Search for variations of the fine-structure constant via the hyperfine electronic bridge in highly charged ^{229}Th ions

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The search for variations of the fine-structure constant α by using atomic clocks has driven the development of novel atomic clock technologies. Among these, the ^{229}Th nuclear clock based on nuclear transition stands out due to its high sensitivity. Previous studies on the α variation in atomic clocks have typically focused purely either on a nuclear or electronic transition but not on the hyperfine electronic bridge (HEB) transitions, which involve simultaneous changes in both nuclear and electronic structures. In this work, we propose to search for the α variation by measuring the 3.14-eV HEB transition between the hyperfine levels $[I_e, (4f^{12})J_e = 4, F_e]$ and $[I_g, (4f^{12})J_e = 2, F_g]$ of ^{229}Th^{32+} ions, and the 2.39-eV HEB transition between $[I_e, (4f^4)J_g = 4, F_e]$ and $[I_g, (4f^4)J_e = 2, F_g]$ of ^{229}Th^{40+} ions. These two HEB transitions exhibit remarkably large sensitivity factors K_α of about -2.2×10^4 and -2.9×10^4, respectively. Compared to the nuclear-clock transition (I_e to I_g) in bare ions, the sensitivity factors are significantly enhanced by factors of 2.7 and 3.5. Moreover, both the 3.14- and 2.39-eV HEB transitions are well supported by current high-precision laser technology, making our approach promising for improving the precision of α variation detection.

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I. INTRODUCTION

Since Dirac first questioned whether "fundamental constants" are truly constant [1], the investigation of their potential variations has received considerable experimental and theoretical attention [2,3]. In particular, a time variation of the fine-structure constant α has been one of the central focuses in the scientific community, with direct impact upon theories beyond the Standard Model, including higher-dimensional theories (see, e.g., Refs. [2,3] and the references therein) and dark matter models [4–6]. Therefore, detailed measurements on possible variations of α can provide valuable insights in the search for new physics beyond the Standard Model.

Various physical systems have been used to detect the variation of α, including cosmological observations [7–11], the Oklo natural nuclear reactor [12–16], and atomic clocks [17–21]. Atomic clocks are valued for their high precision and repeatability in laboratory measurements. However, most (present) atomic clocks are not sensitive to the α variation [22]. This limitation has stimulated the development of novel atomic clock technologies, such as highly charged ions clocks [23–26] and nuclear clocks [27,28]. These novel clocks combine robustness against external perturbations with enhanced sensitivity to the α variation. Among these, the 8.36-eV isomeric transition in the ^{229}Th nucleus [29–31] is particularly sensitive to the α variation, with a sensitivity factor of about -8.2×10^3 [32–36] [as illustrated in Fig. 1(a)]. This remarkable property is attributed to its extremely low isomeric transition energy. In contrast, isomeric transitions in other nuclear systems typically exhibit much smaller sensitivity factors, due to their higher transition energies [37].

Previous studies using atomic-clock concepts to detect α variations typically treated nuclear and electronic transitions as two independent processes [17–28,32–37], neglecting the possibility that these two processes may affect each other. Therefore, it is unclear how the performance of atomic clocks is affected when the nuclear and electronic transitions become correlated. We find that the hyperfine electronic bridge (HEB) process introduced in our previous work [38] provides a natural framework to address this question. The HEB process has been proposed as the basis of a quantum optical scheme for the efficient and precise manipulation of the ^{229}Th isomeric state [38].

