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Asymptotically safe Hilbert–Palatini gravity in an on-shell reduction scheme

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Abstract We study the renormalization flow of Hilbert–Palatini gravity to lowest non-trivial order. We find evidence for an asymptotically safe high-energy completion based on the existence of an ultraviolet fixed point similar to the Reuter fixed point of quantum Einstein gravity. In order to manage the quantization of the large number of independent degrees of freedom in terms of the metric as well as the connection, we use an on-shell reduction scheme: for this, we quantize all degrees of freedom beyond Einstein gravity at a given order that remain after using the equations of motion at the preceding order. In this way, we can straightforwardly keep track of the differences emerging from quantizing Hilbert–Palatini gravity in comparison with Einstein gravity. To lowest non-trivial order, the difference is parametrized by fluctuations of an additional abelian gauge field. The critical properties of the ultraviolet fixed point of Hilbert–Palatini gravity are similar to those of the Reuter fixed point, occurring at a smaller Newton coupling and exhibiting more stable higher order exponents.

1 Introduction

For the approach to quantizing gravity both the degrees of freedom to be quantized as well as the correct quantization method are a matter of intense debate. A priori different choices could lead to different potentially consistent theories of quantum gravity ultimately requiring experimental data to single out the theory realized in nature.

It is well known that – already on the classical level – very different choices for the degrees of freedom can lead to the same classical equation of motion, namely Einstein's

equation [1–4]. This includes of course the maybe simplest choice being the metric, but also the vierbein, possibly in combinations with various forms of connections. Of course, classical equivalence does not entail quantum equivalence, therefore different choices might rather be expected to lead to different quantum theories.

In turn, different choices of degrees of freedom or even the quantization procedure could finally describe the same quantum theory if they lead to the same universality class as for instance identified by a renormalization group analysis. In discrete approaches this can be indicated by the presence of a second order phase transition or even quantified in terms of critical exponents of a corresponding quantum critical point; in the context of gravity, cf. [5–8].

Research in recent years has accumulated evidence that such a quantum critical point exists for gravity when using the metric as the fundamental degree of freedom together with a (standard) quantization procedure that is able to capture non-perturbative information. The latter is necessary, since the quantum critical point corresponds to a fixed point of the renormalization group at a finite coupling, realizing Weinberg's asymptotic safety scenario for Einstein gravity [9,10]. This Reuter fixed point has been discovered by applying functional renormalization group (RG) methods to gravity [11], and confirmed in many refined studies, see [12–17] for recent reviews.

Functional RG methods and a classification of universality classes [18] are not limited to the metric as the quantum degree of freedom. In fact, pioneering studies have been performed for Einstein–Cartan theory with the Hilbert–Palatini action being generalized to the Holst action [19–21] or constrained to self-dual connections [22], or for "tetrad only" formulations [23]. All these works find indications for the existence of UV fixed points supporting asymptotic safety of such quantum gravity theories, with the fixed points likely

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representing universality classes different from that of quantum Einstein gravity. In fact rather complex phase diagrams are partly found being paralleled by the complexity of the computations involving such a large number of gauge degrees of freedom. On the other hand, it is interesting to observe that also reduced versions such as unimodular gravity [24–28] or conformally reduced gravity [29–32] exhibit UV complete renormalization group trajectories (see however [33] for a critical view on conformally reduced gravity.)

A major motivation to study formulations of gravity based on the metric and the connection is the greater similarity to gauge theories of particle physics. In addition to this structural resemblance, also a larger technical toolkit developed for gauge theories may become available for concrete calculations; specifically lattice formulations for gravity become accessible [34,35].

In the present work, we suggest to reduce the amount of complexity introduced by the large number of gauge degrees of freedom of a metric-affine formulation by an on-shell reduction scheme: at a given expansion order, we only quantize those degrees of freedom which remain after using the equations of motion of the preceeding order. For instance at lowest order in the curvature (Einstein–Hilbert level), different choices of degrees of freedom boil down to the Einstein equation for the metric; hence the on-shell reduction suggests to quantize only the metric at this level. At higher order in the curvature, connection degrees of freedom typically develop their independent dynamics. In the present work, we will focus on the second-order curvature level where a co-vector field remains as an independent degree of freedom in the connection after on-shell reduction. The corresponding additional action to be quantized is of Maxwell type.

In the asymptotic safety scenario, this on-shell reduction scheme helps to monitor the quantitative modifications of the RG flow and of the universality class associated with RG fixed points in a controlled and systematic way. We observe a UV-attractive fixed point of Reuter type with more stabilized critical exponents at a smaller value of the Newton coupling G.

The present paper is organized as follows: in Sect. 2, we review Hilbert–Palatini gravity to second order in the curvature and apply on-shell reduction in order to identify the degrees of freedom to be quantized. The correspondingly quantized theory is studied in Sect. 3 using the functional RG. Here, we focus on the UV fixed-point structure and determine the critical parameters in comparison to those of metric gravity. We conclude in Sect. 4.

2 Hilbert–Palatini gravity

In the Einstein formulation of gravity, the connection is linked to the metric in the form of the Levi-Civita connection.

By contrast, the Hilbert–Palatini formulation of gravity treats the metric and the connection as independent degrees of freedom. Therefore, the connection can a priori carry additional degrees of freedom that may or may not be fully linked to the metric via the equations of motion. In the following, we start with a general connection and study the on-shell constraints at increasing orders of curvature.

2.1 General connection on smooth manifolds

Suppose we have a vector field V on a smooth manifold with metric g and connection $\tilde{\Gamma}$. The covariant derivative $\tilde{\nabla}$ of this vector field reads

$$\tilde{\nabla}_\mu V^\alpha = \partial_\mu V^\alpha + \tilde{\Gamma}^\alpha_{\mu\nu} V^\nu. \tag{1}$$

In the most general case, the connection $\tilde{\Gamma}$ can be decomposed into the expressions for the Levi-Civita connection Γ (Christoffel symbols), contorsion K and displacement L (though the latter does not have a collectively agreed upon name [36]),

$$\tilde{\Gamma}^\alpha_{\mu\nu} = \Gamma^\alpha_{\mu\nu} + K^\alpha_{\mu\nu} + L^\alpha_{\mu\nu}. \tag{2}$$

The Levi-Civita connection is constructed from the metric, $\Gamma = \Gamma[g]$ and can be used to define the standard covariant derivative ∇ of Einstein gravity

$$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha_{\mu\nu} V^\nu. \tag{3}$$

The Levi-Civita part of the connection Γ accounts for curvature through the Riemann tensor defined below. The contorsion tensor K is related to Cartan torsion T which corresponds to the anti-symmetric part of the connection with respect to the lower indices,

$$T^\alpha_{\mu\nu} = \tilde{\Gamma}^\alpha_{\mu\nu} - \tilde{\Gamma}^\alpha_{\nu\mu}. \tag{4}$$

The displacement tensor L induces non-metricity Q which is symmetric in the last two indices,

$$\tilde{\nabla}_\mu g_{\alpha\beta} = -Q_{\mu\alpha\beta}. \tag{5}$$

the linear relation between contorsion K and torsion T, and analogously for the displacement L and the non-metricity Q, can be found in the literature [36–38], but are not of relevance for what follows.

