

Regular Articles

Recent developments in the understanding and passive mitigation of transverse mode instability[☆]

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ABSTRACT

In this article we look at the newest developments in the understanding and mitigation of TMI in single-core fibers. This includes recent quantitative measurements that reveal the dependence of the TMI threshold on the modal content of the seed, systematic measurements on the dependence of the TMI threshold on the fiber core size, as well as the study of TMI in PM fibers including a novel passive mitigation strategy and the static modal energy transfer recently observed in these fibers.

1. Introduction

Fiber laser systems have demonstrated an unprecedented average-power scaling capability. This is due to their ability to efficiently evacuate heat thanks to their thin and long geometry, which offers a large surface to volume ratio. This capacity to operate at very high powers, together with their essentially hands-free operation and a nearly diffraction-limited emitted beam, has made them one of the fastest growing laser technologies to date. Their application portfolio ranges from material processing to basic science [1]. Furthermore, with the advent of multicore fiber technology, fiber laser systems are being seriously considered for visionary applications such as laser-driven fusion [2] and particle acceleration [3], just to mention a few. This clearly illustrates the success of a technology that began as a mere laboratory curiosity and has emerged as one serious contender for the development of tomorrow's high-power, high-energy lasers.

Despite this success, the development path of fiber lasers was nothing but challenging, since they faced and had to overcome many limitations along the way. One of such limitations was to solve the problem of coupling high intensity pump radiation in the optical fiber. This led to the development of the double-clad concept [4] and has been one of the main drivers for a continuous improvement of the brightness of diode

lasers. Another classic limitation of fiber lasers is created by non-linear effects, which onset threshold is particularly low in these systems due to the tight confinement of the light in the fiber core and the long interaction lengths of high-intensity radiation with the fiber material. Thus, fibers had to, yet again, evolve to reduce the intensity of the light in the fiber core. This led to the development of large-mode-area (LMA) fiber designs [5], which have been taken to the extreme with the so-called rod-type fiber designs [6]. In fact, rod-type fibers, such as the large-pitch fiber (LPF) [7], have already demonstrated effective single-mode operation with mode field diameters in excess of one hundred times the wavelength [8,9]. At the same time that the fiber cores grew larger, their length became shorter, which further mitigates non-linear effects. The problem of this design strategy is that it inadvertently increased the heat load in the fibers under high power operation. In fact, it can be safely stated that the requirements to mitigate non-linear effects are partially opposed to those needed to mitigate thermal issues [10].

Thus, with this early emphasis in the mitigation of non-linear effects, it is not surprise that, as the average power of fiber laser systems continued to increase, at some point thermal limitations should arise. This point was around 2010, when two manifestations of thermal-related limitations were reported. The first one, the least severe of them, is that of mode shrinking [11], in which the mode diameter of the

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beam propagating in a fiber shrinks with increasing output power due to thermal lensing [12–14]. This effect becomes more pronounced the larger the core diameter is, and it increases the impact of non-linear effects in the system. The second thermal limitation has become one of the most hindering effects for this technology and it is the main topic of this manuscript: transverse mode instability (TMI) [15,16]. This term describes the sudden uncontrolled fluctuations that the output beam of a fiber laser system undergoes once that a certain average power threshold has been reached [17,18]. Even though TMI has been mostly observed in Yb-doped fibers, it is a general phenomenon of fiber laser systems, regardless of their emission wavelength. This has been recently demonstrated by the observation of this effect in Tm-doped fibers in the 2 μm emission region [19], something that had been theoretically predicted before [20].

There has been a sustained worldwide effort in the fiber laser community to unravel the physics behind TMI for well over a decade now. As a result, there has been a significant progress in the understanding of the physical mechanisms behind this effect both from the theoretical point of view [21–28], and from the experimental side [29–34]. However, in spite of this significant effort, there are still some points that remain open and are still a source of discussion in the scientific community. Some of these open points will be addressed in this manuscript.

TMI is a particularly harmful effect for fiber laser systems since it hampers them exactly in what they are best at: scaling the average output power. Therefore, it is understandable that there is a great deal of interest in mitigation strategies for TMI. This interest has sparked the development and proposal of many techniques to increase the TMI threshold, both passive (i.e. which do not require any extra actuation elements) [35] and active (i.e. requiring extra actuation elements) [36–38]. All these techniques have been successful in one way or another and have attracted the attention of the scientific community. However, it is fair to say that the passive mitigation strategies are the most attractive from the practical point of view since they do not increase the complexity of the system. Therefore, this manuscript will focus on the recent progress in passive techniques.

Finally, it should be mentioned that, recently, the advent of parallelization [39] and, in particular, of multicore fibers (MCFs) [40], has unveiled a path to simultaneously shift the onset threshold of all the limitations of fibers by the number of cores/channels [41]. This has shown a practical way to get around the challenges currently limiting the performance scaling of these systems. In fact, some theoretical studies have revealed a very large scaling potential of MCFs both in the average power and in the pulse energy [42]. Despite this, each single core in a MCF still remains limited by TMI. Therefore, understanding the dependencies of this effect in such a way that the threshold of individual cores can be optimized/increased will have a high impact in multicore fibers (since the improvement will be multiplied by the number of cores, which can be in the hundreds). Thus, the development of multicore fibers has not made the study of TMI redundant but, instead, its full understanding has become even more pressing.

In this article we will look at the newest developments in the understanding and mitigation of TMI in single-core fibers. The first section provides a short summary and overview of the physical origin of TMI, which will be required to understand the following sections. Section 2 will then review recent quantitative measurements that reveal the dependence of the TMI threshold on the modal content of the seed. Section 3 presents very recent systematic measurements that provide a conclusive answer to one of the most long-standing discussions on this topic: the dependence of the TMI threshold on the fiber core size. Section 4 presents recent results on the study of TMI in PM fibers including a novel passive mitigation strategy and the static modal energy transfer recently observed in these fibers. Finally, some conclusions are drawn.

