interaction, while the FG maintains gyroscopic stability due to the conservation of the total angular momentum (dominated by the intrinsic spin). Another way to understand the sensitivity of an FG is to note that the ferromagnetic spin-lattice interaction spreads the quantum fluctuations over a broad frequency band ( $\gtrsim 1\text{--}100 \text{ GHz}$ ). Due to the gyroscopic stability, one can still measure slow changes of the average direction of the FG spin. Integrating over long periods of time averages the quantum fluctuations, acting as a ‘low-pass filter’ for the quantum noise. Thus a high sensitivity to comparatively low-frequency spin precession can be achieved.

As an example of the potential of FGs as tools for testing fundamental physics, we consider an experimental search for yet-to-be-discovered (exotic) spin-dependent interactions mediated by new bosons [57–59]. In particular, axions and axionlike particles (ALPs) mediate a pseudoscalar ( $P$ ) interaction between electrons described by the potential

$$\begin{aligned} \mathcal{V}_{PP}(\mathcal{R}) = & \frac{(g_P^e)^2}{4\pi\hbar c} \frac{\hbar^3}{4m_e^2 c} \left[ \mathbf{S}_1 \cdot \mathbf{S}_2 \left( \frac{m_b c}{\hbar \mathcal{R}^2} + \frac{1}{\mathcal{R}^3} + \frac{4\pi}{3} \delta^3(\mathcal{R}) \right) \right. \\ & \left. - (\mathbf{S}_1 \cdot \hat{\mathcal{R}}) (\mathbf{S}_2 \cdot \hat{\mathcal{R}}) \left( \frac{m_b^2 c^2}{\hbar^2 \mathcal{R}} + \frac{3m_b c}{\hbar \mathcal{R}^2} + \frac{3}{\mathcal{R}^3} \right) \right] e^{-m_b c \mathcal{R}/\hbar}, \end{aligned} \quad (31)$$



**Figure 7.** Comparison between the existing experimental constraints (solid lines and shaded regions) on a pseudoscalar-mediated dipole–dipole interaction between electron spins and the projected sensitivity of an experiment using a one micron radius spherical FG levitated above a type-I SC (dotted red line). The projected sensitivity is based on the gas-collision limit [equation (28)], comparable to the expected technical limit due to vibrations and microphonic noise, equation (30)]. Constraints shown with the black line and dark blue shaded region are based on He spectroscopy [63]; constraints shown with the blue line and light blue shaded region are from an experiment using a spin-polarized torsion pendulum [60]. The proposed experiment with the levitated FG assumes as a polarized spin source a 1 mm radius  $\text{SmCo}_5$  sphere positioned 1 mm away from the FG and an integration time of  $t = 10^6$  s.

where  $(g_P^e)^2 / (4\pi\hbar c)$  is the dimensionless pseudoscalar coupling constant between electrons,  $m_e$  is the electron mass,  $\mathbf{S}_{1,2}$  are the electron spins,  $m_b$  is the mass of the hypothetical pseudoscalar boson,  $c$  is the speed of light, and  $\mathcal{R} = \mathcal{R}\hat{\mathcal{R}}$  is the separation between the electrons.

One could search for spin precession induced by the pseudoscalar-mediated dipole–dipole interaction, equation (31), by modulating the distance between a polarized spin source and a levitated FG. Some of the most stringent laboratory constraints on such exotic dipole–dipole interactions have been achieved using spin-polarized torsion balances [60, 61] with  $\text{SmCo}_5$  as a polarized spin source. In  $\text{SmCo}_5$ , the orbital magnetic moment of the  $\text{Sm}^{3+}$  electrons nearly cancels their spin moment, and so  $\text{SmCo}_5$  possesses a high spin polarization while having a relatively small magnetic moment, thus reducing magnetic-field-related effects. The spin-polarized source in such an experiment could be positioned underneath the SC to further shield the FG from the magnetic field due to the spin source. Although the SC will shield the FG from the magnetic field of the  $\text{SmCo}_5$  spin source, it turns out that the pseudoscalar interaction (31) is unshielded by the SC [62]. This is a consequence of the fact that SC shielding relies on the coupling of magnetic fields to currents rather than to electron spins. Thus, since the Meissner effect is unrelated to interactions with the electron spins, the SC shield has no effect on the pseudoscalar-mediated dipole–dipole interaction described by equation (31) [61, 62]. Note that effects due to exotic interactions manifest as external torques  $\tau_{\text{ext}}$  as described by equation (26), and therefore their influence on the precession frequency is suppressed by the same factor appearing in equations (23) and (27).

An experiment using a one-micron-radius FG levitated above an SC would be sensitive to the region of parameter space bound from below by the dotted red line in figure 7. We assume that the  $\text{SmCo}_5$  spin source is a one-mm radius sphere positioned one mm away to the FG to allow space for the SC. A one-mm-radius  $\text{SmCo}_5$  sphere would contain  $\sim 5 \times 10^{19}$  polarized electron spins. The FG sensitivity to spin precession is given by equation (28). For comparison, figure 7 shows the most stringent laboratory constraints in this region of parameter space, which are based on spin-polarized torsion-balance measurements [60] and He spectroscopy [63, 64]; related experiments are discussed in the review [9] and references [16, 65–71]. Compared to these existing constraints, our proposed experiment with a levitated FG can explore many decades of unconstrained parameter space. This illustrates the potential of FGs as tools to search for exotic spin-dependent interactions, which could open a window to beyond-the-Standard-Model physics. We also note that new bosons such as axions and ALPs are candidates to explain the nature of dark matter [9], and much like other types of precision mechanical sensors [72], FGs can be useful tools for the potential detection of bosonic, wavelike dark matter.

## 6. Conclusion

In summary, we present a roadmap for experimental realization of a FG. In essence, an FG is a ferromagnet that precesses under the influence of external torques. A FG is a new type of sensor that can be particularly useful as a tool for precision tests of fundamental physics.

We model and explain the dynamics of an FG freely floating in space and propose a strategy to experimentally realize an FG. The librational mode in the magnetization dynamics serves as a calibration tool for the applied magnetic field. This enables the magnetic field to be tuned to sufficiently small magnitudes so that the FG precession mode can be observed.

We also compare the dynamics of a freely floating FG to that of an FG levitated above a type-I SC. The effect of the SC is modelled using an image dipole field. We find that the SC has a significant effect on the FG dynamics: the image dipole field generates a ‘negative feedback’ that effectively suppresses the response of an FG to external torques as compared to the case of a freely floating FG. The effective magnetic field threshold below which precession is dominant is thus higher in the case of an FG levitated above an SC as compared to a freely floating FG [equation (24)] while the observed precession frequency for a given field strength is lower [equation (23)].


## Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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