Eur. Phys. J. C (2024) 84:477
https://doi.org/10.1140/epjc/s10052-024-12766-x

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Fermionic fixed-point structure of asymptotically safe QED with a Pauli term

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Received: 13 March 2024 / Accepted: 4 April 2024

Abstract We test the physical viability of a recent proposal for an asymptotically safe modification of quantum electrodynamics (QED), whose ultraviolet physics is dominated by a non-perturbative Pauli spin-field coupling. We focus in particular on its compatibility with the absence of dynamical generation of fermion mass in QED. Studying the renormalization group flow of chiral four-fermion operators and their fixed points, we discover a distinct class of behavior compared to the standard picture of fixed-point annihilation at large gauge couplings and the ensuing formation of chiral condensates. Instead, transcritical bifurcations, where the fixed points merely exchange infrared stability, are observed. Provided that non-chiral operators remain irrelevant, our theory accommodates a universality class of light fermions for $N_\mathrm{f} > 1$ irreducible Dirac flavors. On the contrary, in the special case of $N_\mathrm{f} = 1$ flavor, this comes only at the expense of introducing one additional relevant parameter.

1 Introduction

Quantum electrodynamics (QED) is an extremely well-tested theory, exhibiting remarkable agreement with precision experiments at low energies [1–5]. Of course, high-energy tests are also passed by the theory though at lower precision [6,7] and ultimately require the embedding of QED into the electroweak sector of the standard model. Still, the high-energy behavior of pure QED remains of interest in its own right, as it has constituted a puzzle since the early days of quantum field theory: perturbation theory predicts a divergence of the minimal gauge coupling at a finite Landau pole [8,9]. While this may simply signal the expected breakdown of perturbation theory in the strong-coupling regime, the conclusion of the existence of a finite scale of maximum ultraviolet (UV) extension is supported by lattice simulations [10–12] and functional methods [13].

The picture obtained from such nonperturbative methods is, however, decisively different from simple perturbation theory: a strong coupling phase of QED – even if it existed – can generically not be connected by a line of constant physics to physical QED because of chiral symmetry breaking. Strong gauge interactions induce fermion mass generation with masses on the order of the high scale being incompatible with the observed existence of a light electron. In continuum computations, the symmetry breaking can be traced back to fermionic self-interactions turning into relevant operators at strong coupling and triggering condensate formation [13–16]. The corresponding long-range limit of such a theory would then be a free photon theory.

As a resolution, a recent proposal has been based on the observation that the Pauli spin-field coupling term $\bar{\psi}\sigma_{\mu\nu}F^{\mu\nu}\psi$ has the potential to screen the Landau pole – and thus the strong-coupling regime – within an effective field theory [17]. In fact, a self-consistent analysis of pure QED with a Pauli term has provided evidence for the existence of interacting fixed-points potentially rendering QED asymptotically safe [18,19] and thus high-energy complete. As a dimension-5 operator with only a single derivative with respect to the photon, the Pauli term represents the unique next-to-leading-order contribution in a combined derivative and power-counting operator expansion of the effective action.

By the techniques of functional renormalization, the extended theory space has been shown to include two nontrivial fixed points $\mathcal{B}$ and $\mathcal{C}$ at vanishing gauge coupling [18], each of which provides an ultraviolet (UV) completion of QED as an asymptotic safety scenario. Specifically, the fixed point $\mathcal{C}$ occurring at a finite Pauli coupling κ is compatible

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Published online: 08 May 2024

with a renormalization group (RG) trajectory reproducing the long-range values of phenomenological QED. As it features three relevant directions, the long-range physics is fully predictive, once three parameters have been fixed by experiment (e.g., the electron mass, the fine structure constant, and the anomalous magnetic moment of the electron). (By contrast, fixed point $\mathcal{B}$ predicts unphysically large values of the anomalous magnetic moment in the infrared (IR); while being potentially consistent and UV complete, this universality class is observationally not viable.)

In view of the impossibility to connect conventional strong-coupling QED with the observed existence of light electrons, an obvious question needs to be answered: does an asymptotically safe UV completion based on the Pauli term preserve chiral symmetry along its RG trajectories towards the infrared (IR)? This is not at all evident, since fixed point $\mathcal{C}$ – though featuring a vanishing mass – occurs at a deeply nonperturbative value of the Pauli coupling $\kappa^* = 3.82$, independently of fermion flavor number [19].