2. The physical origin of TMI

In order to develop mitigation strategies for TMI, it is necessary to

understand its physical origin. This will allow identifying dependencies that can be exploited to increase the onset threshold of this effect. Therefore, in this section we will briefly review the physics behind TMI, which will provide the necessary foundation to understand the following sections.

As already mentioned, TMI is an effect which onset is related to the average output power of the system or, better said, to its heat load. Once this threshold has been reached, the output beam starts to fluctuate in ms time-scales [29]. Soon after the first reported observations of TMI, it was identified that the beam fluctuations were the result of an uncontrolled energy transfer between different transverse modes of the fiber. This observation allowed the development of the first theoretical models to explain the effect [21,25,43], which pointed out that the modal energy transfer is assisted by a thermally-induced index grating (RIG). The steps that lead to the creation of this grating are depicted in Fig. 1 and explained in detail elsewhere [18]. There it can be seen that, if there are two or more mutually coherent transverse modes in the fiber core, they will beat with each other giving rise to a modal interference pattern (MIP) (Fig. 1 upper left). The main feature of this MIP is that, at most positions along the fiber, it exhibits an asymmetric intensity distribution in the core. This, in turn, leads to an asymmetric depletion of inversion in the active fiber core, which, along the fiber, gives rise to an inversion grating that mimics the MIP (Fig. 1 upper right). This semi-periodic, transversally inhomogeneous inversion depletion results in a quasi-periodic heat-load generation, which features resemble that of the MIP. This heat-load profile, over time, develops in a quasi-periodic temperature profile (Fig. 1 lower right) which, through the thermo-optic effect, is translated into a thermally-induced refractive index grating (Fig. 1 lower left). Since this RIG has been ultimately written by the MIP, it has all the characteristics (in terms of periodicity and refractive index shape) required to potentially transfer energy between the transverse modes of the fiber.

However, as first pointed out by A. Smith [21], for this modal energy transfer to happen it is necessary that there is a phase shift between the MIP and the RIG. In active fibers such a phase shift is possible due to the slow thermal response of the system [44]. In fact, it has been theoretically shown that, depending on the sign of this phase shift, the energy flows in a different direction between the transverse modes [18,43]. For example, if the MIP is shifted downstream the fiber with respect to the RIG, the energy flows from the fundamental mode (FM) into the higher order modes (HOM) (this situation is called negative phase shift). Conversely, if the MIP is shifted upstream the fiber with respect to the RIG, then the energy flows from the higher order modes towards the fundamental mode (this situation is referred to as positive phase shift). In TMI the beam fluctuations are the result of the uncontrolled changes in direction of the energy flow between the transverse modes, resulting from a dynamic phase shift between the RIG and the MIP.

This theory of the phase-shift and its crucial role in the dynamics of TMI has been experimentally demonstrated with some systematic experiments involving the modulation of the pump power [37]. There it was shown that, if the pump power increases suddenly, there is a certain time period in which the temperature increases and results in a positive phase shift, thus leading to the energy flowing towards the fundamental mode. As a result, a beam cleaning could be demonstrated even at (instantaneous) powers significantly higher than the TMI threshold. Likewise, if the pump power drops, the progressive cooling of the fiber leads to the generation of a negative phase shift and to the energy flowing from the fundamental mode into the higher order modes. This leads to a degradation of the beam quality even at low output powers. These experimental observations have led to the development of effective active TMI mitigation strategies based on the modulation of the heat load and the seed in an optical fiber [38,45–47]. However, interesting as these techniques are, they fall outside the scope of this paper which, as already mentioned, will focus exclusively on passive TMI mitigation strategies. In this context, one of the most attractive “passive” mitigation strategies is, certainly, to develop new glass compositions with a higher