In this work, we propose an approach to detect the α variation using HEB transitions. This is feasible because we demonstrate that sensitivity factors of HEB transitions can be significantly enhanced compared to that of the bare nuclear transition. Specifically, we identify that the 3.14-eV HEB transition between the hyperfine levels $[I_e, (4f^{12})J_e = 4, F_e]$ and $[I_g, (4f^{12})J_e = 2, F_g]$ in ^{229}Th^{32+} ions [see Figs. 1(b)

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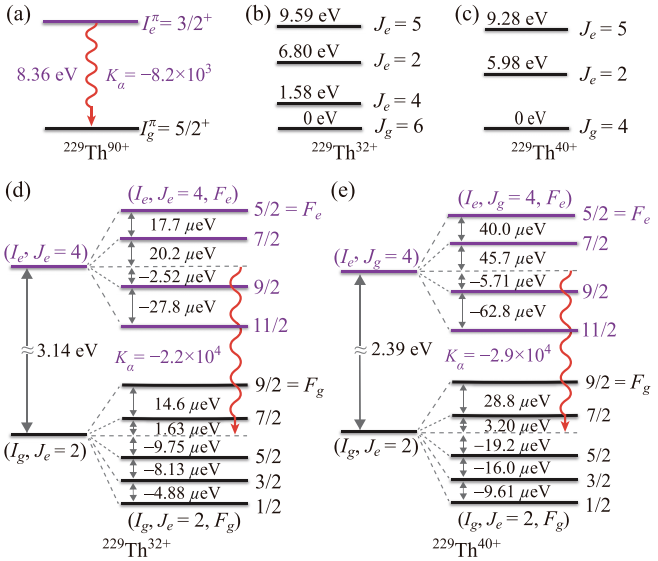

FIG. 1. (a) The ground state and isomeric state in the bare ^{229}Th nucleus ($^{229}Th^{90+}$). (b), (c) Partial electronic energy levels of $^{229}Th^{32+}$ ions (b) and $^{229}Th^{40+}$ ions (c). (d) 3.14-eV HEB transition between hyperfine levels $(I_e, J_e = 4, F_e)$ and $(I_g, J_e = 2, F_g)$ in $^{229}Th^{32+}$ ions. (e) 2.39-eV HEB transition between hyperfine levels $(I_e, J_g = 4, F_e)$ and $(I_g, J_e = 2, F_g)$ in $^{229}Th^{40+}$ ions. The hyperfine splitting is calculated by using a magnetic dipole moment of 0.36 μ_N ($-0.37\ \mu_N$) and an electric quadrupole moment of 3.11 eb (1.74 eb) for the bare nuclear ground (isomeric) state [39,40].

and 1(d)] as well as the 2.39-eV HEB transition between $[I_e, (4f^4)J_g = 4, F_e]$ and $[I_g, (4f^4)J_e = 2, F_g]$ in $^{229}Th^{40+}$ ions [see Figs. 1(c) and 1(e)] are highly sensitive to the α variation. The corresponding sensitivity factors reach up to -2.2×10^4 and -2.9×10^4, respectively, representing enhancements by factors of 2.7 and 3.5 compared to the bare nuclear transition. Moreover, the 3.14- and 2.39-eV HEB transitions lie within the visible spectral range, making them accessible to current high-precision laser spectroscopic techniques. This approach offers a promising method for enhancing the precision of the α variation detection and can be extended to more nuclear systems.

II. HEB TRANSITION

For an atomic system consisting of nucleus and electrons, the Hamiltonian is written as

$$H = H_n + H_e + H_{en}, \tag{1}$$

where H_n and H_e are the nuclear and electronic Hamiltonians, respectively, and H_{en} is the hyperfine interaction [41]. In the presence of the hyperfine interaction, the eigenstates of the combined nucleus-electron system are the dressed hyperfine states $|[I\gamma J]FM\rangle$ [38], which generally refer to entangled states between different nuclear and electronic states. They are characterized by the total angular momentum F and its projection M, and can be explicitly described as

$$|[I\gamma J]FM\rangle = a|I\gamma J; FM\rangle + \sum_t b_t |I_t\gamma_t J_t; FM\rangle, \tag{2}$$

where $|I\gamma J; FM\rangle$ and $|I_t\gamma_t J_t; FM\rangle$ are different hyperfine-coupled bases [42], b_t is the mixing coefficient, and $a = \sqrt{1-\sum_t |b_t|^2}$ is a normalized factor. Here, I is the nuclear spin, J is the electronic angular momentum, and γ denotes all other electronic quantum numbers. The summation involving I_t in Eq. (2) can be typically limited to the nuclear ground spin I_g and isomeric spin I_e if there are large energy gaps between these two states and other nuclear states. For example, the energy of the isomeric level (the first excited state) in the ^{229}Th nucleus is 8.36 eV, while the energy of its second excited state is 29 keV [43].