To further scrutinize the physical relevance of this continuum theory, we go beyond the Pauli term in the truncation of the effective action. Operators of particular interest are given by dimension-6 four-fermion channels of the Nambu–Jona-Lasinio (NJL) type, which appear in an effective theory of spontaneous chiral symmetry breaking in quantum chromodynamics [20,21]. Just as the formation of chiral condensates is responsible for the constituent quark masses, the focus of this work is to investigate whether the strong coupling regime at fixed point $\mathcal{C}$ dynamically generates mass at a UV scale, for example at the Planck scale, which would be in contradiction to the observation of light fermions of the Standard Model. Similar problems are known to impede non-trivial formulations of pure QED [10,13–15].

In QED, chiral symmetry is broken explicitly by the mass term. In the same manner, the Pauli spin-field coupling is also a source of explicit breaking, both of which we consider as small in agreement with observation. While such small breakings allow for the appearance of many further four-fermion interactions, we concentrate here on an otherwise Fierz-complete basis of NJL-type channels, assuming that they play a dominant role in the case of interaction-induced dynamical chiral symmetry breaking. This assumption is similar to low-energy effective theories for QCD where explicit chiral breaking terms can be treated as a small perturbation.

In this setting, we discover that the distinct coupling of the two NJL-type channels by the Pauli term qualitatively alters the bifurcation behavior known from strong QED or QCD: instead of annihilation upon collision, the NJL fixed points merely exchange stability such that an IR attractor persists at arbitrarily strong Pauli coupling for more than $N_\mathrm{f} = 1$ fermion flavor. In such cases, there exists a universality class where the RG flow remains bounded and mass generation can be avoided without further fine-tuning. A similar conclusion can be drawn from our initial analysis for fixed point $\mathcal{B}$, despite the different role played by the aforementioned bifurcation.

The structure of this paper is as follows: In Sect. 2, we introduce the abelian gauged NJL model with a Pauli term. Section 3 then presents the corresponding RG flow equation. Section 4 is allocated to analyzing the fixed point structure in the four-fermion sector of our theory, drawing comparisons to previous results in relation to the pure NJL model (4.1) and the introduction of a gauge field (4.2).

2 Gauged NJL model with Pauli term

We consider the massless limit of an abelian gauged NJL model with a Pauli term. Satisfying Osterwalder–Schrader reflection positivity in Euclidean spacetime, the effective action reads

$$\begin{aligned}\Gamma_k = \int d^4x \Big\{ &\bar{\psi}^a \left(i Z_\psi \partial\!\!\!/ + \bar{e} A\!\!\!/ + i \bar{\kappa} \sigma_{\mu\nu} F^{\mu\nu} \right) \psi^a \\ &+ \frac{Z_A}{4} F_{\mu\nu} F^{\mu\nu} + \frac{Z_A}{2\xi} \left(\partial_\mu A^\mu \right)^2 \\ &+ \frac{1}{2} \bar{\lambda}_+ \,(\mathrm{V}+\mathrm{A}) + \frac{1}{2} \bar{\lambda}_- \,(\mathrm{V}-\mathrm{A}) \Big\}. \end{aligned} \tag{1}$$

Here $a = 1, \ldots, N_\mathrm{f}$ labels the Dirac flavors ψ^a interacting with a U(1) gauge field A_μ. The couplings $\bar{e}$, $\bar{\kappa}$ and $\bar{\lambda}_\pm$, as well as the wave function renormalizations $Z_{\psi,A}$, are dependent on the RG scale k. We further work in the Landau gauge $\xi = 0$ as a fixed point of the RG flow [22,23]. For our purposes, it suffices to consider the point-like approximation where the four-fermion couplings $\bar{\lambda}(p_1, p_2, p_3) \to \bar{\lambda}(0,0,0)$ are approximated by their low-momentum limit [24]. Neglecting the explicit breaking of a chiral $\mathrm{SU}(N_\mathrm{f})_\mathrm{L} \otimes \mathrm{SU}(N_\mathrm{f})_\mathrm{R}$ symmetry by the Pauli term, the four-fermion channels

$$(\mathrm{V} \pm \mathrm{A}) \equiv \left(\bar{\psi}^a \gamma_\mu \psi^a \right)^2 \pm \left(\bar{\psi}^a i \gamma_\mu \gamma_5 \psi^a \right)^2, \tag{2}$$

would form a Fierz-complete basis under the RG flow. The $(\mathrm{V}+\mathrm{A})$ channel is Fierz equivalent to the conventional NJL channel. In the limit of vanishing $\bar{\kappa}$, $\bar{\lambda}_\pm$ (and upon inclusion of an explicit fermion mass term), the present model is identical to QED. If RG trajectories exist that match the QED long-range behavior, then a high-energy complete trajectory in the present model can be viewed as a UV-complete version of QED. In the search for scale-invariant fixed points facilitating UV-complete trajectories, it is convenient to define further dimensionless renormalized couplings

$$\lambda_\pm = \frac{k^2 \bar{\lambda}_\pm}{Z_\psi^2}, \quad e = \frac{\bar{e}}{Z_\psi \sqrt{Z_A}}, \quad \kappa = \frac{k \bar{\kappa}}{Z_\psi \sqrt{Z_A}}. \tag{3}$$