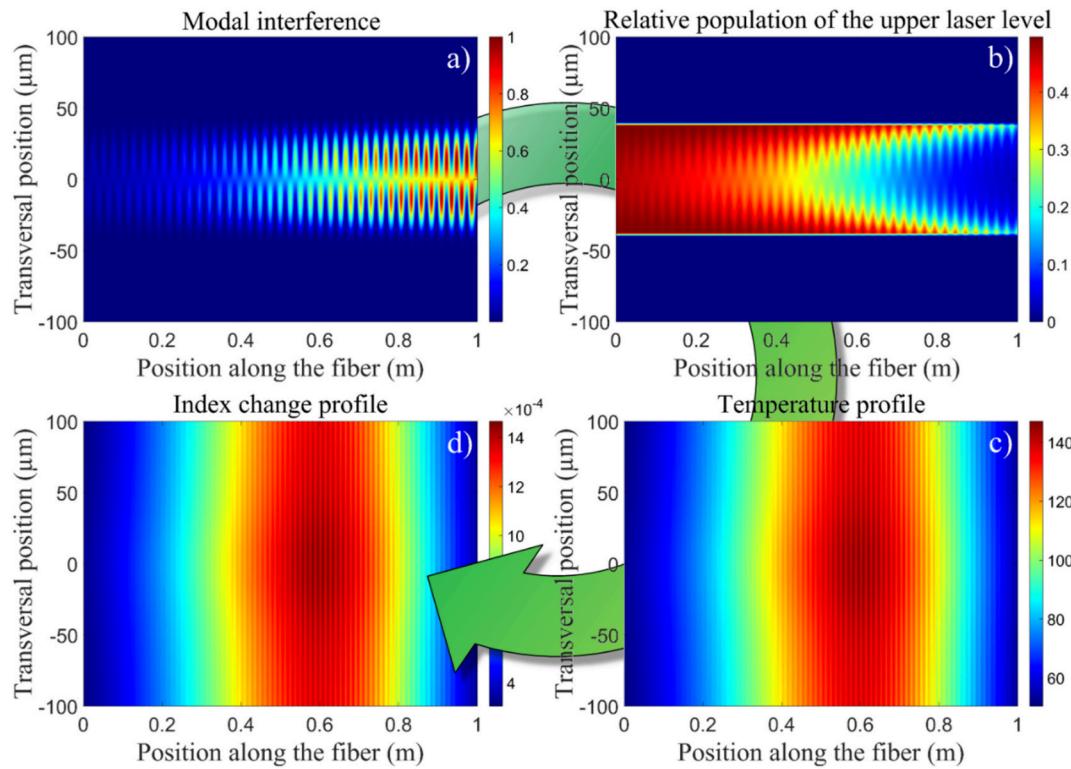


Fig. 1. Schematic illustration of the physical origin of TMI. A modal interference pattern (upper left) depletes the inversion giving rise to a pattern that mimics it (upper right). This, in turn generates a quasi-periodic temperature profile (lower right) that, through the thermo-optic effect leads to the creation of a refractive index grating (lower left). Adapted from [17].

resilience to TMI [48–51]. In fact, finding a material solution to what ultimately is a material problem appears, in principle, to be the best and most sensible strategy. However, we consider this approach to be outside of the scope of this paper and, certainly, outside the expertise of the authors of this manuscript. In spite of this, there is a lot that can be done and learnt about TMI by optimizing the fiber and laser system parameters, which will be the main focus of this work.

3. Dependence of TMI on the modal content of the seed

Since TMI is ultimately induced by the MIP between the fundamental mode and one or more higher order modes (as explained in the previous section), it seems logic to think that there must be a dependency between its onset threshold and the modal content of the seed signal in an amplifier. Indeed, there have been reports time and again pointing in that direction [31,34,52–57]. Many of these reports are experimental in nature and offer only anecdotal or, at least, non-systematic evidence. Additionally, the largest proportion of them investigate the dependence of the TMI threshold on the HOM loss (which can be understood as an indirect measure of modal purity), usually due to fiber bending. In any case, all these experimental reports are supported by theoretical models that can explain the observations [23,26–28,58–60]. Besides, there are some systematic works that study the influence of HOM content on TMI [61,62], but their focus is set more on the dynamic behaviour of the effect and not on the threshold.

As already mentioned above, in spite of all this mounting experimental and theoretical evidence, a detailed systematic measurement of the impact of the seed modal content on the TMI threshold has only been presented very recently [63]. The reason is that this is a very difficult measurement to carry out, that requires a carefully aligned and calibrated setup as well as a flexible way to change the modal content of the seed beam. In this experiment such conditions have been achieved by using a fiber mode multiplexer [64]. This device has eight input single-

mode fibers, each one associated with a different transverse mode in the single output multimode fiber. This way, by changing the power in the different input fibers, the modal content in the output multimode fiber can be accurately controlled. Then, this multimode output beam is relayed into the test fiber using a concatenation of 4-f optical setups to preserve both the amplitude and phase information of the seed beam. The test fiber is a 76 cm long, index-guiding, photonic-crystal, rod-type fiber [65] which has 85 μ m core and 200 μ m cladding (which consists of a hexagonal array of holes possessing a diameter to pitch ratio of ~ 0.19). The core is formed by 19-missing holes surrounded by 4 rings of air-holes. This fiber has been purposely chosen because it has a very low TMI threshold and does not offer any type of HOM filtering that could falsify the results. The beams at the fiber input and output are sampled and the modal content is evaluated using a modal decomposition algorithm [66]. In these experiments the HOM was selected to be exclusively the LP₁₁ and its content in the seed beam could be varied between 8 % and 92 %.

The results are presented in Fig. 2. The first part of the plot (up to a HOM content of 50 %) corresponds to the region usually explored in most of the experimental reports mentioned before. As can be seen, our result confirms the experimental observations repeatedly reported worldwide: a higher HOM (LP₁₁) content leads to a lower TMI threshold. The added value of this measurement is that, for the first time, there is a direct quantitative relation between the HOM content and the reduction of the TMI threshold. Additionally, our theoretical model, described in [37], is able to accurately predict the behaviour of the TMI threshold with the HOM content. Note that in our simulations the test fiber has been modelled as a step-index core with a diameter of 85 μ m, a V-parameter of 7, a length of 76 cm and an Yb-doping concentration of 6.26e-25 ion/m³. Additionally, the model considers photodarkening-induced degradation in this fiber. Where the plot becomes more interesting and enters a largely unexplored territory is for HOM contents larger than 50 % (please note that this regime is mostly of academic