The general connection (2) can be used to define a generalized Riemann tensor analogously to the pure metric formulation,

$$\left[\tilde{\nabla}_\mu, \tilde{\nabla}_\nu\right] V_\sigma = \tilde{R}^\rho{}_{\sigma\mu\nu} V_\rho, \tag{6}$$

taking the familiar form

$$\tilde{R}^\rho{}_{\sigma\mu\nu} = \partial_\mu \tilde{\Gamma}^\rho_{\nu\sigma} - \partial_\nu \tilde{\Gamma}^\rho_{\mu\sigma} + \tilde{\Gamma}^\rho_{\mu\lambda} \tilde{\Gamma}^\lambda_{\nu\sigma} - \tilde{\Gamma}^\rho_{\nu\lambda} \tilde{\Gamma}^\lambda_{\mu\sigma}. \tag{7}$$

In contrast to the standard case, where the Riemann tensor is composed out of the metric, we can view $\tilde{R}$ as dependent

on the metric, the contorsion and the displacement, $\tilde{R} = \tilde{R}[g, K, L]$. Since the general connection $\tilde{\Gamma}$ is not symmetric in the lower two indices anymore, the generalized Riemann tensor $\tilde{R}^{\cdot}{}_{\cdots}$ is not anti-symmetric in the first two indices, as will be important below.

Analogously, we can construct a generalized Ricci tensor by contracting the first and the third index

$$\tilde{R}_{\sigma\nu} = \tilde{R}^{\rho}{}_{\sigma\mu\nu}\, g^{\mu}_{\rho}, \tag{8}$$

which – contrary to Einstein gravity – is not purely symmetric anymore. Contracting this tensor further leads to a generalized Ricci scalar,

$$\tilde{R} = \tilde{R}_{\sigma\nu} g^{\sigma\nu}. \tag{9}$$

As in Einstein gravity, we can use these generalized curvature forms to construct curvature invariants and formulate an action S governing the dynamics of a theory, with the metric, the contorsion/torsion and the displacement/non-metricity as fundamental degrees of freedom, $S = S[g, K, L]$. In a quantized version, all these degrees of freedom have to be integrated out, requiring appropriate gauge fixing also for the connection degrees of freedom, cf. for instance [19].

2.2 Einstein–Hilbert–Palatini action

Let us focus on a Palatini formulation of gravity starting with the lowest nontrivial order in the curvature. This corresponds to the Einstein–Hilbert action (also referred to as Einstein–Hilbert–Palatini action in order to emphasize the dependence on the general connection),

$$S[g, \tilde{\Gamma}] = \int d^4x \sqrt{g} \frac{1}{16\pi G} \left(\Lambda - 2\tilde{R} \right). \tag{10}$$

Classically, the corresponding fields (the metric g and the connection $\tilde{\Gamma}$) are constrained by their equations of motions, namely

$$\frac{\delta S}{\delta g_{\mu\nu}} \overset{!}{=} 0 \tag{11}$$

which is a partial differential equation for the metric, and additionally in the Palatini formulation

$$\frac{\delta S}{\delta \tilde{\Gamma}^{\alpha}_{\mu\nu}} \overset{!}{=} 0. \tag{12}$$

This new equation can be interpreted as an equation of motion for the contorsion K and displacement L. As the derivative terms turn out to be total derivatives, Eq. (12) is a purely algebraic equation for K and L. Its general solution in terms of the general connection $\tilde{\Gamma}$ can be written as

$$\tilde{\Gamma}^{\alpha}_{\mu\nu} = \Gamma^{\alpha}_{\mu\nu} + A_{\mu}\delta^{\alpha}_{\nu}, \tag{13}$$

with a general co-vector field A as the independent degree of freedom [39]. This field contributes, for instance, to the trace

of the Cartan torsion, $T^{\alpha}_{\mu\alpha} = 3A_{\mu}$ [40,41]. For the discussion of a relation to the affine Weyl connection, see [42,43].

The generalized curvature tensors in the Palatini formulation can now be expressed in terms of curvature quantities familiar from ordinary Einstein gravity which derive from the the Levi-Civita connection and additional terms that depend on the new vector field A,

$$\tilde{R}_{\rho\sigma\mu\nu} = R_{\rho\sigma\mu\nu} + g_{\rho\sigma}F_{\mu\nu} \tag{14}$$

$$\tilde{R}_{\sigma\nu} = R_{\sigma\nu} + F_{\sigma\nu} \tag{15}$$

$$\tilde{R} = R, \tag{16}$$

with the tensor F acquiring the form of a Maxwellian field strength,

$$F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}. \tag{17}$$

In Eq. (16) we observe that the generalized Ricci scalar reduces to the standard Ricci scalar on shell. Therefore, the classical action (10) is on-shell equivalent to the Einstein–Hilbert action of classical GR, see [39,44,45] for recent detailed discussions. Since each component $A_{\mu} \in \mathbb{R}$, the Einstein–Hilbert–Palatini action has an $\mathbb{R}^4$ gauge invariance.

Interestingly, the A field appears in the form of a Maxwell-type field strength tensor F as the anti-symmetric part of the generalized Ricci tensor. At higher orders in the curvature, we can thus expect that more general gravity theories of higher order in the curvature will exhibit Maxwellian gauge invariance. This implies that the $\mathbb{R}^4$ invariance for this part of the general connection reduces to a $U(1)$ invariance at higher orders.