The mixing coefficient b_t can be calculated perturbatively and expressed as [44]

$$b_t = \sum_{\tau K} \frac{(-1)^{I+J_t+F}}{E_0 - E_t} \begin{Bmatrix} I_t & J_t & F \\ J & I & K \end{Bmatrix} \times \langle I_t||\mathcal{M}^{(\tau K)}||I\rangle\langle\gamma_t J_t||T^{(\tau K)}||\gamma J\rangle, \tag{3}$$

where $\mathcal{M}^{(\tau K)}$ and $T^{(\tau K)}$ are the nuclear and electronic multipole operators of rank K associated with the hyperfine interaction H_{en}, respectively. The specific expressions of operators $\mathcal{M}^{(\tau K)}$ and $T^{(\tau K)}$ can be found, e.g., in Refs. [41,44]. E_0 and E_t represent the energies of the states of $|I\gamma J; FM\rangle$ and $|I_t\gamma_t J_t; FM\rangle$, respectively. $\tau = E$ or M is used to distinguish whether the operators $\mathcal{M}^{(\tau K)}$ and $T^{(\tau K)}$ are of electric or magnetic type [38,44].

The HEB transition describes a transition between two different hyperfine levels, e.g., $(I_i, \gamma_i J_i, F_i)$ and $(I_f, \gamma_f J_f, F_f)$, where both the nuclear and electronic structures undergo a transition. This means that the initial (leading) nuclear and electronic levels $(I_i, \gamma_i J_i)$ both differ from the final levels $(I_f, \gamma_f J_f)$. If the initial and final nuclear levels differ while the electronic levels remain unchanged, the process is referred to as the hyperfine-induced nuclear transition [44,45]. The minimal coupling of these two hyperfine levels due to the interaction with the radiation field gives rise to a photon emission. These two hyperfine levels are described by the dressed hyperfine states $|[I_i\gamma_i J_i]F_i M_i\rangle$ and $|[I_f\gamma_f J_f]F_f M_f\rangle$. The radiative rate of the type of order τL (L is an integer) for this HEB transition follows a derivation similar to that of the hyperfine-induced nuclear transition, and is given by

$$A_{\rm HEB} = \frac{2[L][F_f]}{[L]!!^2}\left(\frac{\omega}{c}\right)^{[L]}\frac{L+1}{L} \times |\langle[I_f\gamma_f J_f]F_f||\mathcal{O}^{(\tau L)}||[I_i\gamma_i J_i]F_i\rangle|^2, \tag{4}$$

where ω is the transition energy and $\mathcal{O}^{(\tau L)}$ is the electronic transition operator of rank L, with its specific form detailed in Ref. [44]. The notation $[L]$ denotes $2L+1$, and the reduced matrix element of $\mathcal{O}^{(\tau L)}$ is given by

$$\begin{aligned}&\langle[I_f\gamma_f J_f]F_f||\mathcal{O}^{(\tau L)}||[I_i\gamma_i J_i]F_i\rangle \\ &\approx \sum_t \left[a_i b^*_{f,t} \begin{Bmatrix} J_t & J_i & L \\ F_i & F_f & I_i \end{Bmatrix} \langle\gamma_t J_t||\mathcal{O}^{(\tau L)}||\gamma_i J_i\rangle \right. \\ &\quad + a^*_f b_{i,t} \begin{Bmatrix} J_f & J_t & L \\ F_i & F_f & I_f \end{Bmatrix} \langle\gamma_f J_f||\mathcal{O}^{(\tau L)}||\gamma_t J_t\rangle \\ &\quad \left. \times (-1)^{J_t - J_i + I_f - I_i} \right].\end{aligned} \tag{5}$$