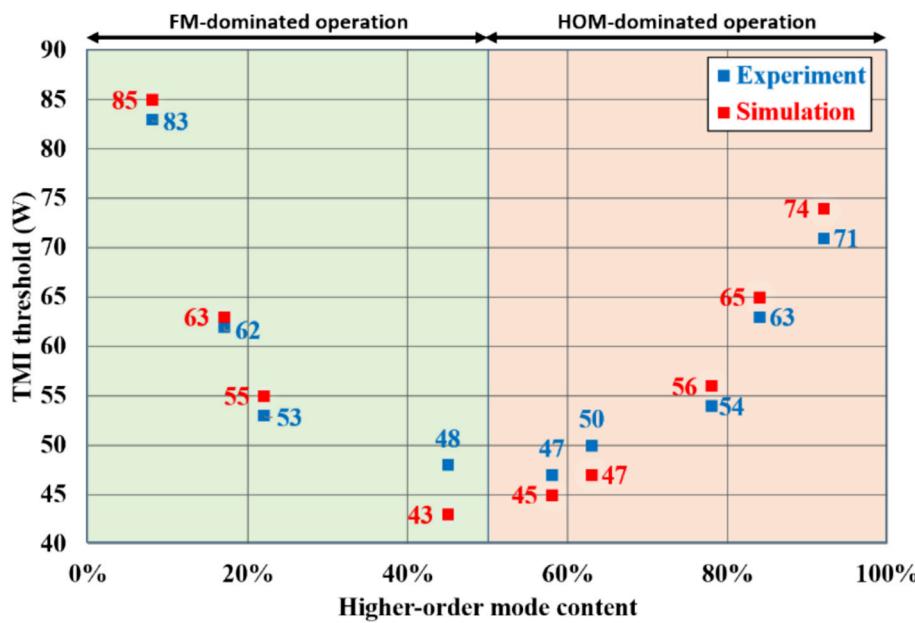


Fig. 2. Experimental (blue points) and simulated (red points) results of the dependency of the TMI threshold in a rod-type fiber amplifier as a function of the HOM content in the seed beam. The graph is divided in two regions: the green shaded one corresponds to an output beam dominated by the FM and the red-shaded one corresponds to an output beam dominated by the HOM. Plot adapted from [63].

interest since the related degradation in beam quality makes it less attractive for applications). It can be observed that, after reaching a minimum at $\sim 50\%$ HOM content, the TMI threshold increases again when the HOM content grows beyond that point. This, to the best of our knowledge, is a behaviour that has not been reported prior to these measurements and points out that the degree of single-mode operation in a fiber is the key to a high TMI threshold. Interesting as this observation is, it is not unexpected or entails new physics, since the theoretical model can very accurately reproduce this behaviour. In fact, the qualitative explanation of this observation is rather simple and has to do with the waning contrast of the thermally-induced RIG when a dominant mode is propagating in the fiber (regardless of which mode it is), which leads to a higher TMI threshold. Interestingly, the measurements of Fig. 2 also reveal that the TMI threshold is slightly higher when the FM content is 92 % than when the HOM (LP_{11}) content is 92 %. This, again, is accurately predicted by the theoretical model, which indicates that this difference is most likely due to the lower gain saturation of the amplifier when operating with a dominant HOM mode (due to their larger A_{eff}). As it had already been established that a stronger gain saturation leads to higher TMI thresholds [10,23,35,67–69], the result seen in Fig. 2 is also not surprising. So, to conclude, it can be deduced that the highest TMI threshold in a fiber will be reached by keeping single mode operation in the fundamental mode in a fiber as long as possible (i.e. for as higher powers as possible). This has, for example, been emphatically demonstrated in [70] in which ~ 3 kW of output power could be extracted out of a 4 m long fiber. As far as we are aware, this is the highest TMI-free average heat load demonstrated so far in Yb-doped fiber laser systems and could be achieved by strongly suppressing higher order modes (i.e. by keeping single mode operation even at high average powers).

4. Dependence of TMI on the mode-field diameter

Fewer topics have been more heatedly discussed in the context of TMI than the dependence of the threshold on the core size of the active fiber. This is understandable since the core size (or, more correctly, the mode-field diameter – MFD) is an extremely important design parameter that is typically leveraged to mitigate nonlinear effects [71]. In fact, the first observations of TMI [15,16] coincided with a design trend that

optimized fibers in terms of nonlinearity by making them shorter and with larger cores [72]. As a matter of fact, the TMI threshold in these fibers was lower than the average power already demonstrated with long fibers and smaller cores [73]. Therefore, it was concluded that something in these fiber designs promoted the onset of TMI. It was soon established that shortening the fiber leads to an increase of the average heat load and, therefore, to a drop in the TMI threshold [10,74]. The question that remained was whether increasing the core size also contributed to a drop of the TMI threshold. In principle there are enough physical reasons to expect such a behaviour, since it is known that the impact of a thermal lens becomes stronger for larger beam diameters [12,27]. The interesting thing, however, is that the different theoretical models developed over the years provided widely different answers to this question: some indicated that the dependence should be weak or almost non-existent under certain circumstances [10,24,75–77], whereas others pointed out to a very strong dependence [27,28,78]. In fact, the most widely accepted theory to date [27,79] claims that the dependence of the TMI threshold on the MFD should follow a $\sim 1/MFD^2$ function, thus strongly decreasing for larger fibers.

These widely different theoretical predictions have given rise to a lively discussion in the scientific community that has lasted until present. The reason why this has not been settled before is the lack of careful, systematic, experimental evidence on this topic, which is due to the intrinsic difficulty of achieving truly comparable measurements for fibers with different core sizes. For example, in conventional step-index fibers, scaling the core size will result in a change of the guiding properties (i.e. the number of modes guided by the fiber, their overlap with the core, their bend losses, etc). Since the amount of HOMs present in the fiber affects the TMI threshold (as we have seen in the previous section), it becomes nearly impossible to do a fair comparison between two such fibers with different core sizes. This has hampered getting sound experimental evidence on the dependence of the TMI threshold on the core size.

However, there is a family of fibers which show a so-called proportional scaling, i.e. the number and relative size of the modes do not change with the core size. Therefore, these type of fibers, known as large pitch fibers (LPFs) [7,9,80], are the perfect candidates to do a study on the impact of the core size on the TMI threshold. Besides, these are so-called, rod-type fibers [6], which are stiff waveguides that cannot

(and should not) be bent and have a device length usually between 1 m and 2 m. This additional feature is helpful in avoiding any inaccuracies introduced by bend losses. However, finding the right fiber to do the experiments is just the first step. Then the measurements have to be meticulously carried out to rule out other sources of error. For example, as already hinted out above, TMI is very sensitive to sources of heat in the fiber other than the quantum defect. One such heat source is photodarkening [81]. This effect has to be considered when performing careful measurements of the TMI threshold since it leads to a temporally changing TMI threshold that stabilizes/saturates after a certain time (usually some hours). Additionally, as already mentioned, the degree of gain saturation in the fiber also affects the TMI threshold. This means that the seed power has to be carefully controlled in these experiments since, under some circumstances, even small changes in the seed power can lead to strong TMI threshold variations [10,23,68].