For the present study, we use the functional RG based on the effective average action Γ_k which interpolates between the classical bare action $\Gamma_{k\to\Lambda} = S$ and the full quantum effective action $\Gamma_{k\to 0} = \Gamma$ [24–29]. Defining the RG time as $t = \ln k$, the flow through theory space is governed by the Wetterich equation [30–33]

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{STr}\left[\partial_t R_k \left(\Gamma_k^{(2)}[\phi] + R_k\right)^{-1}\right], \tag{4}$$

where $R_k(p^2)$ acts as a momentum-dependent regulator, screening the contribution of IR modes with momenta below the cutoff k.

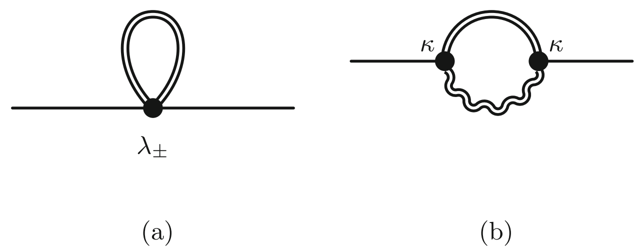

Fig. 1 1PI Feynman diagrams contributing to the fermionic anomalous dimension η_ψ. **a** The tadpole diagrams vanish in the point-like limit where the momentum dependence of the NJL couplings $\lambda_\pm$ is neglected. **b** The self-energy diagrams result in the value of $\eta_\psi = -1$ at fixed point $\mathcal{C}$, which renders both Pauli coupling κ and NJL coupling $\lambda_\pm$ perturbatively marginal in the absence of higher-order terms

3 Flow equations

With the effective action Γ_k expressed in terms of the operators in (1), the Wetterich equation (4) can be projected onto the four-fermion sector to yield the beta functions

$$\begin{aligned}
\partial_t \lambda_+ = {} & (2 + 2\eta_\psi)\,\lambda_+ + 4v_4\, l_4^{(\mathrm{F})}(0,0) \\
& \times \left[6\lambda_+^2 + \left(d_\gamma N_f + 4\right)\lambda_+\lambda_-\right] \\
& - 24 v_4\, l_4^{(\mathrm{B,\,F})}(0,0)\, e^2\lambda_+ - 48 v_4\, l_4^{(1,\,\mathrm{B,\,F})}(0,0)\,\kappa^2\lambda_- \\
& + 6 v_4\, l_4^{(\mathrm{B}^2,\,\mathrm{F})}(0,0)\, e^4 - 48 v_4\, l_4^{(1,\,\mathrm{B}^2,\,\mathrm{F})}(0,0)\, e^2\kappa^2 \\
& + 96 v_4\, l_4^{(2,\,\mathrm{B}^2,\,\mathrm{F})}(0,0)\,\kappa^4,
\end{aligned} \tag{5}$$

$$\begin{aligned}
\partial_t \lambda_- = {} & (2 + 2\eta_\psi)\lambda_- + 2v_4\, l_4^{(\mathrm{F})}(0,0) \\
& \times \left[\left(d_\gamma N_f - 4\right)\lambda_-^2 + d_\gamma N_f \lambda_+^2\right] \\
& + 24 v_4\, l_4^{(\mathrm{B,\,F})}(0,0)\, e^2\lambda_- - 48 v_4\, l_4^{(1,\,\mathrm{B,\,F})}(0,0)\,\kappa^2\lambda_+ \\
& - 6 v_4\, l_4^{(\mathrm{B}^2,\,\mathrm{F})}(0,0)\, e^4 + 48 v_4\, l_4^{(1,\,\mathrm{B}^2,\,\mathrm{F})}(0,0)\, e^2\kappa^2 \\
& - 96 v_4\, l_4^{(2,\,\mathrm{B}^2,\,\mathrm{F})}(0,0)\,\kappa^4.
\end{aligned} \tag{6}$$