Let us therefore consider terms to second order in the curvature. More specifically, we concentrate on terms that can be constructed from a Ricci-like tensor. Since the generalized Riemann tensor $\tilde{R}^{\cdot}{}_{\cdots}$ is not anti-symmetric in the first two indices, we can construct a second Ricci-like tensor of rank two $\tilde{L}$ by instead contracting the second and the fourth index

$$\tilde{L}_{\sigma\nu} = g^{\rho\mu}\tilde{R}_{\sigma\rho\nu\mu} = R_{\sigma\nu} - F_{\sigma\nu}. \tag{18}$$

The tensors $\tilde{L}$ and $\tilde{R}$ obviously coincide in the limit $A \to 0$, reducing to the ordinary Ricci tensor R. In the general case, we can use both curvature tensors for the construction of invariants, yielding

$$\tilde{R}_{\sigma\nu}\tilde{R}^{\sigma\nu} = \tilde{L}_{\sigma\nu}\tilde{L}^{\sigma\nu} = R_{\sigma\nu}R^{\sigma\nu} + F_{\sigma\nu}F^{\sigma\nu}, \tag{19}$$

$$\tilde{R}_{\sigma\nu}\tilde{L}^{\sigma\nu} = R_{\sigma\nu}R^{\sigma\nu} - F_{\sigma\nu}F^{\sigma\nu}. \tag{20}$$

We observe that only two combinations are independent. A general contribution to the action can thus be spanned by the linear combination of the two independent invariants. On shell, we have the equivalence for general couplings σ^1, σ^2:

$$\sigma^1 \tilde{R}_{\mu\nu}\tilde{R}^{\mu\nu} + \sigma^2 \tilde{R}_{\mu\nu}\tilde{L}^{\mu\nu} = \sigma^R R_{\mu\nu}R^{\mu\nu} + \sigma^F F_{\mu\nu}F^{\mu\nu}. \tag{21}$$

The corresponding couplings in front of the standard Ricci-squared and Maxwell terms satisfy

$$\sigma^R = \sigma^1 + \sigma^2 \tag{22}$$

$$\sigma^F = \sigma^1 - \sigma^2. \tag{23}$$

Equation (21) illustrates that a second order curvature theory built from the generalized Ricci-like tensors is on-shell equivalent to a second-order metric theory plus an abelian gauge field. Of course, a further independent second-order invariant can be formed by suitably squaring the generalized Riemann tensor. As is obvious from Eq. (14), this boils down to a square of the Riemann tensor and a Maxwell term as well. In the following, we ignore such terms to quadratic order in the Riemann tensor for simplicity.

3 Quantum Hilbert–Palatini gravity

The preceding observations on the classical level suggest to study the quantized version of Hilbert–Palatini gravity in the on-shell reduction scheme: we use the degrees of freedom of the on-shell form found for the general connection to first order in the curvature, i.e., Eq. (13), to quantize the theory to second order in the curvature. In practice, this corresponds to extending results for quantum Einstein gravity to this order by including a Maxwell-type gauge field.

3.1 Renormalization flow of Hilbert–Palatini gravity

We now investigate the renormalization flow of the gravitational effective action $\Gamma_k[g, \tilde{\Gamma}]$ in the theory space spanned by the Hilbert–Palatini action including the terms to quadratic order in Ricci-like curvature tensors as discussed above,

$$\Gamma_{\text{gr},k}[g, \tilde{\Gamma}] = \int d^4x \sqrt{g} \frac{1}{16\pi \bar{G}_k}$$
$$\times \left[\bar{\Lambda}_k - 2\tilde{R} + \bar{\sigma}_k^1 \tilde{R}^{\mu\nu} \tilde{R}_{\mu\nu} + \bar{\sigma}_k^2 \tilde{L}^{\mu\nu} \tilde{R}_{\mu\nu} \right]. \tag{24}$$

Here, k denotes a renormalization scale at which the theory is considered, and all coupling constants are considered to be k dependent. Now, instead of considering all degrees of freedom of the general connection $\tilde{\Gamma}$, we perform the on-shell reduction of Eq. (13) which allows us to understand the action as a functional of the metric and the abelian gauge field,

$$\Gamma_{\text{gr},k}[g, A] = \int d^4x \sqrt{g} \frac{1}{16\pi \bar{G}_k}$$
$$\times \left[\bar{\Lambda}_k - 2R + \bar{\sigma}_k^R R^{\mu\nu} R_{\mu\nu} + \bar{\sigma}_k^F F^{\mu\nu} F_{\mu\nu} \right]. \tag{25}$$

We are interested in the scale dependence of the running Newton coupling $\bar{G}_k$, the cosmological parameter $\bar{\Lambda}_k$, the higher curvature coupling $\bar{\sigma}_k^R$, and the wave function renormalization Z_k^A of the abelian field strength defined by

$$Z_k^A = \frac{\bar{\sigma}_k^F}{4\pi \bar{G}_k}. \tag{26}$$

For a treatment of the gauge degrees of freedom, we use the background field formalism and perform a linear split of the metric g and the gauge field A into fluctuations around their respective background fields which are denoted by a bar

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \bar{\kappa} h_{\mu\nu}, \tag{27}$$

$$A_\mu = \bar{A}_\mu + a_\mu, \tag{28}$$

with the abbreviation

$$\bar{\kappa}^2 = 32\pi \bar{G}. \tag{29}$$

The rescaling of the metric fluctuation h by the quantity $\bar{\kappa}$ ensures a standard canonical mass dimension of the field. For the Faddeev–Popov quantization, we include gauge-fixing terms

$$\Gamma_{\text{gf},k} = \frac{1}{2} \int d^4x \sqrt{\bar{g}} \left(\frac{1}{\alpha_{\text{gr}}} \mathcal{F}^\mu \mathcal{F}_\mu + \frac{1}{\alpha_A} \mathcal{G}\mathcal{G} \right) \tag{30}$$

with gauge parameters α_{gr} and α_A. As gauge-fixing conditions for the metric sector $\mathcal{F}$ and the abelian gauge sector $\mathcal{G}$, we use

$$\mathcal{F}^\mu = \sqrt{2}\bar{\kappa} \left(\bar{g}^{\mu\kappa} \bar{\nabla}^\lambda - \frac{1+\beta}{4} \bar{g}^{\kappa\lambda} \bar{\nabla}^\mu \right) h_{\kappa\lambda} \tag{31}$$

$$\mathcal{G} = \sqrt{Z_k^A} \left(\bar{\nabla}_\mu a^\mu \right), \tag{32}$$

where β denotes another gauge parameter of the metric sector. Also including the corresponding ghost terms $\Gamma_{\text{gh},k}$ for both sectors, the total effective (average) action reads

$$\Gamma_k[\bar{\Phi}, \Phi] = \Gamma_{\text{gr},k} + \Gamma_{\text{gf},k} + \Gamma_{\text{gh},k}. \tag{33}$$

Here, $\bar{\Phi}$ and Φ denote collective field variables, representing the background and fluctuations fields, respectively,

$$(\Phi) = \left(h, a, \bar{c}, c, \bar{b}, b \right) \tag{34}$$

$$(\bar{\Phi}) = \left(\bar{g}, \bar{A} \right) \tag{35}$$

with $\bar{c}$ and c being the (anti-)ghost fields for the gravitational sector and $\bar{b}$ and b being the (anti-)ghost fields for the abelian gauge sector.

