

INSTABILITY MITIGATION USING OCTUPOLES IN BUNCHES WITH SPACE-CHARGE

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Abstract

Octupole magnets are a central mitigation method against the transverse collective instabilities in synchrotrons. For the beam parameters of the SIS synchrotrons, the self-field space-charge effect dominates the betatron footprint, and strongly modifies the instability drive and the Landau damping properties. We consider all these effects and study Landau damping of head-tail modes due to the combination of octupoles and space-charge.

INTRODUCTION

Octupole magnets should be used to suppress the transverse collective instabilities expected for the high-intensity operation of the SIS18 and SIS100 synchrotrons [1] in the FAIR project [2]. On the other hand, an octupole field, as a nonlinearity, can reduce the dynamic aperture [3, 4]. This can restrict the tolerable octupole magnet power, or gives a demand to minimise the octupole magnet power.

The classical symmetric pattern of a head-tail mode [5] is observed if the instability growth rate $\Gamma = \text{Im}(\Delta Q_k)$ is small in comparison to the synchrotron tune Q_s . For a faster instabilities, the driving wake field can strongly distort the eigenmode [6, 7] and thus the observed instability pattern. Other interactions can cause tune shifts and also affect the mode functions and spectra [8, 9]. This work considers the mode modifications due to space-charge. In the report [10], the main focus was the effect on Landau damping. A quite fast $k = 1$ mode was considered in the simulations, $\Gamma/Q_s = 0.7$, due to numerical reasons. The main findings in [10] were the impact of Landau damping due to space-charge for the $k \neq 0$ modes, and the loss of Landau damping due to space-charge and thus the need for stronger octupoles. Here we use the same simulation setup [11–16] and consider a weaker head-tail mode case, $\Gamma/Q_s = 0.09$, as a fixed growth rate for both $k = 0$ mode and $k = 1$ mode. A closer focus is given to the mode modifications due to space-charge, as a counterpart to the effect of space-charge on Landau damping.

SPACE CHARGE AND LANDAU DAMPING

Space-charge in a bunch is characterised by the parameter

$$q = \frac{\Delta Q_{sc}}{Q_s} \quad (1)$$

where ΔQ_{sc} is the modulus of the tune shift for the rms-equivalent K-V beam in the peak of the line density (usually the bunch middle). Space-charge in a Gaussian bunch provides a specific tune spread: the tune shift depends on the transverse amplitudes, during the synchrotron oscillation the

tune shift changes due to the line density variation along the bunch. This is referred as the transverse detuning and the longitudinal detuning.

Octupole magnets provide reliable Landau damping in many hadron synchrotrons. An octupole configuration generates the incoherent tune shifts [4],

$$\begin{aligned} \Delta Q_x &= \kappa_x J_x - \kappa_{xy} J_y, \\ \Delta Q_y &= \kappa_y J_y - \kappa_{xy} J_x, \end{aligned} \quad (2)$$

where J_x, J_y are the horizontal, vertical action variables. The effect of octupoles in a beam can be characterised by the parameter

$$q_4 = \frac{\Delta Q_\sigma}{Q_s}, \quad (3)$$

where ΔQ_σ is the tune shift ΔQ_x of a particle with the amplitudes $J_x = \varepsilon_x/2, J_y = 0, \varepsilon_x$ is the rms emittance.

In a coasting beam, there is no Landau damping due to space-charge alone. But, stability thresholds given by other damping mechanisms are changed by space-charge. The loss of Landau damping happens when space-charge shifts the collective frequency away from the incoherent spectrum [17–19].

The space-charge implications are more complicated for the case of head-tail modes in a bunch. Landau damping due to space-charge alone is effective for the modes $k \neq 0$. Properties of Landau damping provided by other sources, for example by the octupoles, are modified by space-charge.