Here we have adopted the notation for threshold functions introduced in [18]. The anomalous dimension

$$\eta_\psi = -\partial_t \ln Z_\psi \tag{7}$$

implements an RG improvement by resummation of 1PI diagrams contributing to the propagator, as depicted in Fig. 1. In the point-like limit, four-fermion corrections to η_ψ must vanish, as momentum conservation in the tadpole diagram of Fig. 1a ensures the independence of the loop momentum from external momentum. As such, the fermionic anomalous dimension at fixed point $\mathcal{C}$ remains at the value of $\eta_\psi^* = -1$ due to Pauli contributions (Fig. 1b). This precisely renders the dimension-5 Pauli operator marginally relevant in $d = 4$, even before considering higher-order diagrams.

Likewise, the scaling terms in Eqs. (5)–(6) vanish, and thus the relevance of the NJL channels is decided entirely by the higher-order terms of the beta functions, as represented by Fig. 2.

4 Fixed point analysis

4.1 Pure NJL model

Before we examine the fate of the NJL channels in the presence of the Pauli term, we first review the RG flow of the pure NJL model ($e = \kappa = \eta_\psi = 0$) [24,34,35], shown in Fig. 3 for the case of $N_\mathrm{f} > 1$ irreducible flavors, for which the Dirac representation has dimension $d_\gamma = 4$. As the beta functions (5)–(6) form a pair of quadratic functions of the NJL couplings $\lambda_\pm$, there exist in general four fixed points $\mathcal{F}_i = \left(\lambda_+^{*i}, \lambda_-^{*i}\right)$, $i = 1, \ldots, 4$. We quantify the fixed-point properties in terms of their critical exponents θ which are related to the eigenvalues of the stability matrix $B_{ij} := \partial(\partial_t\lambda_i)/\partial\lambda_j|_{\lambda^*}$: $\theta = -\mathrm{eig}(B)$. Positive values of θ denote RG relevant directions that correspond to physical parameters to be fixed. Negative exponents, in turn, characterize RG irrelevant directions that do not exert an influence on the long-range IR physics.

As expected from power counting, the Gaussian fixed point $\mathcal{F}_1$ is purely IR attractive with critical exponents both being $\theta = -2$. The interacting fixed points $\mathcal{F}_2$ and $\mathcal{F}_3$ each has one relevant direction, while the fourth fixed point $\mathcal{F}_4$ is purely IR repulsive, i.e. relevant. Each of the fixed points $\mathcal{F}_{2,3,4}$ has one relevant eigendirection ($\theta = 2$) pointing along the line that connects the fixed point $\mathcal{F}_{i\geq 2}$ with the Gaussian fixed point $\mathcal{F}_1$. This is in line with general theorems [34,36]. It is straightforward to also compute the remaining critical exponents analytically.

We observe that the purely IR-repulsive $\mathcal{F}_4$ moves towards infinity for $N_\mathrm{f} \to 1$ flavor. This is because the beta function (6) becomes linear in λ_-, with the vanishing fermionic loop contribution $\sim \lambda_-^2$ of Fig. 2a.

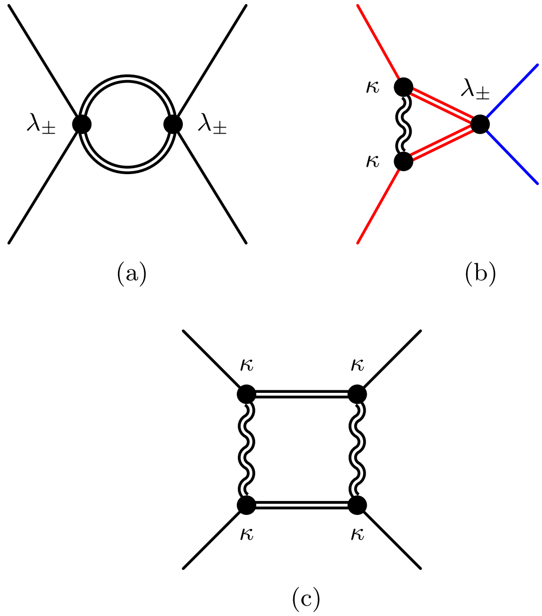

Fig. 2 1PI Feynman diagrams contributing to the flow of the four-fermion vertex. **a** The fermionic loops carry flavor number dependence such that the λ_-^2 contribution vanishes for a single irreducible flavor. **b** The triangular diagrams only contribute to the RG flow in the chirally invariant NJL subspace when the photon is exchanged between fermions of identical flavor. Moreover, the flows of $\lambda_\pm$ are maximally coupled by these diagrams, unlike their gauge coupling counterparts. **c** The box diagrams can induce a finite NJL coupling $\lambda_\pm$ purely through photonic fluctuations