We quantize the system, using the Wetterich equation [46–49],

$$\partial_t \Gamma_k[\bar{\Phi}, \Phi] = \frac{1}{2} \text{STr} \left[\left(\Gamma_k^{(2)}[\bar{\Phi}, \Phi] + \mathcal{R}_k \right)^{-1} \partial_t \mathcal{R}_k \right], \tag{36}$$

to compute the renormalization flows of the renormalized, dimensionless couplings denoted without a bar

$$G_k = k^2 \bar{G}_k, \qquad \Lambda_k = \frac{1}{k^2} \bar{\Lambda}_k, \qquad \sigma_k^R = k^2 \bar{\sigma}_k^R, \qquad (37)$$

For simplicity, we focus on the Landau gauge, choosing

$$\alpha_A \to 0, \qquad (38)$$
$$\alpha_{\mathrm{gr}} \to 0, \qquad (39)$$
$$\beta = 0, \qquad (40)$$

see [50–53] for studies of gauge or parametrization dependencies in the metric context. For the computation of the traces and the identification of the corresponding operators on both sides, we use a spherical background $\bar{g}$, and a covariantly constant background field $\bar{A}$. For the details of the regularization around the scale k controlled by the regulator $\mathcal{R}_k$ in Eq. (36), we choose a Type I regularization scheme, following the computation of [54]. Computations that include further invariants and higher order curvature terms are, in principle, possible, e.g., along the lines of [54–63].

Using the flows for the dimensionless, renormalized couplings, the wave function renormalization Z_k^A occurs only through the corresponding anomalous dimension

$$\eta_A = -\frac{k \partial_k Z_k^A}{Z_k^A}, \qquad (41)$$

which is determined by an algebraic equation. The flows of the couplings as driven by the metric fluctuations has been computed in [54]. These are amended by contributions from the abelian vector field which we evaluate analogously to [64], but to second order in the curvature. The anomalous dimension of the abelian gauge field subject to metric fluctuations has been computed in [26]; see the Appendix.

We collect all running couplings into $\vec{u}$ which is a vector in the truncated theory space

$$\vec{u}_k = \begin{pmatrix} G_k \\ \Lambda_k \\ \sigma_k \end{pmatrix}, \qquad (42)$$

allowing for a compact notation for the flow equations

$$\vec{\beta}(\vec{u}) = \begin{pmatrix} \beta_G \\ \beta_\Lambda \\ \beta_\sigma \end{pmatrix} = \begin{pmatrix} k \partial_k G_k \\ k \partial_k \Lambda_k \\ k \partial_k \sigma_k \end{pmatrix}. \qquad (43)$$

The explicit flows are summarized in the Appendix. We are specifically interested in fixed points $\vec{u}_\star$ of the RG flow which satisfy

$$\vec{\beta}(\vec{u}_\star) \overset{!}{=} 0. \qquad (44)$$

In order to characterize the fixed points, we linearize the flow equations around the fixed point and determine the critical

Table 1 Fixed-point solutions and critical exponents in second order truncation for Hilbert–Palatini gravity (this work) and metric gravity for the present truncation [54]

	Palatini gravity (this work)	Metric gravity [54]
$G_\star$	1.132	1.467
$\Lambda_\star$	0.214	0.171
$\sigma_\star$	0.326	0.339
$\theta_{1,2}$	$2.057 \pm 3.195 \cdot i$	$1.627 \pm 2.570 \cdot i$
θ_3	12.780	21.232
$\eta_{A,*}$	-0.0924	–

exponents related to the eigenvalues of the Jacobian (stability matrix) of the expansion,

$$\{\theta_1, \theta_2, \theta_3\} = -\mathrm{eig}\left(\vec{\nabla}_{\vec{u}} \otimes \vec{\beta}\right)\Big|_{\vec{u} = \vec{u}_\star}. \qquad (45)$$

Positive critical exponents characterize RG relevant directions which are attracted by the fixed point towards the UV. These directions determine the long-range properties of the theory towards the IR and correspond to physical parameters.

3.2 Results

The fixed point equations (44) turn out to be rational equations in the couplings, see the Appendix, and can be solved analytically. In addition to the Gaußian fixed point, we find five non-Gaussian fixed points. Discarding those with a negative Newton coupling for physical reasons, those with $\Lambda_* > \frac{1}{2}$ which is beyond a singularity in the graviton propagator, and those with very large values for G_* which we consider as artifacts of the approximations involved, we end up with one viable fixed point the quantitative results of which are listed in Table 1.

For comparison, we also list the results for the Reuter fixed point in pure metric gravity obtained in the analogous approximation as obtained in [54]. In general, we observe that the results are rather similar to one another which we interpret as evidence that a direct analogue of the Reuter fixed point in metric gravity also exists in on-shell reduced Hilbert–Palatini gravity with additional dynamical degrees of freedom in the general connection. For comparison, we plot the fixed point positions projected onto the G, Λ plane in Fig. 1. The fixed point labeled as "EH" marks the Reuter fixed point in metric gravity in the lowest-order Einstein–Hilbert truncation. Upon inclusion of terms quadratic in the Ricci tensor, this fixed point moves a bit to larger values of the coupling parameters (labeled by "Ric^2" in the figure and listed in the second column of Table 1). The position of the corresponding fixed point in Hilbert–Palatini found in this work is labeled by "HP" in Fig. 1.
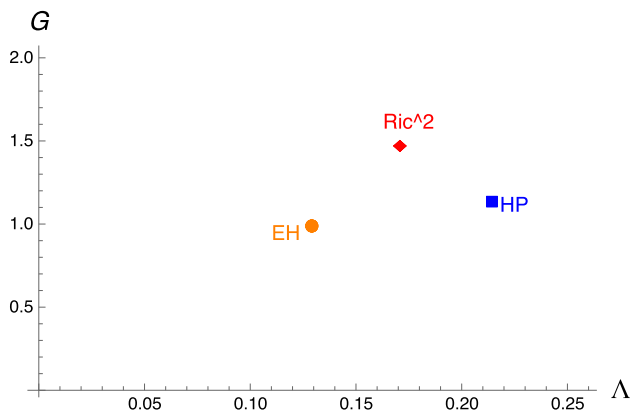
Fig. 1 Fixed-point positions projected onto the G, Λ plane. The orange circle and red diamond represent metric gravity in the Einstein–Hilbert truncation (EH) and its extension to quadratic order in the Ricci tensor (Ric^2) [54], while the blue square represents Hilbert–Palatini gravity (HP) to the second order in the Ricci tensor found in this work using on-shell reduction

Fig. 2 Flow diagram in the theory space spanned by the couplings Λ and G with the third coupling set to its UV fixed-point value $\sigma_\star$. The red dot represents the non-Gaussian UV fixed-point and the blue dot the Gaussian IR fixed-point. The arrows flow towards the IR

Inspecting the results of Table 1 more closely, we observe that specifically the fixed-point value of the Newton coupling is somewhat smaller. This can serve as an indication that a quantum gravity theory with independent connection variables may more easily be compatible with weak-gravity bounds [65–75] which arise from the demand for gravity-matter systems to be compatible with particle-physics observations.

