

IMPACT OF THE NEUTRAL MOLECULE TRAPPING ON BEAM LIFETIME AND BEAM PROFILE*

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Abstract

Recent investigations have shown that neutral molecules may be trapped by the electromagnetic field of a charged particle beam. This effect changes the residual gas distribution according to a characteristic temperature of the system consisting of beam and residual gas. Here, we investigate the effect of the modified gas density profile on the beam lifetime.

INTRODUCTION

The possibility of trapping neutral molecules with a nonzero dipole moment by the beam field was studied in Ref. [1], and a few theoretical aspects were further elaborated in Ref. [2]. Possible mitigations of the unwanted trapping were discussed in Ref. [3]. At the ECloud22 workshop, yet another aspect was explored, namely, Ref. [4] addressed the potential impact on the beam lifetime of a modified residual gas distribution, and, for the case of flakes formation, it illuminated the danger of such aggregates. Here, we expand those preliminary studies.

RESIDUAL GAS DISTRIBUTION AND T/T^*

From the theory of the dynamics of neutral molecules [1], we know that a gas composed of molecules whose dipole moments are aligned with the beam field is subject to a slow change of its distribution, which evolves towards a stationary transverse density profile depending only on T/T^* , where T denotes the temperature of the gas (or beam pipe) and T^* is the characteristic trapping temperature of the combined system “beam + residual gas”. We consider the residual gas distribution for the axisymmetric case, which, in full generality, can be described as

$$n_g(r, z) = \rho_g\left(\frac{r}{\sigma}\right) \bar{n}_g, \quad (1)$$

where σ designates the rms transverse beam size, and the function $\rho_g(\cdot)$ characterizes the transverse gas distribution. We may call ρ_g the “gas density profile”. The symbol $\bar{n}_g$ is the average particle density in the beam pipe. For a uniform residual gas distribution $\rho_g(r/\sigma) = 1$. Figure 1 shows two examples of stationary density gas profiles corresponding to two different values of T/T^* . The modification of the density profile under the influence of the beam field is quite evident.

MODELING THE BEAM LIFETIME

The beam distribution as a function of r is also described by a (beam) density profile function $\rho_b(r/\sigma)$, so that the

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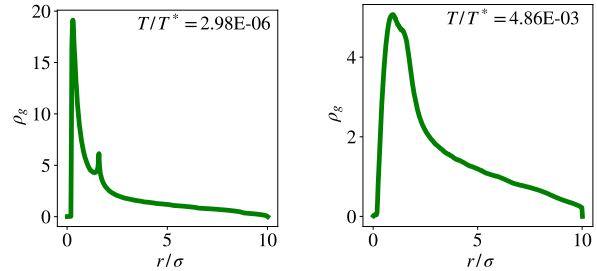


Figure 1: Example of two radial density gas profile distributions obtained from the numerical simulations for molecules with an electrical dipole moment (EDM) for two different values of T/T^* ; see Ref. [1].

radial beam particle distribution reads $n_b(r) = \rho_b(r/\sigma) \bar{n}_b$, with, e.g., $\rho_b(0) = 1$, and $\bar{n}_b$ the average beam particle density on axis.

The simulations of the beam dynamics normally use a surrogate of the beam composed of macro-particles. In this study, we assign to each macro-particle a weight w , that evolves according to the laws determining the beam lifetime. Hence, the beam distribution will be represented by the distribution of the “weighted” macro-particles. In this approach the beam loss is not modelled by the elimination of a macro particle, but through the change of its associated weight. The initial value of the weight of each macro-particle is $w_0 = 1$. The fraction of the surviving beam particles at any time will then be the sum of all weights divided by the number of macro-particles.

As a beam particle travels through the accelerator, on each turn it encounters the modified residual gas density. For example, if the modified residual gas is located in a region of length Δz_g , the weight of a macro-particle passing once through that region changes according to $\Delta w/w = -\rho_g(\tilde{r}) \bar{n}_g \Sigma_c \Delta z_g$, where Σ_c is the cross-section of the interaction of the beam particle with the residual gas particles, and $\tilde{r} = \sqrt{\tilde{x}^2 + \tilde{y}^2}$, with $\tilde{x} = x/\sigma$, and $\tilde{y} = y/\sigma$, and (x, y) denoting the macro-particle’s transverse position. We also assume that the phase advance of the beam particles through the residual gas region is small. Therefore, in this model each macro-particle is characterized by the five coordinates $\mathbf{X} = (\tilde{x}, \tilde{x}', \tilde{y}, \tilde{y}', w)$, where we have also introduced $\tilde{x}' = x'/\sigma$, and $\tilde{y}' = y'/\sigma$. These five coordinates evolve from turn n to turn $n+1$ according to

$$\mathbf{X}_{n+1} = \begin{pmatrix} M & 0 \\ 0 & 1 - \rho_g(\tilde{r}_{n+1}) \bar{n}_g \Sigma_c \Delta z_g \end{pmatrix} \mathbf{X}_n \quad (2)$$

with M the 4×4 one-turn map.