In the derivation above, only the leading-order terms in the perturbation expansion are kept. Here, a_i (a_f) and $b_{i,t}$ ($b_{f,t}$) represent the normalized factor and mixing coefficient for the initial (final) hyperfine levels $(I_i, \gamma_i J_i, F_i)$ [$(I_f, \gamma_f J_f, F_f)$]. The nuclear spin I_t in the mixing coefficient $b_{i,t}$ ($b_{f,t}$) takes the value of I_f (I_i). Unlike the formula for the hyperfine-induced nuclear transition [44,45], there is no direct nuclear transition term (involving the nuclear transition operator $\mathcal{M}^{(\tau K)}$) in Eq. (4) because the electronic levels in the initial and final states are now different as well.

III. SENSITIVITY FACTOR FOR THE HEB

For the HEB transition between hyperfine levels $(I_i, \gamma_i J_i, F_i)$ and $(I_f, \gamma_f J_f, F_f)$, the transition energy ω is calculated to be

$$\omega = \langle [I_i \gamma_i J_i] F_i || H || [I_i \gamma_i J_i] F_i \rangle - \langle [I_f \gamma_f J_f] F_f || H || [I_f \gamma_f J_f] F_f \rangle \equiv \langle\langle H \rangle\rangle, \tag{6}$$

where the notation $\langle\langle O \rangle\rangle$ is introduced to represent the difference between the expectation values of a scalar operator O for the initial and final states. By applying the Hellmann-Feynman theorem, the relative temporal variation of the transition frequency ω induced by the temporal variation of α is given as [33]

$$\frac{\dot{\omega}}{\omega} = K_\alpha \frac{\dot{\alpha}}{\alpha}. \tag{7}$$

Here, $K_\alpha = \alpha \langle\langle \partial H / \partial \alpha \rangle\rangle / \omega$ is the (so-called) sensitivity factor or enhancement factor (if larger than 1), which measures how sensitive the system is to the variation of α. The larger the absolute value of K_α is, the greater the sensitivity of the system to the variation of α is.

The electronic Hamiltonian H_e consists of the electronic kinetic energy and the electronic Coulomb interaction $V_{\rm eC}$ which includes both the nucleus-electron and electron-electron potentials. Only $V_{\rm eC}$ depends on α in H_e. Following the method in Refs. [13,33,34], we split the nuclear Hamiltonian H_n into two parts: one consisting of the nuclear kinetic energy and the strong interaction, and the other representing the nuclear Coulomb interaction $V_{\rm nC}$. To a good approximation, only $V_{\rm nC}$ depends on α. Since the Coulomb interactions $V_{\rm eC}$ and $V_{\rm nC}$ are linearly proportional to α, and the strength of $V_{\rm nC}$ is much stronger than those of both the electronic Coulomb interaction $V_{\rm eC}$ as well as the hyperfine interaction H_{en}, K_α is simplified to

$$K_\alpha = \frac{\langle\langle V_{\rm nC} \rangle\rangle}{\omega}. \tag{8}$$

The matrix elements of $V_{\rm nC}$ are diagonal with respect to the nuclear levels because $V_{\rm nC}$ commutes with both the nuclear angular momentum operators I^2 and I_z, i.e., $[V_{\rm nC}, I^2] = [V_{\rm nC}, I_z] = 0$. Therefore, by substituting the expansion expressions of the dressed hyperfine states $|[I_i \gamma_i J_i] F_i m_i\rangle$ and $|[I_f \gamma_f J_f] F_f m_f\rangle$ [given in Eq. (3)] into Eq. (8), one obtains

$$K_\alpha = \frac{\Delta E_C}{\omega} \left[1 - \sum_t (|b_{i,t}|^2 + |b_{f,t}|^2) \right], \tag{9}$$

where $\Delta E_C = \langle I_i || V_{\rm nC} || I_i \rangle - \langle I_f || V_{\rm nC} || I_f \rangle$ is the nuclear Coulomb energy difference between the initial and final nuclear states. The above equation provides the general expression for calculating the sensitivity factor K_α for HEB transitions. For the bare nuclear transition, mixing coefficients vanish and our expression in Eq. (9) reduces to that presented in Refs. [33,34,37]. For a given nucleus, the Coulomb energy difference ΔE_C is fixed. Since the mixing is typically quite weak, qualitatively speaking, the smaller the HEB transition energy ω is, the larger the sensitivity factor K_α is.