It is worth mentioning that there has been an attempt to measure the dependence of the TMI threshold on the core size using fully aperiodic LPFs [82] in 2017. These early results are very interesting but, unfortunately, they only used three different core sizes, which are too few to discern a well-defined dependence. Additionally, it is not clear in the manuscript whether the saturation level, photodarkening losses, polarization, etc have been controlled and accounted for in the measurements. Therefore, in spite of these early experimental results, there is still a need for more comprehensive and systematic measurements of the dependence of the TMI threshold on the core size. Very recently such measurements have been made in our labs using a set of ~ 1.3 m long, ytterbium-doped, LPFs which cores ranged from ~ 45 μm to ~ 90 μm . All the fibers were pumped at 976 nm in the counter-propagating direction. After some refinements of the early measurements presented in [83], we have managed to account for all the effects mentioned above (e.g. the fibers were operated at high power over several tens of hours to ensure that the TMI threshold had stabilized, also the experiments were carried for different seed powers to be able to evaluate the case with constant input power and with constant input intensity, etc). With these refinements we have obtained a conclusive answer to this particular question, as shown in Fig. 3. At this point it is important to remark that the core size of a fiber is not a self-sufficient parameter in this context (i.e. the same core size can lead to different mode sizes, depending on, e.g. the V-parameter). This is why the results in Fig. 3 are plotted as a function of the mode field diameter at the output of the fiber (hot MFD) and not against the core size.

In Fig. 3 we can see the measured TMI extracted-power threshold (i.e. output power minus seed power) for LPFs with different core sizes (plotted against the MFD of the output beam) when the seed power is

kept constant at a value of 30 W. What these results show is that, as predicted by most models, the TMI threshold decreases for larger core sizes. Crucially, though, this dependency is weaker than anticipated by some models and is best fit by a function $\sim 1/\text{MFD}$. In the case in which the input seed intensity is kept constant (i.e. for a constant saturation level in the amplifier), similar results are obtained but the dependency is even slightly weaker (and seems to become even flatter the higher the seed intensity). This result means that care and attention have to be put in the fiber design when trying to mitigate non-linear effects at high average powers. In general, it seems that, for a desired output peak power, there will be an optimum fiber core size that allows maximizing the average power.

5. TMI in PM fibers

Recently researchers worldwide have started to explore TMI and related phenomena in specialty fiber laser systems such as double-pass amplifiers [84,85], dual-core fibers [86], multimode fibers [87] and polarization maintaining (PM) fibers [88–90], just to mention a few. The case of PM fibers is particularly interesting in this context because it has shown a rich palette of phenomena related to TMI that not only allows understanding the physics behind this effect better, but it has also led to the development of an effective TMI mitigation technique. The reason for this rich behaviour is the fact that, in the context of TMI, PM fibers act as two collocated but independent waveguides (one for each polarization axis). Thus, even though the radiation fields propagating in each polarization axis cannot interfere with those of the orthogonal polarization, their thermally induced RIGs offer an interaction path between the two polarization axes. This, in turn, leads to a complex dynamic behaviour. Some of the most interesting phenomena reported so far will be summarized in this section.

5.1. TMI mitigation in PM fibers

As already mentioned, PM fibers have two main polarization axes as a result of a relatively strong linear birefringence. Crucially, the birefringence experienced by each transverse mode in a PM fiber is different [90,91]. In the context of TMI this means that the period of the MIP will be slightly different in the slow and fast axis of the fiber. As proposed in [90] and experimentally demonstrated in [89], this fact can be exploited to mitigate TMI. The operating principle is shown in Fig. 4.

As can be seen (and has already been mentioned above), in a PM fiber the MIPs in the two main polarization axes have slightly different periods. Therefore, when exciting both simultaneously by injecting a seed

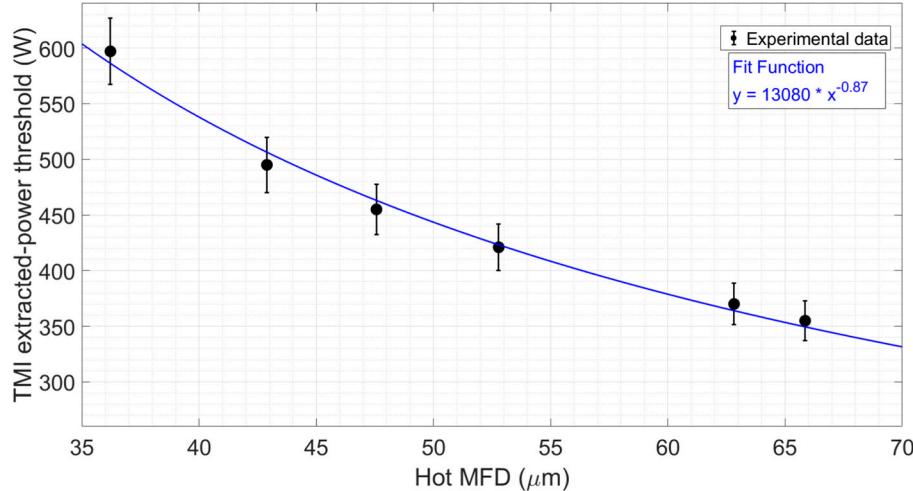


Fig. 3. Dependence of the TMI threshold on the MFD at the end of the fiber at high power operation (hot MFD). The 1.3 m long LPFs were seeded at 1030 nm with 30 W of power. The figure is an updated version of that presented in [83].

