

Study of $D_s^+ \rightarrow f_0(980)\rho^+$ and $\phi\pi^+$ Decays through $D_s^+ \rightarrow \pi^+\pi^+\pi^-\pi^0$

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We perform the first amplitude analysis of $D_s^+ \rightarrow \pi^+\pi^+\pi^-\pi^0$ decays based on data samples of electron-positron collisions recorded with the BESIII detector at center-of-mass energies between 4.128 and 4.226 GeV, corresponding to an integrated luminosity of 7.33 fb^{-1} . We report the observation of $D_s^+ \rightarrow f_0(980)\rho(770)^+$ with a statistical significance greater than 10σ and determine the branching fractions $\mathcal{B}(D_s^+ \rightarrow \pi^+\pi^+\pi^-\pi^0|_{\text{non-}\eta}) = (2.04 \pm 0.08_{\text{stat}} \pm 0.05_{\text{syst}})\%$ and $\mathcal{B}(D_s^+ \rightarrow \eta\pi^+) = (1.56 \pm 0.09_{\text{stat}} \pm 0.04_{\text{syst}})\%$. Moreover, we measure the relative branching fraction between $\phi \rightarrow \pi^+\pi^-\pi^0$ and $\phi \rightarrow K^+K^-$ to be $[\mathcal{B}(\phi(1020) \rightarrow \pi^+\pi^-\pi^0)/\mathcal{B}(\phi(1020) \rightarrow K^+K^-)] = 0.230 \pm 0.014_{\text{stat}} \pm 0.010_{\text{syst}}$, which deviates from the world average value by more than 4σ .

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The exploration of charmed-meson $D_{(s)}$ hadronic decays allows the interplay of short-distance weak-decay matrix elements and long-distance quantum chromodynamics interactions to be studied. Moreover, measurements of the branching fractions (BFs) of charmed mesons can provide valuable insights for understanding the amplitudes and phases induced by the strong force [1–4]. The amplitudes describing the weak decays of charmed mesons are dominated by two-body processes, i.e., $D_{(s)} \rightarrow VP$, $D_{(s)} \rightarrow PP$, $D_{(s)} \rightarrow SP$, and $D_{(s)} \rightarrow VV$ decays, where V , S , and P denote vector, scalar, and pseudoscalar mesons, respectively. Significant progress has been achieved through a series of amplitude analyses on hadronic charmed-meson decays [1,5–8]. However, there have been fewer studies of $D_{(s)} \rightarrow SV$ decays [1], which means that the theoretical understanding of this process is less advanced compared to other types of two-body decays. Among $D_{(s)} \rightarrow SV$ decays, $D_s^+ \rightarrow f_0(980)\rho^+$ is of particular importance as it mainly involves a W -external-emission channel, the BF of which can be precisely calculated in the absence of final-state interactions, such as quark exchange or resonance formation [9–12]. Final-state interactions are key ingredients in the production of light scalar mesons, i.e., $f_0(500)$, $f_0(980)$, and $a_0(980)$, which are of particular interest given the lack of consensus on whether these particles are members of the normal scalar meson nonet or four-quark states. In addition, the BESIII

Collaboration recently observed abnormally large BFs for the $D_s^+ \rightarrow a_0(980)^{0(+)}\pi^{+ (0)}$ [6] and $D_s^+ \rightarrow a_0(980)^{0(+)}\rho^{+ (0)}$ [13] decays, which could potentially be explained by final-state rescattering effects [9,10]. Therefore, studying $D_s^+ \rightarrow f_0(980)\rho^+$ through an amplitude analysis of $D_s^+ \rightarrow \pi^+\pi^+\pi^-\pi^0$ decays can experimentally constrain the contribution from final-state interactions and help in understanding of the nature of the $f_0(980)$ meson.

Another interesting intermediate decay $D_s^+ \rightarrow \phi\pi$ can be studied through $\phi \rightarrow \pi^+\pi^-\pi^0$. As the key reference channel for D_s^+ decays, $D_s^+ \rightarrow \phi\pi^+$ is typically measured through $\phi \rightarrow K^+K^-$ [1]. However, studies of ϕ decays have primarily been conducted in e^+e^- annihilation and $K-p$ scattering experiments [1,14–18], which often encounter challenges from complex backgrounds and various interferences. The measurement of the BF of $D_s^+ \rightarrow \phi(\rightarrow \pi^+\pi^-\pi^0)\pi^+$, along with $\mathcal{B}[D_s^+ \rightarrow \phi(\rightarrow K^+K^-)\pi^+]$ [19], can provide a new method to determine the relative BF of $R_\phi = \mathcal{B}(\phi \rightarrow \pi^+\pi^-\pi^0)/\mathcal{B}(\phi \rightarrow K^+K^-)$ in a more controlled environment. Precise measurements of the BFs of ϕ decays are crucial not only for studying the strong interaction [20,21] but also for investigating B decays that involve ϕ mesons [22–25].