IV. RESULTS FOR $^{229}Th^{32+,40+}$

For the ^{229}Th nucleus, the spin parities of its ground and isomeric states are $5/2^+$ and $1/2^+$, respectively [as depicted in Fig. 1(a)]. The allowed nuclear transition types include the magnetic dipole ($M1$) and electric quadrupole ($E2$) transitions. The nuclear reduced matrix element $\langle I_g || \mathcal{M}^{(\tau K)} || I_e \rangle$ that appears in mixing coefficients [see Eq. (3)] is determined by the reduced nuclear transition probability $B(\tau K, I_e \to I_g) = [K] |\langle I_g || \mathcal{M}^{(\tau K)} || I_e \rangle|^2 / (4\pi [I_e])$. In the current work, the values of $B(M1, I_e \to I_g)$ and $B(E2, I_e \to I_g)$ are taken to be 0.022 [29] and 30 Weisskopf units [46], respectively. The ^{229}Th isomer exhibits a long radiative half-life, estimated to be about 1.7×10^3 s.

For the HEB transitions, the initial and final electronic levels are different, and thus at least one electronic excited level must be involved. The lifetime of the electronic excited level is typically much shorter than that of the ^{229}Th isomeric level. As a result, the precision of a clock based on the HEB transition is mainly limited by the lifetime of the electronic excited level. In this work, we focus on the $^{229}Th^{32+}$ and $^{229}Th^{40+}$ ions since they possess low-lying electronic excited levels with relatively long lifetimes on the order of several seconds.

The electronic ground state configuration of the $^{229}Th^{32+}$ ion is $[\mathrm{Kr}]4d^{10}4f^{12}$ with an angular momentum $J_g = 6$. Its partial electronic energy levels are calculated by using the Jena Atomic Calculator (JAC) [47]. For $^{229}Th^{32+}$, the first and second excited electronic states have angular momenta of $J_e = 4$ and 2, with corresponding excitation energies of 1.58 and 6.80 eV, respectively, as shown in Fig. 1(b). These states are relatively long lived, with half-lives of approximately 5.7×10^3 s for the first excited state and 8.3 s for the second.

In $^{229}Th^{32+}$ ions, the HEB transition between the hyperfine levels $[I_e, (4f^{12})J_g = 6, F_e]$ and $[I_g, (4f^{12})J_e = 2, F_g]$ exhibits a larger sensitivity factor K_α than that between $[I_e, (4f^{12})J_e = 4, F_e]$ and $[I_g, (4f^{12})J_e = 2, F_g]$. However, the transition rate for $[I_e, (4f^{12})J_g = 6, F_e] \to [I_g, (4f^{12})J_e = 2, F_g]$ is very small, on the order of $10^{-16}\,\mathrm{s}^{-1}$, making experimental detection very challenging. We therefore concentrate on the HEB transition $[I_e, (4f^{12})J_e = 4, F_e] \to [I_g, (4f^{12})J_e = 2, F_g]$ with a transition energy of about 3.14 eV. The calculated transition rates are presented in Table I. For the sake of simplicity, the transition from $[I_e, (4f^{12})J_e = 4, F_e = i]$ to $[I_g, (4f^{12})J_e = 2, F_g = j]$ is denoted as $F_e = i \to F_g = j$ in this table and in the following discussions. The HEB transition rates are dominated by the $M1$ transition and lie in the range of 10^{-8} to $10^{-10}\,\mathrm{s}^{-1}$. These rates are comparable to that of the electronic electric octupole ($E3$) transition (on the order of $10^{-9}\,\mathrm{s}^{-1}$) in the $^{171}Yb^+$ optical clock [48,49]. These results are obtained

TABLE I. The HEB transition rates for different channels in $^{229}Th^{32+}$ ions.