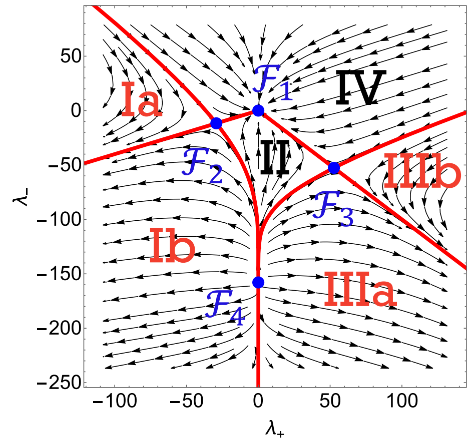

Fig. 3 Phase diagram of the NJL theory subspace spanned by the (V ± A) channels for $N_\mathrm{f} = 2$ irreducible Dirac flavors. Arrows indicate flow towards the infrared. Separatrices (red curves) flowing between fixed points $\mathcal{F}_i$ (blue points) delineate a universality class of light fermions (regions II and IV; black) as observed in nature. On the contrary, the RG flow in regions I and III diverge, heralding the onset of dynamical mass generation. For $N_\mathrm{f} = 1$, $\mathcal{F}_4$ lies at infinity

The universality class defined by the Gaussian fixed point $\mathcal{F}_1$ corresponds to a chirally symmetric phase (regions II and IV) with massless fermions, as the NJL couplings $\lambda_\pm$ remain finite under the RG flow and approach zero in the long-range limit. Meanwhile, initial conditions within regions I and III lead to divergence at a finite RG scale k_{SB}, signaling the formation of a condensate which dresses the fermions with a mass $m_\psi \sim k_{SB}$. This phase of the model is used in low-energy QCD effective models.

In the simplest incarnation of the NJL model, the coupling λ_- is set to zero, and the coupling λ_+ corresponds to (-2) times the more familiar scalar-pseudoscalar channel (S − P). The fixed-point $\mathcal{F}_2$ (projected on the λ_+ axis) then defines the NJL critical coupling that separates the chirally symmetric weak-coupling phase from the chirally broken phase at strong coupling.

4.2 Finite gauge coupling

With a nonzero gauge coupling e, the beta functions (5)–(6) reproduce the known result where the Gaussian fixed point $\mathcal{F}_1$ is annihilated by collision with $\mathcal{F}_2$ at a critical value e_crit. This – in a nutshell – illustrates the relevant mechanism that screens the perturbative Landau pole and inhibits a UV completion of long-range QED: even if QED were UV complete in the strong coupling region, it would exhibit high-scale chiral symmetry breaking and mass generation in contradiction to the observed light mass of the electron [11–14]. In analogous nonabelian settings, the similar mechanism involving the strong gauge coupling triggers the dynamical mass generation in the IR limit of quantum chromodynamics [24,37–45].

Such an effect is already captured by the Fierz-incomplete single-channel approximation $\lambda_- \equiv 0$. The remaining beta function, now represented by a parabola, is shifted vertically by the finite gauge coupling until the fixed points undergo a saddle-node bifurcation into the complex plane.

4.3 Finite Pauli coupling

In contrast to the theory space spanned by minimally coupled QED, the inclusion of the Pauli coupling has provided evidence for the existence of a new universality class governed by a non-Gaussian fixed point, called fixed point $\mathcal{C}$ in [18]. This fixed point occurs at $e^* = 0$, $\kappa^* \simeq 3.82$ and $\eta_\psi = -1$ with the Pauli coupling and the minimal coupling corresponding to relevant directions (in addition to the massive perturbation).

With regard to Eqs. (5) and (6), we note that the only qualitative difference lies in the terms $\partial_t \lambda_\pm \sim \kappa^2 \lambda_\mp$ corresponding to the exchange of a single photon (Fig. 2b). Unlike their gauge coupling counterparts, these terms are non-diagonal in the $\lambda_\pm$ basis. Ultimately, this is due to the anticommutativity of all Dirac matrices with the γ_5 from the axial vector channel in Eq. (2).