For a uniform gas distribution, after N_t turns, a generic macro-particle carries the weight

$w_{N_t} = w_0 \exp[-\bar{n}_g \Sigma_c \Delta z_g N_t]$. We observe that the evolution of the weight depends on the quantities $\mathcal{L} = \bar{n}_g \Sigma_c \Delta z_g$, and N_t , which reflect the dependence on 1) residual gas density, 2) cross-section, 3) length of the region with significant gas density Δz_g , and 4) the number of turns. Note that the dimensionless scale factor $\mathcal{L}$ represents the beam loss per storage-ring turn: The number of particles after N_t turns is $N(N_t) = N_0 \exp(-\mathcal{L}N_t)$, and the average beam density becomes $\bar{n}_b(N_t) = \bar{n}_b(0) \exp(-\mathcal{L}N_t)$. The beam lifetime is given by the number of turns $N_l = 1/\mathcal{L}$ after which the intensity has decreased by $N(N_l)/N_0 = 1/e$. Applying Eq. (2) for N_t turns to all particles, and using the weights w_{N_t} , we obtain the new beam distribution as $n_b(\tilde{r}, N_t) = n_b(\tilde{r}, 0) \exp(-\mathcal{L}N_t)$. Finally, the new density beam profile function of the beam at turn N_t is $\rho_b(\tilde{r}, N_t) = \rho_b(\tilde{r}, 0)$. Therefore, in case of a uniform gas density the transverse beam profile does not change with N_t , but only the fraction of surviving beam particles.

EFFECT OF RESIDUAL GAS TRAPPING

For a given value of T/T^* , the associated stationary radial gas-density profile function $\rho_g(\tilde{r})$ can be obtained from simulations of the molecule dynamics under the influence of the beam electromagnetic field. If the beam is represented by N_0 macro-particles, with a transverse Gaussian distribution, we will compute its evolution by repeatedly applying the map Eq. (2) to all the macro-particles. The value of $\mathcal{L}$ translates into a certain beam lifetime, as observed from the beam loss after N_t turns.

When the gas temperature pushes T/T^* towards the “cold gas limit” [1], the corresponding $\rho_g(\tilde{r})$ is not constant anymore, and we must repeatedly apply the map of Eq. (2). After having applied the map for N_t turns to all particles, from the weights w_{N_t} we compute the new macro-particle distribution $n_b(\tilde{r}, N_t)$ and the new beam density profile $\rho_b(\tilde{r}, N_t)$. Note that these quantities are not too sensitive to N_t as long as $\mathcal{L}N_t$ is constant. In fact, one macro-particle’s dynamics induce its weight’s dynamics according to $\log \frac{w(N_t)}{w(0)} = -\mathcal{L}N_t \left[\frac{1}{N_t} \sum_{j=1}^{N_t} \rho_g(\tilde{r}_j) \right]$. The quantity in the square bracket is the average of ρ_g over all oscillations of this particle. For $N_t \rightarrow \infty$ the square bracket (only the quantity in the square bracket) becomes a function $\Lambda(\tilde{\epsilon}_x, \tilde{\epsilon}_y)$ with $\tilde{\epsilon}_x$ and $\tilde{\epsilon}_y$ denoting the “single-particle emittances” (i.e., twice the action variables) normalized by the rms beam emittances. This function may also depend on the machine tunes Q_x, Q_y , if close to a resonance. Therefore, over many turns, i.e. for large N_t , we can write to good approximation

$$w(N_t) = w(0) e^{-\mathcal{L}N_t \Lambda(\tilde{\epsilon}_x, \tilde{\epsilon}_y)}. \quad (3)$$

The important conclusion is that, if the residual gas density is modified by the beam, we expect the weight dynamics to depend on the quantity $\mathcal{L}N_t$, almost independently of the individual values of $\mathcal{L}$ and N_t as long as N_t remains a large number and the tunes Q_x, Q_y are not satisfying a low-order resonance condition. Equation (3) also reveals that the evolution of the weights is not the same for all particles, as it

depends on $\tilde{\epsilon}_x, \tilde{\epsilon}_y$ through $\rho_g(\tilde{r})$; hence the density beam profile $\rho_b(\tilde{r})$ will become an intricate function, that can be determined mainly through computer simulations. We can visualize the effect of T/T^* for the case of $\mathcal{L}N_t = 0.2$ by comparing, after N_t turns, the beam distribution for a uniform gas transverse distribution with the beam distribution subjected to the influence of a trapped residual gas distribution for the same $\mathcal{L}N_t$, see Fig. 2. In order to vi-