Transition	Type	Rate (s^{-1})
$F_e = 5/2 \to F_g = 3/2$	$M1$	1.3×10^{-9}
$F_e = 5/2 \to F_g = 5/2$	$M1$	9.8×10^{-9}
$F_e = 5/2 \to F_g = 7/2$	$M1$	4.4×10^{-9}
$F_e = 7/2 \to F_g = 5/2$	$M1$	1.5×10^{-9}
$F_e = 7/2 \to F_g = 7/2$	$M1$	3.7×10^{-8}
$F_e = 7/2 \to F_g = 9/2$	$M1$	8.8×10^{-9}
$F_e = 9/2 \to F_g = 7/2$	$M1$	1.2×10^{-10}
$F_e = 9/2 \to F_g = 9/2$	$M1$	6.5×10^{-8}
$F_e = 11/2 \to F_g = 9/2$	$M1$	2.0×10^{-8}

using a newly developed module for HEB transitions implemented within the JAC toolbox.

The absolute values of mixing coefficients for the dressed hyperfine states $|[I_e(4f^{12})J_e = 4]F_eM_e\rangle$ and $|[I_g(4f^{12})J_e = 2]F_gM_g\rangle$ are less than 2×10^{-5} and the corresponding hyperfine splittings lie in the range of several to tens of μeV [as depicted in Fig. 1(d)]. As a result, the sensitivity factor K_α remains nearly constant for the HEB transition $[I_e, (4f^{12})J_e = 4, F_e] \to [I_g, (4f^{12})J_e = 2, F_g]$ across different values of F_e and F_g. Compared to the bare nuclear transition [$K_\alpha = -8.2 \times 10^3$, see Fig. 1(a)], the transition energy is reduced by a factor of about 2.7, resulting in a proportional enhancement of K_α, which reaches -2.2×10^4. Furthermore, the intrinsic quality factor Q ($\equiv \omega/\Gamma$ with transition energy ω and linewidth Γ) for this HEB transition is estimated to be about 5.7×10^{16}, exceeding that of the $^{40}Ca^+$ optical clock ($Q \approx 3.0 \times 10^{15}$) [50,51] and comparable to that of the $^{27}Al^+$ optical clock ($Q \approx 1.5 \times 10^{17}$) [52,53].

Similarly, the electronic ground state configuration of the $^{229}Th^{40+}$ ion is $[Kr]4d^{10}4f^4$ with an angular momentum of $J_g = 4$. Its first excited electronic state with an angular momentum of $J_e = 2$ lies at an energy of 5.98 eV, as illustrated in Fig. 1(c). This state exhibits a long half-life of about 10.5 s. Consider the HEB transition between the hyperfine levels $[I_e, (4f^4)J_g = 4, F_e]$ and $[I_g, (4f^4)J_e = 2, F_g]$, with a transition energy of about 2.39 eV. The corresponding transition rates, listed in Table II, are dominated by $M1$ transition and lie in the range of 10^{-9} to 10^{-10} s^{-1}, comparable to the electronic $E3$ transition rate in the $^{171}Yb^+$ optical clock.

TABLE II. Same as Table I, but for $^{229}Th^{40+}$ ions.

Transition	Type	Rate (s^{-1})
$F_e = 5/2 \to F_g = 3/2$	$M1$	9.1×10^{-10}
$F_e = 5/2 \to F_g = 5/2$	$M1$	2.1×10^{-9}
$F_e = 5/2 \to F_g = 7/2$	$M1$	5.7×10^{-10}
$F_e = 7/2 \to F_g = 5/2$	$M1$	2.1×10^{-9}
$F_e = 7/2 \to F_g = 7/2$	$M1$	5.5×10^{-9}
$F_e = 7/2 \to F_g = 9/2$	$M1$	8.7×10^{-10}
$F_e = 9/2 \to F_g = 7/2$	$M1$	7.6×10^{-10}
$F_e = 9/2 \to F_g = 9/2$	$M1$	6.0×10^{-9}
$F_e = 11/2 \to F_g = 9/2$	$M1$	4.0×10^{-9}