At the same time, space-charge affects the instability drive itself. The eigenfrequencies of the head-tail modes are shifted by space-charge, which has been observed experimentally [20] and numerically [12] in a good agreement with theoretical predictions [21, 22]. Recently, physics of the tune shift variations along the bunch has been studied [8, 9] in terms of effective impedance modifications for the Landau damping devices, for example a radio frequency quadrupole or a pulsed electron lens. Space-charge also provides a longitudinal detuning due to the line density variation along the bunch, so there should be a similar effect from space-charge. However, the shifts of the head-tail modes are not related to the dynamic part of the longitudinal detuning and to the associated effective impedance modifications, because it exists and is very well described within the airbag bunch model [12, 14, 21]. The reason is probably related to the beam-internal nature of space-charge.

The modifications of the eigenmode itself and the resulting growth rate due to space-charge should be disentangled from the effect of space-charge on Landau damping, which also changes the observable growth rate. For a beam stability study, both aspects must be taken into account.

SPACE CHARGE AND INSTABILITY DRIVE

For our study of the instability mitigation, we are interested in an instability with a fixed growth rate and compare different space-charge conditions. First, we examine simulation scans with a fixed driving wake for varying space-charge strength. Figure 1 shows the resulting growth rates for the $k = 0$ mode (black) and for the $k = 1$ mode (red). The $k = 0$ mode growth rate stays nearly constant. The $k = 1$ mode is unstable at the no-space-charge condition, the driving wake for the results in Fig. 1 is chosen such that the growth rate is equal to the $k = 0$ case. With space-charge, the $k = 1$ mode is completely stabilised by Landau damping due to space-charge until $q \approx 22$. It is unstable at higher q values but with different growth rates with respect to the no-space-charge case. For a study with a fixed growth rate, this can be compensated by adjusting the driving wake.

Even though the growth rate of the $k = 0$ mode is not affected by space-charge, some mode modifications can be recognised. Figure 2 shows examples for the transverse dipole moment traces of the developed $k = 0$ instability. The top plot with $q = 8$ demonstrates the symmetric pattern close to the usual no-space-charge case. In the stronger space-charge case, the bottom plot with $q = 28$, the pattern is broader and less symmetric.

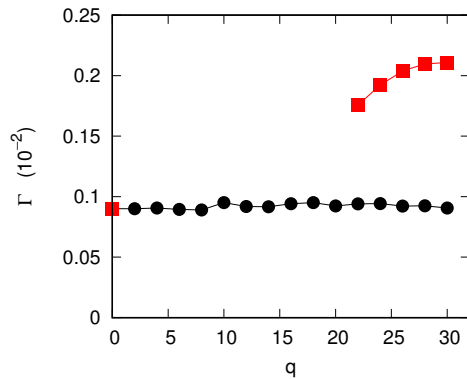


Figure 1: Instability growth rate in a dependency from the space-charge parameter q . The black circles are for the $k = 0$ head-tail mode, the red squares are for the $k = 1$ mode.

OCTUPOLES FOR INSTABILITY SUPPRESSION

The complete octupole field kicks are included into our particle tracking simulations in order to determine the minimum octupole magnet power for the instability suppression. The simulation scans are performed in the similar way as presented in [10]. Figure 3 presents the results for the $k = 0$ mode. The different space-charge conditions, given by the parameter q , correspond to varying space-charge tune shifts. All the other beam and machine parameters are fixed, including the driving wake. Without the octupoles, the beam is unstable with the growth rate $\Gamma = 0.9 \times 10^{-3}$ for the each