While the leading critical exponents become somewhat larger in Hilbert–Palatini gravity, the most decisive change occurs for the third critical exponent θ_3 which becomes much smaller by almost a factor of 2. The story of this critical exponent is somewhat involved: already in the first analysis of the asymptotic safety scenario at the quadratic curvature order [55], this exponents was found to be rather large which seemed to contradict the expected hierarchy of decreasing critical exponents for higher order operators. In fact, subsequent higher-order truncations revealed that this large value $\theta \gtrsim 20$ is a truncation artifact [76–78], stabilizing at $\mathcal{O}(1)$ if computed at higher order. In the light of these findings, we interpret the reduction of θ_3 by a factor of 2 as a hint that Hilbert–Palatini gravity may not be so severely affected by the truncation artifact.

It is interesting to observe that the anomalous dimension of the U(1) vector field at the fixed point $\eta_{A,*}$ is negative. This is in agreement with studies of the influence of gravitational fluctuations on (non-)abelian gauge fields, where (depending on the matter sector) $\eta_{A,*} < 0$ can go along with either (i) an asymptotically free gauge sector even for abelian gauge theories or (ii) an asymptotically safe gauge sector with a higher degree of predictivity [79–81]. Both scenarios indicate that the fluctuations of the additional degrees of freedom in the connection do not induce new UV problems such as
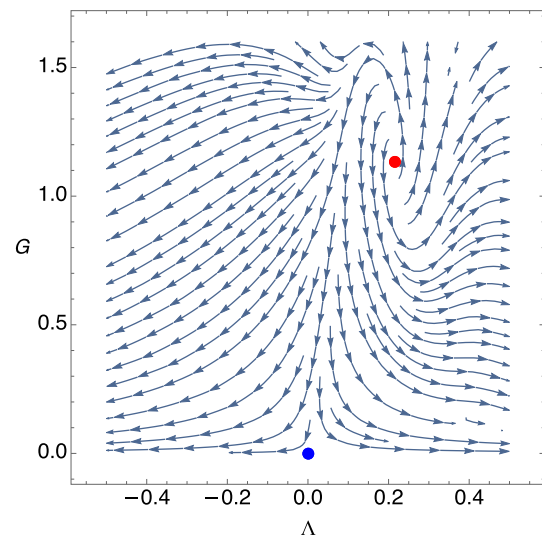
Landau pole singularities despite their similarity to abelian gauge theories in the on-shell reduction scheme.

For the physical validity of the fixed point, a crucial question is as to whether an RG trajectory exists that connects the high-energy fixed-point regime with the regime of classical gravity where the dimensionful renormlized Newton coupling and cosmological constant are indeed constant over a wide range of scales (higher order curvature couplings are not tightly constraint by observations). For this, an RG trajectory must exist that emanates from the UV fixed point and passes by sufficiently near the Gaussian fixed point for G and Λ such that they satisfy canonical scaling. The fact that such trajectories exist is illustrated by the stream plot in the G, Λ plane (evaluated at $\sigma = \sigma_*$) in Fig. 2 where the arrows indicate the RG flow towards the IR. We conclude that our findings support the existence of a UV-complete RG trajectory in quantum Hilbert–Palatini gravity that features a long-range regime where classical GR holds as an effective low-energy theory.

4 Conclusions

We have analyzed the renormalization flow of Hilbert–Palatini gravity using the functional renormalization group. Our study provides evidence for the existence of a non-Gaussian UV fixed point similar to the Reuter fixed point of metric gravity. This result is based on an analysis of an expansion of the action in terms of curvature invariants including squares of generalized Ricci-like tensors and uses an on-shell reduction scheme that allows to gradually include the addi-

tional degrees of freedom introduced by a general connection in comparison to a pure metric formulation.

The discovered fixed point supports the existence of UV-complete RG trajectories in Hilbert–Palatini gravity within an asymptotic safety scenario. Quantitatively, the fixed point occurs at coupling values similar to those of metric gravity. A similar comment applies to the critical exponents – although we even find indications for a larger degree of stability under the increase of the expansion order. Importantly, there exist RG trajectories emanating from this fixed point which can be connected to a low-energy regime with the long-range limit corresponding to Einstein's classical GR.

Within our on-shell reduction scheme, the additional degrees of freedom in the general connection reduce to a vector field that is related to the trace of the Cartan torsion. In our present truncation, the vector field features a local U(1) invariance and thus contributes similarly to an abelian gauge field. This observation holds true for any truncation built from local curvature invariants of the generalized Riemann tensor.

To higher-orders in the on-shell reduction scheme and the curvature expansion, we expect that further additional degrees of freedom acquire their own dynamics and start to contribute to the flow. At the present order, they drop out, because their equation of motion is algebraic and their action corresponds to that of simple quadratic mass terms. Therefore, we expect that their dynamics at higher orders corresponds to that of massive modes. This does not only suggest that they decouple towards the IR, but are also likely to contribute to the UV only at the prize of corresponding mass suppression factors. This observation justifies to consider the on-shell reduction scheme as a quantitatively controlled expansion scheme, provided the underlying curvature expansion scheme has satisfactory convergence properties for observables.

While the differences to metric gravity as found in this work are comparatively small, the question remains as to whether the additional connection degrees of freedom exert a stronger influence on other sectors. In particular, since the additional field after on-shell reduction is a vector field resembling an abelian gauge field, a possible impact on the matter sector in the fixed-point regime or beyond is conceivable. Towards low-energies, this degree of freedom has been discussed as a candidate for dark matter (dark photon) [82]. A particularly relevant question for the high-energy regime where gravity is non-perturbative refers to possible consequences of this degree of freedom for the realization of symmetries such as chiral symmetry of fermionic matter particles [83–88]. The latter is closely related to the existence of light fermions in nature which are an observational fact that needs to be supported also by the quantum gravitational sector.