The absolute values of mixing coefficients in the dressed hyperfine states $|[I_e(4f^4)J_g = 4]F_eM_e\rangle$ and $|[I_g(4f^4)J_e = 2]F_gM_g\rangle$, as well as the associated hyperfine splittings [see Fig. 1(e)], are of the same order of magnitude as those in the $^{229}Th^{32+}$ case. Therefore, the sensitivity factor K_α remains again nearly constant for the HEB transition $[I_e, (4f^4)J_g = 4, F_e] \to [I_g, (4f^4)J_e = 2, F_g]$ across different values of F_e and F_g. Due to the reduction in the transition energy by a factor of 3.5 compared to the bare nuclear transition, K_α becomes -2.9×10^4. In addition, the intrinsic quality factor Q is calculated to be 5.5×10^{16}, close to that obtained for the $^{229}Th^{32+}$ case.

V. FURTHER DISCUSSION

Our approach can be readily extended to other nuclear systems, such as the ^{235}U nucleus. This nucleus possesses an isomeric state with an energy of about 76.74 eV [54], which is the second-lowest nuclear excitation energy among all known isomers. The isomer exhibits an extremely long radiative half-life on the order of 10^{24} s, rendering its direct detection via precision spectroscopic methods infeasible. In contrast, the electronic bridge (EB) process offers a feasible way to manipulate the transition of this isomer, thereby enabling its potential application for establishing another nuclear-clock transition [55]. In the Supplemental Material of Ref. [37], the authors claimed that the EB process cannot enhance K_α of ^{235}U but without any proof. By taking the hyperfine structure into account, our present work clearly demonstrates that the EB mechanism can significantly enhance the sensitivity to the α variation, primarily due to the reduction of the transition energy. It also indicates that the claim given in Ref. [37] is incorrect.

In experiments, the α variation is constrained by measuring the relative change in the frequency ratio of two atomic clocks. Specifically, let the two clock frequencies be ω_1 and ω_2, and define their ratio as $R = \omega_1/\omega_2$. The relative change of R is given by

$$\frac{\dot{R}}{R} = (K_{\alpha,1} - K_{\alpha,2})\frac{\dot{\alpha}}{\alpha}, \tag{10}$$

where $K_{\alpha,1}$ and $K_{\alpha,2}$ are the sensitivity factors corresponding to the two clocks. The difference $(K_{\alpha,1} - K_{\alpha,2})$ determines how sensitive a specific experimental system is to the α variation. Therefore, enhancing the sensitivity factor of one clock and designing an appropriate comparison scheme can improve the experimental precision in detecting the α variation. Several experiments on optical clock frequency comparison have been reported, including frequency comparisons between the $^{27}Al^+$ optical clock and the $^{199}Hg^+$ optical clock [17], as well as the $E2$ and $E3$ transitions in the $^{171}Yb^+$ optical clock [21].

VI. CONCLUSION

In summary, we have demonstrated that the 3.14-eV HEB transition between the hyperfine levels $[I_e, (4f^{12})J_e = 4, F_e]$ and $[I_g, (4f^{12})J_e = 2, F_g]$ in $^{229}Th^{32+}$ ions, as well as 2.39-eV HEB transition between $[I_e, (4f^4)J_g = 4, F_e]$ and $[I_g, (4f^4)J_e = 2, F_g]$ in $^{229}Th^{40+}$ ions, exhibit high sensitivity to the variation of α. By developing the theory of the HEB

transition, we calculate the sensitivity factors K_α of these two HEB transitions to be -2.2×10^4 and -2.9×10^4, respectively. Compared to the bare nuclear transition, these values represent enhancements by factors of 2.7 and 3.5. Notably, the 3.14- and 2.39-eV HEB transitions are well within the current laser-accessible range for precise measurement, making the HEB transition-based approach promising for advancing the precision of α variation detection. Finally, our approach can be readily applied to other nuclear systems to further enhance their sensitivity factors K_α.

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DATA AVAILABILITY

The data that support the findings of this article are available within the article.

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