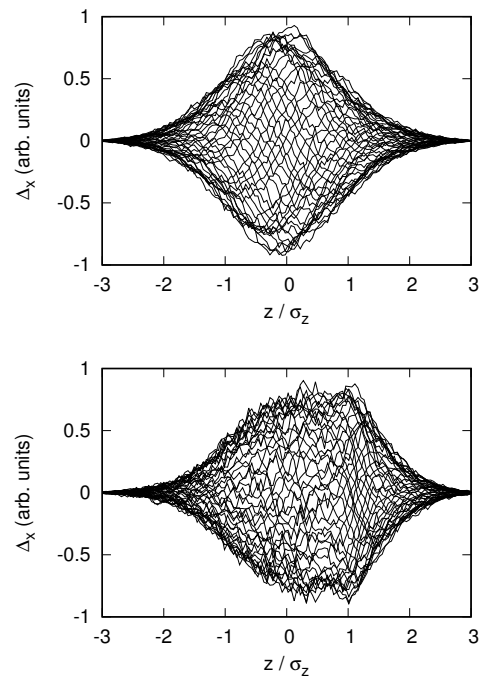


Figure 2: Transverse offset traces along the bunch for the $k = 0$ head-tail mode for $q = 8$ (top plot) and for $q = 28$ (bottom plot).

parameter q . The minimum octupole magnet power for the beam stability is found, both octupole polarities are considered. The space-charge strength up to $q = 30$ is covered, which is the relevant parameter area for the SIS synchrotrons.

We observe from the results of Fig. 3 that with increasing space-charge strength, a higher octupole power is needed. This corresponds to the loss of Landau damping due to space-charge. The maximum is around $q = 16$, where a factor ≈ 6 stronger octupoles are needed in comparison to the no-space-charge case. The q_4 -threshold saturated for q values above $q \approx 20$, with a factor ≈ 4 stronger octupoles with respect to the $q = 0$ case. The decrease of the octupole power threshold above $q = 16$ should be related to the modifications of the eigenmode due to space-charge, discussed in Fig. 2. The differences between the positive and the negative octupole polarity are rather minor, for the largest q values the threshold differences were not resolvable.

The effect of space-charge on the higher-order modes is very different from the $k = 0$ mode case. This is also true for this study. Figure 4 shows the minimum octupole magnet power for the $k = 1$ instability suppression. In addition to the space-charge strength variation, for every q value the driving wake is adjusted such that without the octupoles the growth rate is $\Gamma = 0.9 \times 10^{-3}$. Similarly to the results of Fig. 1, above $q = 0$, there are no data points until $q \approx 22$, because Landau damping due to space-charge suppresses the instability and no octupoles are needed. For stronger space-charge $q \gtrsim 22$, the loss of Landau damping demands more power for octupoles to stabilise the beam. There is a sort of

saturation of the needed octupole power at strongest space-charge. Octupole power with a factor up to ≈ 8 stronger in comparison to the no-space-charge case is necessary.

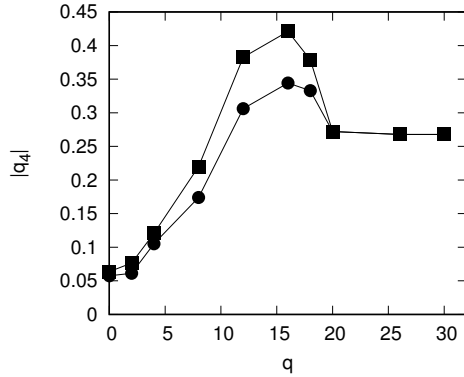


Figure 3: Results of the simulations scans for the $k = 0$ mode: stability thresholds of the octupole power in a dependency from the space-charge parameter q . The circles are for the octupole polarity $q_4 > 0$, the squares are for the octupole polarity $q_4 < 0$.

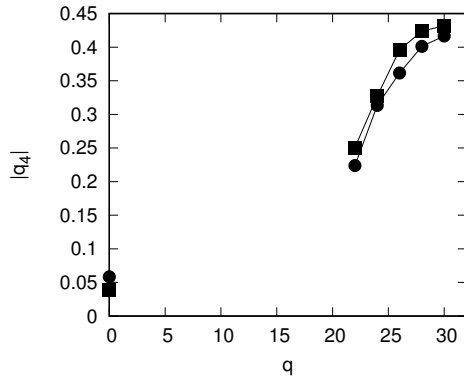


Figure 4: Results of the simulations scans for the $k = 1$ mode: stability thresholds of the octupole power in a dependency from the space-charge parameter q . The circles are for the octupole polarity $q_4 > 0$, the squares are for the octupole polarity $q_4 < 0$.