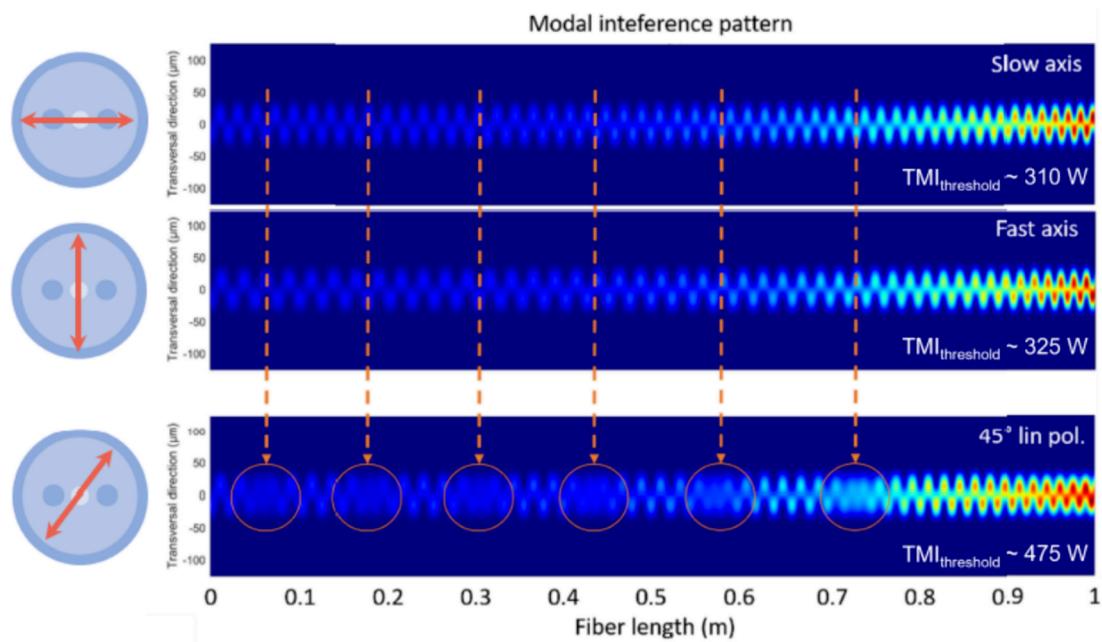


Fig. 4. Modal interference patterns along a 1 m long, 80 μm core fiber when it is illuminated with a seed beam (95 % FM and 5 % HOM) with a linear polarization aligned: parallel to the slow axis (upper plot), parallel to the fast axis (middle plot) or 45° with respect to the main polarization axes (lower plot). The orange lines indicate the positions at which the modal interference patterns of the slow and fast axis are out of phase. Reprinted with permission from [89] © Optica Publishing Group.

with a 45° polarization (lower plot in Fig. 4), the two MIPs overlap in the core. This generates a resulting MIP with washed out regions (that coincide with the positions at which the MIPs have a relative π -phase shift – orange lines and circles). Consequently, this results in a MIP that induces a weaker RIG which, in turn, leads to a higher TMI threshold.

Our simulation tools, described in [89,90], predict that, by rotating the polarization of the seed beam by 45° relative to the fiber axes, the TMI threshold can be increased by at least 50 % (with respect to the threshold of the fast axis). Another prediction of this model is that the TMI threshold of the slow axis should be somewhat lower than that of the fast axis. Even though there is no further explanation offered in [89,90], this is believed to stem from the stronger guiding of the slow axis (i.e. the stronger overlap of the HOMs with the core in this polarization).

Some early observations of the dependence of the TMI threshold on the polarization angle of the seed in a PM fiber were reported in 2020 [88]. Unfortunately, those first measurements were only presented as an observation and lacked in detail. However, they seemed to point out towards a significant increase of the TMI threshold when rotating the

input polarization angle in the vicinity of 45°, as theoretically predicted in [90]. A couple of years later, some thorough systematic investigation on this topic were presented [89], confirming both the previous observations and the theoretical predictions. These experiments used a homemade, 1.2 m long, coiled, few-mode, ytterbium-doped fiber with 36 μm core and 200 μm cladding, and a birefringence $\sim 1.5 \cdot 10^{-4}$. The fiber was seeded by a CW, linearly polarized, 5 W seed and was pumped at 976 nm in the counter-propagating direction. The results of the measurement of the TMI threshold as a function of the input polarization angle is shown in Fig. 5.

What the measurements in Fig. 5 reveal is that the TMI threshold increases significantly when the input polarization angle is detuned from the main polarization axis. In fact, the maximum is found at $\sim 50^\circ$, which is close to the theoretical prediction (the slight difference has to do with the bend losses of the HOM, which were not considered in the theoretical model). In this configuration the TMI threshold is ~ 60 % higher than that of the fast axis (also close to the theoretical prediction) and nearly a factor of 2 higher than that of the slow axis. In these measurements the difference in TMI threshold between the slow and fast