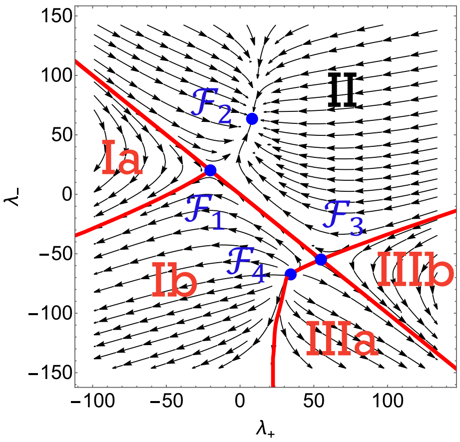

Fig. 4 Phase diagram of the four-fermion subspace spanned by the NJL-type (V $\pm$ A) channels at the Pauli-induced fixed point $\mathcal{C}$ ($\kappa^* = 3.82$, $\eta_\psi = -1$) for $N_\mathrm{f} = 2$ irreducible Dirac flavors. Arrows indicate flow towards the infrared. Compared to Fig. 3, $\mathcal{F}_1$ and $\mathcal{F}_2$ have undergone a transcritical bifurcation and exchanged their stability. There remains a universality class of light fermions (region II)

For $N_\mathrm{f} > 1$ we again observe a collision between fixed points $\mathcal{F}_1$ and $\mathcal{F}_2$ at a critical value κ_crit, but instead of annihilation, they merely exchange stability such that both fixed points continue to exist in the real coupling plane and $\mathcal{F}_2$ is now purely IR attractive. The result of this transcritical bifurcation is shown in Fig. 4. The persistence of this attractor in the strong coupling regime maintains a universality class where mass generation is avoided (region II). This effect is not captured in the single-channel approximation $\lambda_- \equiv 0$.

For an interpretation of the fixed-points as possible routes to UV completion or as critical couplings defining universality classes, we summarize their critical exponents in Table 1. With the gauge system being at fixed point $\mathcal{C}$ with a non-Gaussian Pauli coupling where $\eta_\psi = -1$, the naive non-Gaussian scaling of the four-fermion couplings suggests that the largest critical exponent should be close to zero. We therefore use large deviations from this expectation as an indication for sizable truncation artefacts. This reasoning is analogous to that applied to four-fermion models beyond 2 dimensions [24,34,36,46,47].

From this perspective, $\mathcal{F}_3$ and $\mathcal{F}_4$ represent fixed points with large deviations from the expected scaling that are likely to be dominated by truncation artefacts. While we do not expect them to persist in larger truncations and they should thus not be used for a construction of UV complete trajectories, they and their separatrices may still be used as a qualitative estimate of the boundaries of the chirally symmetric phase II.

By contrast, $\mathcal{F}_1$ and $\mathcal{F}_2$ exhibit small leading exponents close to zero. Fixed point $\mathcal{F}_2$ with two negative exponents is fully IR attractive and thus should be viewed as a *shifted Gaussian fixed point* [35,48], playing the role of the Gaussian fixed point with a location at finite coupling due to the residual non-Gaussian interactions induced by the Pauli coupling. On the other hand, $\mathcal{F}_1$ also exhibits small deviations from the expected scaling with a relevant direction that points approximately along the NJL channel. Whether or not $\mathcal{F}_1$ could be used to define UV complete trajectories should be checked in future investigations. For the present work, we focus on the existence of $\mathcal{F}_2$ as a completely attractive fixed point. This establishes that we find a qualitatively identical phase diagram to Fig. 4 for more than one fermion flavor even for the case that the Pauli coupling κ^* is near fixed point $\mathcal{C}$. This statement holds independently of flavor number N_f [19] with the minor difference that the magnitude of κ^* is insufficient to induce the collision between $\mathcal{F}_1$ and $\mathcal{F}_2$ for larger flavor numbers > 5.25; in such cases, $\mathcal{F}_1$ simply remains the shifted Gaussian fixed point. A strong Pauli coupling phase of QED could thus allow for the construction of UV complete trajectories without being endangered by chiral symmetry breaking in contradistinction to a strong minimal coupling phase. Incidentally, a further transcritical bifurcation occurs for $N_\mathrm{f} \gtrsim 4.94$ between $\mathcal{F}_3$ and $\mathcal{F}_4$. This, however, leaves our conclusions about UV completion in the symmetric phase unaffected.