At this point, it should be emphasized that we have treated all couplings in our truncation as independent. This has facilitated a direct comparison with a large body of literature on metric gravity. However, this implies that potentially inessential couplings [10,89] are treated on the same footing as essential couplings. Recently, a functional RG scheme has been suggested to remove inessential couplings [90] and has already been applied to gravity [91,92]. A similar procedure could be applied to our results, presumably removing one of the couplings in the metric sector.

Finally, we believe that our on-shell reduction scheme can also be useful in further formulations of quantum gravity with different and/or additional degrees of freedom. An example would be given by "tetrad-only" formulations [23] or generalizations of Hilbert–Palatini gravity using the spin-base invariant formalism [93]. In the latter case, it has been shown that on-shell reduction of a generalized spin connection would entail two vector fields [94], presumably with analogous consequences for the construction of an asymptotic safety scenario as found in the present work.

Beyond quantum gravity, on-shell reduction is a rather obvious scheme in functional RG approaches to supersymmetric theories [95]. In this case, on-shell reduction eliminates the auxiliary field(s) that are introduced for a superfield formulation in superspace. While the functional RG can be employed both in the on-shell as well as the off-shell case, the off-shell formulation is advantageous for the description of phase transitions in connection with order parameters related to the off-shell sector [96–99]. As a word of caution, it may therefore be advisable not to use the on-shell reduction scheme for systems in which the off-shell sector is relevant for critical phenomena.

Appendix A: Flow equations

The anomalous dimension for an abelian gauge field η^A has been computed in Appendix D of Ref. [26] and expressed in our notation reads

$$\eta^A = -\frac{G(10 - 40\Lambda + 7\sigma)}{18\pi(1 - 2\Lambda + \sigma)^2}. \tag{A.1}$$

The flow equations for the three remaining couplings Λ, G and σ have been computed along the lines of [54]. The right-hand side of the Wetterich equation – labeled as I in [54] – is extended by the contributions from the abelian gauge field to quadratic order in the curvature according to [12]. The final results for the flow equations are

$$\beta_\Lambda = -\frac{A_\Lambda}{B_\Lambda}, \qquad \beta_G = \frac{A_G}{B_G}, \qquad \beta_\sigma = 2\frac{A_\sigma}{B_\sigma}, \tag{A.2}$$

with

$$A_\Lambda = -432\pi^2 G^2(-2\Lambda + \sigma + 1)^2 \cdot A_\Lambda^{(2)} - 67184640\pi^4\Lambda$$
$$\times (4\Lambda + 6\sigma - 3)^3(-2\Lambda + \sigma + 1)^5 + G^4 \cdot A_\Lambda^{(4)}$$
$$+ 31104\pi^3 G(-2\Lambda + \sigma + 1)^2 \cdot A_\Lambda^{(1)} + 3\pi G^3 \cdot A_\Lambda^{(3)} \tag{A.3}$$

$$A_\Lambda^{(1)} = -27\Lambda\left(16200\sigma^6 + 90996\sigma^5 - 92998\sigma^4\right.$$
$$\left. - 145681\sigma^3 + 120465\sigma^2 - 8599\sigma - 5701\right)$$
$$+ 622080\Lambda^7 + 128\Lambda^6(9000\sigma - 15743) - 32\Lambda^5$$
$$\times \left(71460\sigma^2 + 48998\sigma - 87891\right)$$
$$+ 18\Lambda^2\left(79200\sigma^5 - 188908\sigma^4 - 502556\sigma^3\right.$$
$$\left. + 553149\sigma^2 - 80446\sigma - 25643\right)$$
$$+ 12\Lambda^3\left(101880\sigma^4 + 825560\sigma^3\right.$$
$$\left. - 1166136\sigma^2 + 173151\sigma + 96557\right)$$
$$- 8\Lambda^4\left(483840\sigma^3 - 1212552\sigma^2 + 53236\sigma + 279421\right)$$
$$+ 7290\left(2\sigma^2 + \sigma - 1\right)^2\left(20\sigma^2 + \sigma - 4\right) \tag{A.4}$$

$$A_\Lambda^{(2)} = 5633536\Lambda^6 + 64\Lambda^5(463464\sigma - 478897)$$
$$+ 32\Lambda^4\left(865786\sigma^2 - 3584586\sigma + 1918663\right)$$
$$+ 27\Lambda\left(79720\sigma^5 + 712632\sigma^4\right.$$
$$\left. - 2374666\sigma^3 - 1147514\sigma^2 + 1342848\sigma - 200281\right)$$
$$+ 36\Lambda^2\left(160900\sigma^4 + 1825044\sigma^3\right.$$
$$\left. + 1414833\sigma^2 - 3232750\sigma + 848251\right)$$
$$- 243\left(41500\sigma^5 - 43496\sigma^4 + 14055\sigma^3\right.$$
$$\left. - 31589\sigma^2 + 5968\sigma + 1608\right)$$
$$- 12\Lambda^3\left(2566696\sigma^3 + 5505748\sigma^2\right.$$

$$\left. - 13621200\sigma + 5113385\right) \tag{A.5}$$

$$A_\Lambda^{(3)} = -54\Lambda\left(242816\sigma^6 - 43621136\sigma^5\right.$$
$$- 105741110\sigma^4 - 33856598\sigma^3 + 65125975\sigma^2$$
$$\left. - 58567640\sigma + 19071545\right)$$
$$- 81\left(3694416\sigma^6 + 14436454\sigma^5 + 16191029\sigma^4\right.$$
$$\left. - 889985\sigma^3 - 4324109\sigma^2 + 5270879\sigma - 1603988\right)$$
$$+ 36\Lambda^2\left(3194560\sigma^5 - 152974904\sigma^4\right.$$
$$- 131239628\sigma^3 + 194163574\sigma^2$$
$$\left. - 197711957\sigma + 83407825\right)$$
$$\times 106823680\Lambda^7 + 512\Lambda^6(387008\sigma - 938935)$$
$$- 64\Lambda^5\left(661408\sigma^2 + 23286680\sigma - 1885541\right)$$
$$- 24\Lambda^3\left(12370624\sigma^4 - 146478720\sigma^3 + 172896708\sigma^2\right.$$
$$\left. - 225254630\sigma + 165900405\right)$$
$$+ 16\Lambda^4\left(24289856\sigma^3 + 27106560\sigma^2\right.$$
$$\left. + 18900738\sigma + 133505315\right) \tag{A.6}$$