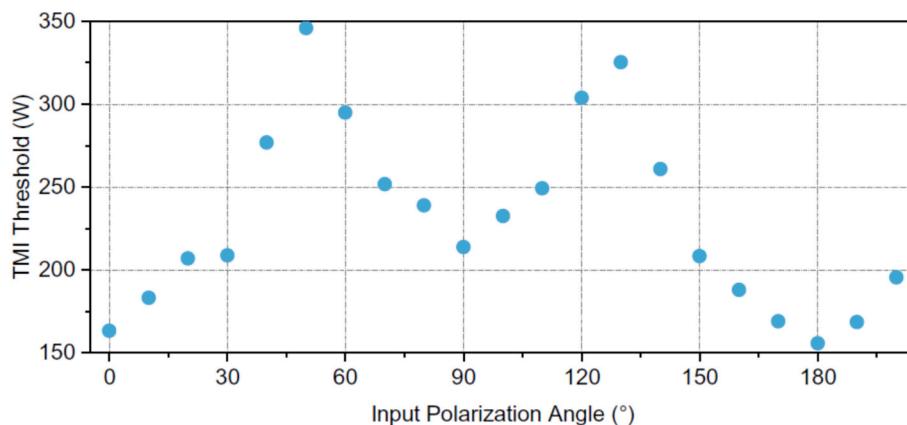


Fig. 5. Dependence of the TMI threshold on the polarization angle of the seed light in a PM fiber. Reprinted with permission from [89] © Optica Publishing Group.

axis is higher than theoretically expected, but this is most likely related to the significantly weaker guidance of the light in the fast axis of this particular PM fiber [89,92].

All in all, this is a simple, attractive and effective mitigation strategy for TMI that has been independently confirmed by at least two groups [88,89]. Its main drawback is that the output light is not linearly polarized. However, since the output beam still has a high degree of polarization, the output polarization state can, in principle, be transformed into a linear one.

5.2. Quasi-static modal energy transfer in PM fibers

The coexistence of two RIGs with slightly different periods in the core of a PM fiber leads to another interesting phenomenon: the quasi-static modal energy transfer below the TMI threshold [93]. This effect has also been reported in double pass amplifiers [84,85], dual core fibers [86,94], and even in fibers which have degraded due to photodarkening [95]. In general, what is observed in all these systems is that there is an energy exchange between the FM and the HOM of a fiber which is static in time but becomes progressively stronger at higher output powers. This usually leads to a progressive decrease of the beam quality emitted by such laser systems with increasing output power. The physical origin of this effect in a PM fiber is schematically explained in Fig. 6.

As can be seen in Fig. 6, which implicitly assumes operation below the TMI threshold, the MIP in the fast axis (in blue) has a slightly longer period than the MIP in the slow axis (red). It is also worth noting that there is no phase shift between a MIP and its corresponding RIG. This implies that the RIGs will not lead to any modal energy transfer in the MIPs that induced them (i.e. the MIP and RIG of the same color). This is the usual situation in single core, non-PM fibers operating below the TMI threshold. However, since the MIP of one polarization axis and the RIG generated by the radiation in the orthogonal polarization axis have slightly different periods (see RIG and MIP of different colors), there is an implicit permanent phase shift between them that leads to a modal energy transfer (which can be quite significant if the difference in periods is not too large). This implies that the RIG generated by the light polarized parallel to the fast axis/slow axis will induce a static modal energy transfer in the light polarized parallel to the slow axis/fast axis. Crucially, since the strength of the RIGs increase with the output power, this static modal energy transfer will become stronger at higher power operation. This can be seen in Fig. 7.

Fig. 7a) shows the theoretical prediction of the quasi-static energy

transfer as a function of the output power in a 1 m long, 80 μm core rod-type, polarization maintaining fiber amplifier. This is seeded with light polarized parallel to the fast axis (28.5 W in the FM and 1.5 W in the HOM). Additionally, a probe beam of 1 mW (only FM) is coupled with its polarization aligned to the slow axis of the fiber. In this situation the only RIG present in the fiber is that induced by the light which polarization is oriented parallel to the fast axis. This way, as expected, below the TMI threshold (~ 325 W in this simulation) the modal content of the light in the fast axis does not change (since there is no phase shift between the MIP and the RIG; see blue lines in Fig. 5). However, there is a progressively stronger modal energy transfer in the probe light polarized parallel to the slow axis as the output power is increased. Fig. 7b) shows the experimental demonstration of this effect in a 7.8 m long, 20 μm core, polarization maintaining fiber. This fiber is seeded with 1 W at 1065 nm and a polarization rotation angle of 85° (so most of the energy is coupled in the fast axis and only a few percent of the power is used to probe the slow axis). This setup shows a TMI threshold of 715 W. As can be seen in Fig. 7b), there is a good qualitative agreement between the experimental results and the theoretical prediction. This is the first experimental demonstration of the phenomenon of static modal energy transfer in PM fibers. Besides, this result delivers an indirect confirmation of the existence of the thermally induced RIG (for an alternative confirmation please refer to [30]) and how it becomes progressively stronger as the output power (and, therefore, the heat load) increases, thus corroborating the theory explained in Section 2.

6. Conclusion

In this work we have reviewed the most recent developments in the understanding and passive mitigation of TMI. Here we address open points in the understanding of TMI and present passive mitigation strategies (i.e. those that do not require any active element) and guidelines to maximize the onset threshold of this effect. One of the first open points that we have identified is the dependence of the TMI threshold on the modal content of the seed. It has been reported time and again that a higher HOM content in the seed leads to a lower TMI threshold, but this has never been directly experimentally quantified. In this work we review some recent measurements that quantify this impact. These measurements reveal that, as expected, the TMI threshold decreases with the HOM content up to a modal split of 50 %. From that moment on, interestingly, the TMI threshold increases again. Thus, these measurements point out and confirm the suspicion that the single mode