In the special case of $N_\mathrm{f} = 1$, as with the pure NJL model of Sect. 4.1, the quadratic term in λ_- in Eq. (6) vanishes. Combined with the vanishing scaling term due to $\eta_\psi = -1$, all dependences on λ_- drop out from the beta function. A transcritical bifurcation between $\mathcal{F}_1$ and $\mathcal{F}_2$ is still observed, but the purely attractive $\mathcal{F}_2$ then lies at infinity along with the purely repulsive $\mathcal{F}_4$ (Fig. 5). While this offers, in principle, a construction of a similar UV complete scenario as in the $N_\mathrm{f} > 1$ case, the inherently large coupling values make it difficult to control the expansion scheme. The remaining two fixed points at finite coupling values $\mathcal{F}_1$ and $\mathcal{F}_3$ show large leading exponents with large deviations from the expected scaling, cf. Table 1. Our present study therefore does not allow us to draw any definite conclusions about the existence of UV complete trajectories controlled by the Pauli coupling for the special case of $N_\mathrm{f} = 1$.

We should note however, that the explicit violation of chiral symmetry by the Pauli term generates further four-fermion channels, e.g., an additional Gross–Neveu channel outside the NJL subspace. This occurs through the exchange of a Pauli-coupled photon between different flavors (in contrast to Fig. 2b). In Eqs. (5)–(6), we have discarded such contributions for simplicity. A Fierz-complete analysis of the RG relevance of such channels is beyond the scope of this work. Nevertheless, the structure of the resulting Fierz-complete equations remains similar to the chirally invariant subspace studied here: the inclusion of n four-fermion channels can potentially entail 2^n fixed points in the corresponding coupling space λ_i. As long as one of these fixed points features properties of a shifted Gaussian fixed point similar to $\mathcal{F}_2$ for

Table 1 Critical exponents θ of the fixed points in the plane of four-fermion interactions with the gauge system being at the fixed point $\mathcal{C}$ with a non-Gaussian Pauli coupling

	$\mathcal{F}_1$	$\mathcal{F}_2$	$\mathcal{F}_3$	$\mathcal{F}_4$
$N_\mathrm{f} = 2$:	(0.102, −4.41)	(−0.101, −4.44)	(4.41, −5.78)	(10.0, 2.75)
$N_\mathrm{f} = 1$:	(3.14, −2.36)	–	(2.36, −6.30)	–

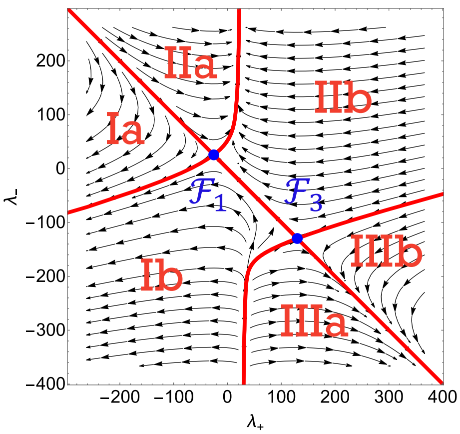

Fig. 5 Phase diagram of the NJL theory subspace spanned by the (V ± A) channels at the Pauli-induced fixed point $\mathcal{C}$ ($\kappa^* = 3.82$, $\eta_\psi = -1$) for $N_\mathrm{f} = 1$ irreducible Dirac flavor. Arrows indicate flow towards the infrared. Compared to Fig. 4, $\mathcal{F}_2$ and $\mathcal{F}_4$ lie at infinity. This is because the beta function (6) no longer depends on λ_-, with the simultaneous vanishing of the scaling term and fermionic loop contribution $\sim \lambda_-^2$

$N_\mathrm{f} > 1$ in the present case, our main conclusions remain unaffected.

For completeness, let us mention that we have applied the same analysis to the compatibility of fixed point $\mathcal{B}$ discovered in [18] with light fermions. As an approximation, we neglect additional terms in the beta functions due to the finite fermion mass m and take into account such threshold effects only through the regulators in Eqs. (5)–(6). The most significant difference lies in the larger fermionic anomalous dimensions $\eta_\psi^* < -1$, which tends to reflect fixed points across the origin and reverse the direction of the RG flow. Once again, we observe a transcritical bifurcation between the now purely IR repulsive would-be Gaussian $\mathcal{F}_1$ and $\mathcal{F}_2$, but the latter is no longer responsible for avoiding heavy fermions. Instead, this role is taken up by the IR-stable $\mathcal{F}_4$, which lies at finite coupling values for $N_\mathrm{f} > 1$.