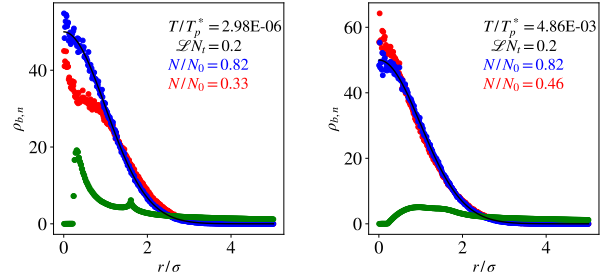


Figure 2: The normalized beam density profile $\rho_{b,n}(\tilde{r})$ for two distinct values of T/T^* . The density profile for uniform residual gas (blue) is compared with the one obtained under the effect of the modified residual gas (red). The green curve shows the gas density profile. The modification of the beam density profile is more visible if the residual gas is closer to the “cold gas limit” (left picture). The beam survival fractions without (blue) and with the effect of trapping (red) are also indicated inside the two pictures.

sualize the effect of the modified residual gas density, we compare the normalized density beam profile defined as $\rho_{b,n}(\tilde{r}) = n_b(\tilde{r}, N_t)/\bar{n}_b(0)$ for two T/T^* .

MODELING THE EFFECT OF FLAKES

In Ref [1] we considered the possibility of flake formation by the clustering of residual gas molecules. Similar to the case of individual molecules above, a stationary distribution of flakes will exhibit a density profile $\rho_f(\tilde{r})$ depending on T/T_f^* , where T_f^* is the trapping temperature of the flakes.

Assume that a residual gas with trapping temperature T^* be placed in the beam pipe at temperature T . Next, consider a residual gas formed by flakes, each of which is composed of N_m molecules of the original residual gas. The flakes, in the more “optimistic” scenario, will have the trapping temperature [1] $T_f^* = N_m T^*$, which means that $T/T_f^* = (1/N_m)(T/T^*)$, where T/T^* is the “temperature ratio” characterizing the original residual gas. The function $\rho_f(\tilde{r})$ will be the same as for any other gas having the same temperature ratio as the flakes. If we take the original residual gas at the temperature of $T_e = T/N_m$, then $T_e/T^* = T/T_f^*$ and so the corresponding low-temperature molecule density profile will be the same as the (higher-temperature) density profile of the flakes. Namely, we find $\rho_f(\tilde{r}) = \rho_g(\tilde{r}, 1/N_m(T/T^*))$. While we can infer this correspondence for the density profile, we have insufficient information for determining the flake density profile $n_f(\tilde{r})$,

as this is related to processes that go far beyond the scope of the present work. Nevertheless, we can incorporate this unknown by simply writing $n_f(\tilde{r}) = \alpha \bar{n}_g \rho_f(\tilde{r})$ with $0 < \alpha < 1$ the ratio of the flake density and the original gas density.

The cross-section of the flakes will also differ from the one of the original residual gas. By assuming a geometric clustering, the cross-section will become $\Sigma_{cf} = N_m^{2/3} \Sigma_c$. Considering instead a linear dependence on the number of molecules per flake yields $\Sigma_{cf} = N_m \Sigma_c$. More generally, we write $\Sigma_{cf} = (N_m)^\beta \Sigma_c$, with β a constant which depends on the clustering process.

Putting everything together, the beam lifetime due to the flakes is controlled by the parameter $\mathcal{L}_f = \alpha (N_m)^\beta \mathcal{L}$, and the weight map for the flakes assumes the following form: $w_{n+1}^f = w_n^f \left[1 - \rho_g \left(\tilde{r}_{n+1}, \frac{1}{N_m} \frac{T}{T^*} \right) \alpha (N_m)^\beta \mathcal{L} \right]$. This map depends on N_m , α , and β , and we can use it in Eq. (2) to compute the weight evolution of each macro-particle. Note that also in this case we can apply Eq. (3) in full generality, but now obtained using the flake density profile ρ_f . For a beam particle of scaled emittances $\tilde{\epsilon}_x$ and $\tilde{\epsilon}_y$, this equation will read $w_{N_t}^f = w^f(0) \exp[-\mathcal{L}_f N_t \Lambda_f(\epsilon_x, \epsilon_y)]$, where Λ_f denotes the Λ obtained through using ρ_f . Therefore, we are now in a position to compute the new beam distribution $n_b(\tilde{r}_j, N_t)$ as a function of α , β , N_m as well as the total number of particles $N(N_t, \alpha, \beta, N_m)$.