$$A_\Lambda^{(4)} = 67230720\Lambda^5 + 128\Lambda^4(578163\sigma - 1781875)$$
$$- 16\Lambda^3\left(2661552\sigma^2 + 8298109\sigma - 18340280\right)$$
$$+ 27\left(1232336\sigma^5 + 5064046\sigma^4 + 5332307\sigma^3\right.$$
$$\left. + 620285\sigma^2 - 500522\sigma - 193960\right)$$
$$- 18\Lambda\left(15761360\sigma^4 + 36304598\sigma^3\right.$$
$$\left. + 6154946\sigma^2 - 3194187\sigma - 2801570\right)$$
$$+ 12\Lambda^2\left(44960536\sigma^3 + 7553626\sigma^2\right.$$
$$\left. + 425125\sigma - 14839910\right) \tag{A.7}$$

$$B_\Lambda = 216\pi^2(-2\Lambda + \sigma + 1)^2 \cdot \left[155520\pi^2\left(8\Lambda^2 + 8\Lambda\sigma\right.\right.$$
$$\left.\left. - 10\Lambda - 6\sigma^2 - 3\sigma + 3\right)^3 + G^2 \cdot B_\Lambda^{(2)} + 144\pi G \cdot B_\Lambda^{(1)}\right] \tag{A.8}$$

$$B_\Lambda^{(1)} = 101248\Lambda^5 + 64\Lambda^4(4693\sigma - 7716) + 8\Lambda^3$$
$$\times \left(27216\sigma^2 - 180968\sigma + 119287\right)$$
$$+ 27\left(3012\sigma^5 + 9284\sigma^4 + 11483\sigma^3 - 26775\sigma^2\right.$$
$$\left. + 15752\sigma - 2887\right)$$
$$- 18\Lambda\left(14444\sigma^4 + 17728\sigma^3 - 99747\sigma^2\right.$$
$$\left. + 94133\sigma - 23561\right)$$
$$+ 12\Lambda^2\left(12640\sigma^3 - 106428\sigma^2 + 201663\sigma - 75629\right) \tag{A.9}$$

$$B_\Lambda^{(2)} = 334208\Lambda^4 + 32\Lambda^3(8839\sigma - 14089) + 48\Lambda^2$$
$$\times \left(487\sigma^2 + 49392\sigma - 11720\right)$$

$$-27\left(51700\sigma^4 + 197344\sigma^3 + 129897\sigma^2\right.$$
$$\left.-108638\sigma + 13984\right)$$
$$+18\Lambda\left(230868\sigma^3 + 155580\sigma^2 - 325499\sigma + 57664\right)$$
$$\tag{A.10}$$

$$A_G = 2G(4\Lambda + 6\sigma - 3)^3\left[-3 \cdot A_G^{(1)} \cdot A_G^{(2)}\right.$$
$$\left.+ G^2 \cdot A_G^{(3)} \cdot A_G^{(4)}\right] \tag{A.11}$$

$$A_G^{(1)} = 30\pi G(4\Lambda + 6\sigma - 3)^2$$
$$\times \left(-6\Lambda(5\sigma + 3) + 9\sigma^2 + 44\sigma + 25\right)$$
$$-54\pi G\left(4\Lambda(4\sigma - 3) + 72\sigma^2\right.$$
$$-46\sigma + 9)(-2\Lambda + \sigma + 1)^2$$
$$+243\pi G(4\Lambda + 6\sigma - 3)^2(-2\Lambda + \sigma + 1)^2$$
$$-432\pi^2(4\Lambda + 6\sigma - 3)^2(-2\Lambda + \sigma + 1)^2$$
$$\times G^2(4\Lambda + 6\sigma - 3)^2(40\Lambda - 7\sigma - 10) \tag{A.12}$$

$$A_G^{(2)} = -6G\left(496\Lambda^2 + 24\Lambda(22\sigma - 31) - 9\right.$$
$$\times \left(276\sigma^2 + 44\sigma - 31\right)\right)(-2\Lambda + \sigma + 1)^3$$
$$+5G(4\Lambda + 6\sigma - 3)^3\left(116\Lambda^2 - 4\Lambda(19\sigma + 129)\right.$$
$$+24\sigma^2 + 118\sigma + 469\right)$$
$$+1080\pi(-4\Lambda - 6\sigma + 3)^3(-2\Lambda + \sigma + 1)^3 \tag{A.13}$$

$$A_G^{(3)} = 24(4\Lambda + 15\sigma - 3)(-2\Lambda + \sigma + 1)^2$$
$$+5(4\Lambda + 6\sigma - 3)^2(20\Lambda - 7\sigma - 22) \tag{A.14}$$

$$A_G^{(4)} = 54\pi\left(16\Lambda^2(62\sigma + 29) + 24\Lambda\right.$$
$$\times \left(4\sigma^2 + 86\sigma - 29\right) + 9\left(-1192\sigma^3\right.$$
$$+788\sigma^2 - 234\sigma + 29\right))(-2\Lambda + \sigma + 1)^3$$
$$+90\pi(-4\Lambda - 6\sigma + 3)^3\left(4\Lambda^2(29\sigma - 1) - 2\Lambda\right.$$
$$\times \left(43\sigma^2 + 351\sigma + 178\right) + 44\sigma^3 + 117\sigma^2$$
$$+822\sigma + 499\right)$$
$$-G(-4\Lambda - 6\sigma + 3)^3(-40\Lambda + 7\sigma + 10)(-2\Lambda + \sigma + 1)$$
$$+7038\pi(-4\Lambda - 6\sigma + 3)^3(-2\Lambda + \sigma + 1)^3 \tag{A.15}$$

$$B_G = 9\pi(-4\Lambda - 6\sigma + 3)^5(-2\Lambda + \sigma + 1)^2$$
$$\times \left[-155520\pi^2\left(8\Lambda^2 + 8\Lambda\sigma\right.\right.$$
$$-10\Lambda - 6\sigma^2 - 3\sigma + 3\right)^3$$
$$\left.+ G^2 \cdot B_G^{(2)} + 144\pi G \cdot B_G^{(1)}\right] \tag{A.16}$$

$$B_G^{(1)} = 101248\Lambda^5 + 64\Lambda^4(4693\sigma - 7716)$$
$$+8\Lambda^3\left(27216\sigma^2 - 180968\sigma + 119287\right)$$
$$+27\left(3012\sigma^5 + 9284\sigma^4 + 11483\sigma^3\right.$$
$$-26775\sigma^2 + 15752\sigma - 2887\right)$$