5 Conclusion

We have studied the renormalization flow of chirally invariant four-fermion operators when subject to a strong-coupling regime as provided by a recently discovered fixed point in QED including a non-Gaussian Pauli spin-field coupling. The flow of these fermionic operators is a crucial litmus test for the viability of an asymptotic safety scenario based on the non-Gaussian fixed point $\mathcal{C}$ as discussed in [18,19]. This is because strong-coupling has the potential to drive chiral symmetry breaking in QED and generate a heavy electron mass incompatible with observation.

In fact, Pauli-induced asymptotic safety at fixed point $\mathcal{C}$ demands that the fermionic anomalous dimension $\eta_\psi = -1$ renders four-fermion couplings perturbatively marginal, at least at the present level of truncation. At first glance, this appears precarious as any prospective fixed points are then maximally susceptible to removal by the Pauli coupling term $\sim \kappa^4$ of the beta function.

However, the flow in the full chirally invariant plane spanned by the pointlike four-fermion interactions known from NJL-type models captures the effects of single-photon exchange (Fig. 2b). The resulting coupled flow of the NJL-type couplings $\lambda_\pm$ exhibits transcritical bifurcations where the fixed points merely exchange stability, in stark contrast to the annihilation observed at strong minimal gauge coupling. As such, for $N_\mathrm{f} > 1$ irreducible Dirac flavors, there remains an infrared attractor at arbitrarily strong Pauli coupling, which prevents dynamical mass generation at a UV scale. This attractor is reminiscent of a shifted Gaussian fixed point. RG trajectories emanating from this fixed point are UV complete and do not introduce further physical parameters. We observe that this scenario is not visible in a Fierz-incomplete chiral truncation based on a single NJL coupling.

On the other hand, for $N_\mathrm{f} = 1$, the simultaneous vanishing of the scaling term and fermionic loop contribution $\sim \lambda_-^2$ conspire to prevent the existence of a fully attractive fixed point at finite coupling. While we do observe two fixed points, they do not satisfy all of our validity criteria and would come with further relevant directions, i.e., require a further physical parameter.

The occurrence of a transcritical bifurcation with a stability exchange of the fixed-point properties is rarely observed in RG flows of similar systems. The generic picture is rather that of a fixed-point merger/collision and subsequent annihilation [40,49–51] specifically for QED [16]. A notable exception is given by tensor O(N) models near $d = 6$, where the annihilation of the fixed points subsequent to a collision is inhibited by the occurrence of an enhanced symmetry [52] that gives rise to the stability-exchange scenario. In the present case, we are not aware of a symmetry enhancement induced by the Pauli term. Understanding the general conditions under which stability exchange can occur in a quantum field theory appears to be a worthwhile future research question.

Our work may be further extended to include a Fierz-complete basis including also the non-chiral four-fermion interactions. This would accommodate the Gross–Neveu channel generated by single-photon exchange between different flavors, resumming all ladder and crossed-ladder diagrams. The inclusion of further channels generically leads to an increase in the number of fixed points [34,36]. Our current scenario remains viable, if one of these fixed points remains fully IR attractive similar to a shifted Gaussian fixed point. While perfectly plausible, this remains to be confirmed.

We have not fully considered the feedback of the four-fermion sector on the running of the gauge/Pauli couplings. While such a feedback on the minimal gauge coupling vanishes at the fermionic fixed points by virtue of the Ward–Takahashi identities [34], a similar mechanism is not expected for the feedback on the flow of the Pauli coupling. But symmetry arguments ensure that such contributions be proportional to the mass m, which vanishes at fixed point $\mathcal{C}$ and can only affect the flow towards the IR.

In summary, our findings provide further evidence for a scenario of an asymptotically safe UV completion of pure QED based on a non-Gaussian Pauli spin-field coupling. We identify a fixed-point collision with a subsequent exchange of stability properties as a crucial mechanism to avoid chiral symmetry breaking in the strong-coupling regime. This mechanism is, however, operative only for $N_{\mathrm{f}} > 1$.

Acknowledgements We thank Jobst Ziebell for valuable discussions. This work has been funded by the Deutsche Forschungsgemeinschaft (DFG) under Grant nos. 398579334 (Gi328/9-1) and 406116891 within the Research Training Group RTG 2522/1, as well as under Grant nos. 392856280, 416607684, and 416611371 within the Research Unit FOR2783/2.

Data Availability Statement This manuscript has associated data in a data repository. [Authors' comment: All tabulated and plotted data can be reproduced directly from the equations in the paper.]

Code Availability Statement This manuscript has associated code/software in a data repository. [Author's comment: Code/software not applicable to this article as no code/software was generated or analysed during the study. All tabulated and plotted data can be reproduced directly from the equations in the paper.]

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