As we do not know α , we can assess the impact of the residual distribution of flakes by deriving the “equivalent flake density”, that would yield the same beam lifetime as the one of the original residual gas. This requires solving the equation $N(N_t, \alpha, \beta, N_m)/N(0, \alpha, \beta, N_m) = N(N_t)/N(0)$ and find α , once we have fixed N_m , β , and N_t . In explicit terms, the equation to be solved is

$$\frac{1}{N_0} \sum_{i=1}^{N_0} e^{-\alpha (N_m)^\beta \mathcal{L} N_t \Lambda_f(\epsilon_{x,i}, \epsilon_{y,i})} = e^{-\mathcal{L} N_t}. \quad (4)$$

This equation reveals that, for fixed value of N_m , the quantity $\alpha (N_m)^\beta$ depends only on $\mathcal{L} N_t$. This result is not obvious, but it is nicely confirmed by simulations in the range $10^{-6} < \mathcal{L} < 10^{-5}$, see Fig. 3 right, which presents solutions of Eq. (4) for Λ_f , using a ρ_f corresponding to the temperature ratio of $T/T^* = 8.89 \times 10^{-6}$. The numerical results show a very weak dependence of $\alpha (N_m)^\beta$ in $\mathcal{L}$ if $\mathcal{L} N_t$ is fixed.

DISCUSSION

After the confirmation of the robustness of the concept of equivalent flake density, we can perform a more specific assessment of the impact of the flakes on the beam lifetime considering more realistic situations. For the sake of example, we fix $\mathcal{L}$ and N_t so that $\mathcal{L} N_t = 0.22$ and consider a residual gas for a temperature ratio of $T/T^* \approx 18$, that is close to the *hot gas limit* [1], namely at a (still cryogenic) temperature where the kinetic energy of the molecules is dominant with respect to the beam trapping forces. This would be the case of H_2O , which has $T^* \approx 0.14$ K, for a cryogenic gas temperature $T \approx 2$ K. For the non-trapped

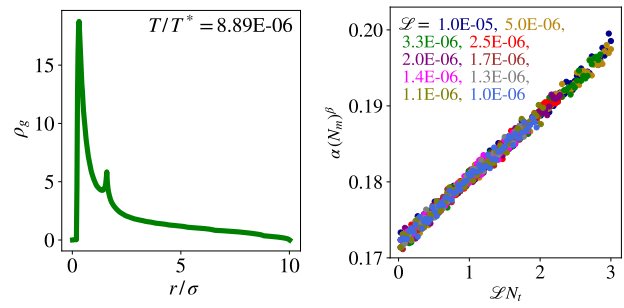


Figure 3: Left: density residual gas profile at $T/T^* = 8.89 \times 10^{-6}$. Right: dependence of $\alpha (N_m)^\beta$ from $\mathcal{L} N_t$ for the Λ_f associated with the density profile on the left picture. For each $\mathcal{L}$ the range of N_t is changed to scan $\mathcal{L} N_t$ along the horizontal axis.

(uniform) residual gas with $\mathcal{L} N_t = 0.22$, the beam survival is 81%. By solving Eq. (4) for a given N_m , we find $\alpha (N_m)^\beta$ and then α . The results of such a computation are shown

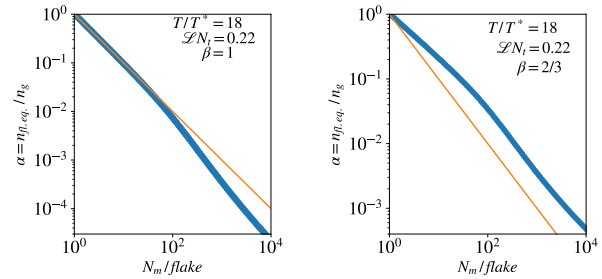


Figure 4: Equivalent flake density as a function N_m . In the left picture, the flake cross-section is assumed to scale linearly with N_m ; in the right picture the flake cross-section scale as $N_m^{2/3}$. The orange straight line $\alpha = 1/N_m$ represents the case that all molecules of the original gas form larger-size flakes uniformly distributed.

in Fig. 4 as a function of N_m and for two different scalings of the flake cross section. If, according to Fig. 4 (right), we replace the original residual gas with a distribution of flakes (each flake composed by N_m molecules), then, in order to again obtain a beam survival fraction of 81%, the flake density must be α -times the original residual gas density. From Fig. 4 (left) we infer that for $N_m = 100$ the beam lifetime reduces by a factor $\alpha N_m \sim 0.73$ with respect to the one of the original gas; for $N_m = 400$ it decreases by ~ 0.47 , and for $N_m = 1000$ by ~ 0.35 . Figure 4 (right) shows that, for this type of cross-section scaling, instead, the beam lifetime with the equivalent flakes is larger. Hence, here the effect of the flakes is less significant. We conclude that, if the geometry of the flakes affects their effective cross-section, the flake geometry could determine the impact of flakes on the beam lifetime.

The treatment presented here is not fully self consistent as dynamic changes in the molecule trapping for decreasing beam intensity are not taken into account, and the shape of the density profile is assumed to be frozen.

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