BEAM STABILITY FOR SIS SYNCHROTRONS

Machine development experiments at SIS18 [23] for the high-intensity operation demonstrated head-tail instabilities in the vertical plane, accompanied by strong and fast beam losses. The vertical tune has been moved from the injection tune $Q_y = 3.6$ to higher values. Around $Q_y = 3.79$, the BPM signals revealed a $k = 3$ intrabunch oscillations, having a clear exponential growth with the growth time $\tau \approx 0.11$ ms for the beam intensity 1.2×10^{10} of N^{7+} ions in four bunches. The observations indicated that the driving impedance is the resistive-wall impedance. The basic growth rate dependencies on the beam parameters result in the equivalent beam intensity 8.6×10^{10} of U^{28+} ion, which

corresponds to $\approx 60\%$ of the FAIR U^{28+} nominal intensity. Installation of octupole magnets in SIS18 is presently under consideration as an option for the instability mitigations.

Particle tracking simulations in this study and [10, 16] consider different instabilities and beam conditions. Still, the predictions for the octupole magnet power needed to suppress an instability are quite similar: $\Delta Q_\sigma / \Gamma$ is between 0.7 and 0.8. This is for the no-space-charge case. As we observe in the present study, the effect of space-charge strongly affects the stability thresholds of the octupole power. In SIS18, the conditions for the expected nominal high-intensity operation in SIS18 correspond to q values up to $q \approx 30$. Results of the present study should help to design a possible octupole configuration for SIS18, with the effect of space-charge taken into account.

A similarly important role of space-charge $q \approx 36$ is expected in the high-intensity operation of the SIS100 synchrotron, presently under construction. The calculations for the expected instabilities in the nominal U^{28+} beams predict the head-tail modes of different order in the coupled-bunch regime as the most dangerous. The growth rates up to $\Gamma \approx 2 \times 10^{-3}$, which relates as $\Gamma / Q_s \approx 0.5$, are anticipated. Using our results, the total octupole power of $\Delta Q_\sigma \approx 10^{-2}$ is predicted to be enough to suppress the instabilities, with the effect of space-charge taken into account. The octupole magnets in SIS100 will provide $\Delta Q_\sigma = 1.3 \times 10^{-2}$ at the maximum current, which is safe for the instability mitigation.

FURTHER ASPECTS

An additional factor for this stability analysis is the elliptic cross-sections of the ion beams. For example, the nominal SIS100 rms emittances for U beams are $\varepsilon_x = 8.7$ mm mrad, $\varepsilon_y = 3.7$ mm mrad. In SIS18, the emittance ratios are similar. For space-charge, the round beam parameter $\varepsilon_\perp$ should be replaced by

$$\varepsilon_\perp = \frac{1}{2} \left(\varepsilon_y + \sqrt{\varepsilon_y \varepsilon_x \frac{Q_{0y}}{Q_{0x}}} \right), \quad (4)$$

here for the vertical (y) plane. For the octupoles, Eq. (2) shows that the κ -coefficients provide a great variety of the tune footprint definition. Here we compared different octupole polarities to show the possible implications. In the synchrotron operation, an optimised configuration should be found which ensures the beam stability and keeps the octupole power minimal for the sake of the dynamic aperture. In a case without coupling between the planes, the incoherent tune distribution in the plane of the instability is relevant, and our consideration with a round beam and effective tune shift parameters is an adequate stability analysis.

Another aspect is the image charges of the vacuum pipe, which can affect and enhance the space-charge related Landau damping [15]. In the case of the SIS synchrotrons, the image charge tune shift is approximately $\Delta Q_{\text{image}} \approx 0.06 \Delta Q_{\text{sc}}$, which means this should have a minor beneficial effect [15, 24] on Landau damping due to space-charge.

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