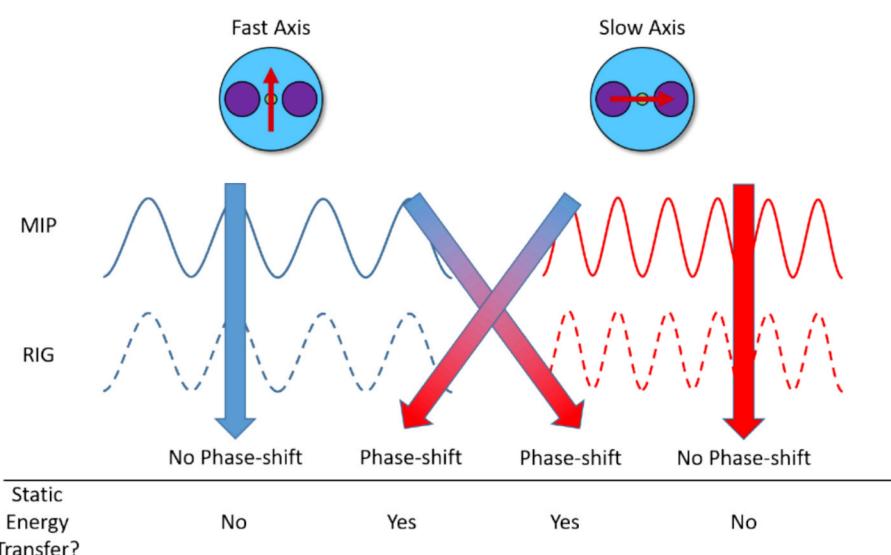


Fig. 6. Schematic diagram of the different interactions between the MIPs and RIGs of a PM fiber that might lead to a static modal energy transfer. Reprinted from [93].

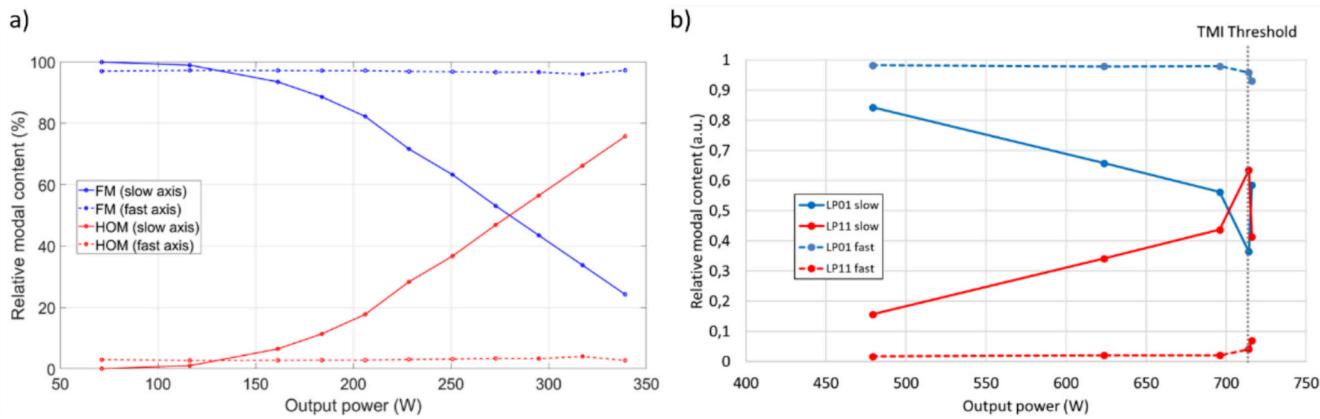


Fig. 7. a) theoretical prediction of the static modal energy transfer in a pm fiber as a function of the output power and b) experimental verification. Adapted from [93] and [96].

behaviour of the fiber is the most important aspect to increase the TMI threshold.

One of the most controversial topics in the context of TMI is the dependence of the threshold with the core size of the fiber. There has been a long-standing discussion in the scientific community on this topic, with opinions ranging from a weak dependence to a very strong one. The reason for this enduring debate is the intrinsic difficulty of the experimental measurement. Therefore, this is one of the most prominent open topics in this area. In this manuscript we have reviewed some very recent experimental measurements, that provide a conclusive answer to this discussion. These measurements reveal that the TMI threshold drops for larger core sizes, but its dependence is noticeably weaker than that predicted by the most widely accepted theories.

A relatively new topic in the context of TMI is the study of this effect in PM fibers. Here we review the use of PM fiber to increase the TMI threshold by rotating the input polarization angle by 45° with respect to the main polarization axes of the fiber. This mitigation strategy has been theoretically predicted and, later, experimentally demonstrated and is very attractive due to its simplicity and effectiveness. Additionally, we also review the theoretical prediction and experimental demonstration of a phenomenon that shares a common physical origin with TMI: the quasi-static modal energy transfer. Hereby, at any given output power below the TMI threshold, there is a constant energy transfer between the FM and the HOM that, usually, leads to a loss of beam quality. Recently, the first experimental demonstration of this effect has been carried out, which shows a very good qualitative agreement with the theoretical predictions. Additionally, the measurements confirm that this effect becomes stronger at higher output powers. This delivers an indirect confirmation of the existence of the thermally induced RIG and corroborates the theory about the origin of TMI.

CRediT authorship contribution statement

César Jauregui: Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Yiming Tu:** Writing – review & editing, Visualization, Investigation, Formal analysis, Data curation, Conceptualization. **Sobhy Kholaif:** Writing – review & editing, Visualization, Investigation, Formal analysis, Data curation. **Friedrich Möller:** Writing – review & editing, Visualization, Investigation, Formal analysis, Data curation. **Gonzalo Palma-Vega:** Writing – review & editing, Visualization, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Nicoletta Haarlammert:** Writing – review & editing, Validation, Supervision, Resources, Project administration, Methodology, Funding acquisition, Conceptualization. **Till Walbaum:** Writing – review & editing, Validation, Supervision, Resources, Project

administration, Methodology, Funding acquisition. **Thomas Schreiber:** Writing – review & editing, Validation, Supervision, Resources, Project administration, Methodology, Funding acquisition, Conceptualization. **Jens Limpert:** Writing – review & editing, Validation, Supervision, Resources, Project administration, Methodology, Funding acquisition, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

The authors do not have permission to share data.

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