$$-18\Lambda\left(14444\sigma^4 + 17728\sigma^3\right.$$
$$-99747\sigma^2 + 94133\sigma - 23561\right)$$
$$+12\Lambda^2\left(12640\sigma^3 - 106428\sigma^2 + 201663\sigma - 75629\right)$$
$$\tag{A.17}$$

$$B_G^{(2)} = -334208\Lambda^4 - 32\Lambda^3(8839\sigma - 14089)$$
$$-48\Lambda^2\left(487\sigma^2 + 49392\sigma - 11720\right)$$
$$+27\left(51700\sigma^4 + 197344\sigma^3\right.$$
$$+129897\sigma^2 - 108638\sigma + 13984\right)$$
$$-18\Lambda\left(230868\sigma^3 + 155580\sigma^2 - 325499\sigma + 57664\right)$$
$$\tag{A.18}$$

$$A_\sigma = -1399680\pi^3\sigma(-2\Lambda + \sigma + 1)^5$$
$$\times (4\Lambda + 6\sigma - 3)^3 + 9\pi G^2(-2\Lambda + \sigma + 1)^2 \cdot A_\sigma^{(2)}$$
$$-648\pi^2 G(-2\Lambda + \sigma + 1)^2 \cdot A_\sigma^{(1)} + G^3 \cdot A_\sigma^{(3)} \tag{A.19}$$

$$A_\sigma^{(1)} = -27\left(16200\sigma^7 + 32644\sigma^6\right.$$
$$-41054\sigma^5 - 133133\sigma^4 + 105227\sigma^3$$
$$+29329\sigma^2 - 41471\sigma + 8812\right)$$
$$+18\Lambda\left(79200\sigma^6 - 182460\sigma^5 - 431284\sigma^4\right.$$
$$+633841\sigma^3 + 11688\sigma^2 - 255583\sigma + 74494\right)$$
$$+12\Lambda^2\left(101880\sigma^5 + 826616\sigma^4\right.$$
$$-1214152\sigma^3 + 27963\sigma^2 + 743483\sigma - 278368\right)$$
$$-8\Lambda^3\left(483840\sigma^4 - 1152328\sigma^3\right.$$
$$-432924\sigma^2 + 1510489\sigma - 629598\right)$$
$$\times 512\Lambda^6(1215\sigma - 1564) + 128\Lambda^5$$
$$\times \left(9000\sigma^2 - 37931\sigma + 23614\right)$$
$$-32\Lambda^4\left(71460\sigma^3 + 126606\sigma^2 - 340887\sigma + 156880\right)$$
$$\tag{A.20}$$

$$A_\sigma^{(2)} = 27\left(79720\sigma^6 + 174652\sigma^5\right.$$
$$-1081722\sigma^4 + 689497\sigma^3 + 249706\sigma^2$$
$$-327888\sigma + 72432\right)$$
$$+18\Lambda\left(321800\sigma^5 + 1596732\sigma^4\right.$$
$$-2697526\sigma^3 + 781767\sigma^2 + 915448\sigma - 386209\right)$$
$$-24\Lambda^2\left(1283348\sigma^4 + 242426\sigma^3\right.$$
$$+987572\sigma^2 - 319735\sigma - 233846\right)$$
$$\times 128\Lambda^5(44012\sigma + 27247) + 128\Lambda^4$$
$$\times \left(231732\sigma^2 + 1394\sigma - 75545\right)$$
$$+8\Lambda^3\left(3463144\sigma^3 - 3908896\sigma^2\right.$$

$$-2731276\sigma + 691313\Big) \tag{A.21}$$

$$
\begin{aligned}
A_\sigma^{(3)} = {} & 27\Big(30352\sigma^7 + 131896\sigma^6 \\
& +424260\sigma^5 + 105248\sigma^4 - 509251\sigma^3 \\
& +49896\sigma^2 + 147724\sigma - 42080\Big) \\
& -18\Lambda\Big(399320\sigma^6 + 1353488\sigma^5 \\
& +1610334\sigma^4 - 4440304\sigma^3 + 753902\sigma^2 \\
& +1625577\sigma - 561570\Big) \\
& +24\Lambda^2\Big(773164\sigma^5 + 1232404\sigma^4 \\
& -6465153\sigma^3 + 3676846\sigma^2 + 2410429\sigma - 1353600\Big) \\
& -8\Lambda^3\Big(3036232\sigma^4 - 10230572\sigma^3 \\
& +19821964\sigma^2 + 2913489\sigma - 6181090\Big) \\
& -5120\Lambda^6(1304\sigma + 189) - 128\Lambda^5 \\
& \times\Big(96752\sigma^2 - 253283\sigma - 94070\Big) \\
& +64\Lambda^4\Big(41338\sigma^3 + 1428338\sigma^2 - 555863\sigma - 578655\Big)
\end{aligned}
\tag{A.22}
$$

$$
\begin{aligned}
B_\sigma = {} & 9\pi(-2\Lambda + \sigma + 1)^2\Big[155520\pi^2 \\
& \times\Big(8\Lambda^2 + 8\Lambda\sigma - 10\Lambda - 6\sigma^2 - 3\sigma + 3\Big)^3 \\
& +G^2\cdot B_\sigma^{(2)} + 144\pi G\cdot B_\sigma^{(1)}\Big]
\end{aligned}
\tag{A.23}
$$

$$
\begin{aligned}
B_\sigma^{(1)} = {} & 101248\Lambda^5 + 64\Lambda^4(4693\sigma - 7716) \\
& +8\Lambda^3\Big(27216\sigma^2 - 180968\sigma + 119287\Big) \\
& +27\Big(3012\sigma^5 + 9284\sigma^4 + 11483\sigma^3 \\
& -26775\sigma^2 + 15752\sigma - 2887\Big) \\
& -18\Lambda\Big(14444\sigma^4 + 17728\sigma^3 \\
& -99747\sigma^2 + 94133\sigma - 23561\Big) \\
& +12\Lambda^2\Big(12640\sigma^3 - 106428\sigma^2 + 201663\sigma - 75629\Big)
\end{aligned}
\tag{A.24}
$$

$$
\begin{aligned}
B_\sigma^{(2)} = {} & 334208\Lambda^4 + 32\Lambda^3(8839\sigma - 14089) \\
& +48\Lambda^2\Big(487\sigma^2 + 49392\sigma - 11720\Big) \\
& -27\Big(51700\sigma^4 + 197344\sigma^3 \\
& +129897\sigma^2 - 108638\sigma + 13984\Big) \\
& +18\Lambda\Big(230868\sigma^3 + 155580\sigma^2 - 325499\sigma + 57664\Big)
\end{aligned}
\tag{A.25}
$$

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