Finally, the decay $D_s^+ \rightarrow \omega\pi^+$ with $\omega \rightarrow \pi^+\pi^-\pi^0$ occurs solely via the W -annihilation process. A precise measurement of its BF can help improve the theoretical understanding, as current calculations suffer from large uncertainties [26–29]. The BESIII Collaboration has reported the BF of this decay to be $\mathcal{B}(D_s^+ \rightarrow \omega\pi^+) = (1.77 \pm 0.32_{\text{stat}} \pm 0.13_{\text{syst}}) \times 10^{-3}$ [30]. In this Letter, we provide a more precise measurement of the BF using a larger dataset through amplitude analysis, which takes the interference effect with other $D_s^+ \rightarrow \pi^+\pi^+\pi^-\pi^0$ processes into account. In addition, the $D_s^+ \rightarrow \pi^+\pi^+\pi^-\pi^0$ decay also

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contains a rich system of other possible intermediate components, such as $D_s^+ \rightarrow \eta\pi^+$, $D_s^+ \rightarrow f_0(500)\rho^+$, $D_s^+ \rightarrow f_0(1370)\rho^+$, $D_s^+ \rightarrow f_2(1270)\rho^+$, $D_s^+ \rightarrow \rho^0\rho^+$, $D_s^+ \rightarrow a_1^+\pi^0$, etc. Studying the relative contributions of these intermediate resonances can benefit the understanding of the strong interaction at low energies and the D_s^+ weak-decay mechanism.

In this Letter, we present the first amplitude analysis of the decay $D_s^+ \rightarrow \pi^+\pi^+\pi^-\pi^0$ using 7.33 fb^{-1} of e^+e^- collision data collected with the BESIII detector at center-of-mass energies between 4.128 and 4.226 GeV. At these energies, $D_s^{*\pm}D_s^\mp$ events provide an ideal environment for the study of D_s^+ physics. Throughout this Letter, charge-conjugated modes and exchange symmetry of two identical π^+ are implied. The resonances $\phi(1020)$, $\omega(782)$, $\rho(770)^{+0}$, and $a_1(1260)^{+0}$ are referred to as ϕ , ω , ρ^{+0} , and a_1^{+0} , respectively.

The BESIII detector [31] records symmetric e^+e^- collisions provided by the BEPCII storage ring [32] in the center-of-mass energy range from 1.85 to 4.95 GeV [33]. Large samples of Monte Carlo (MC) simulated events are produced with GEANT4-based [34] software, and are used to determine the detection efficiency and to estimate the background contributions. The beam-energy spread and initial-state radiation in the e^+e^- annihilation are modeled with the generator KKMC [35]. Inclusive MC samples of 40 times the size of the data sample are used to simulate the background contributions. The inclusive MC sample includes the production of open charm processes, the initial-state radiation production of vector charmonium (like) states, and the continuum processes incorporated in KKMC. All particle decays are modeled with EvtGen [36] using BFs either taken from the Particle Data Group [1], when available, or otherwise estimated with LUNDCHARM [37]. Final-state radiation from charged particles is incorporated using the PHOTOS package [38]. The signal detection efficiencies and signal shapes are obtained from signal MC samples, in which the signal $D_s^+ \rightarrow \pi^+\pi^+\pi^-\pi^0$ decay is simulated using the model from the amplitude analysis introduced in this Letter.

Signal events are from the $e^+e^- \rightarrow D_s^{*+}D_s^- + \text{c.c.} \rightarrow \gamma D_s^+D_s^-$ process, where $D_s^{*+(-)}D_s^{-(+)}$ are produced without additional hadronic particles, which provides a clean environment for amplitude analysis and precise measurement of the absolute BFs of $D_s^\pm$ hadronic decays. We utilize a double-tag (DT) technique [39–41] to study the signal process. In this procedure there are two types of samples: single-tag (ST) events, which are reconstructed with a D_s^- tag; and DT events, which are reconstructed with both a D_s^- tag and signal D_s^+ . In this analysis, the ST tag D_s^- candidates are reconstructed through seven modes: $D_s^- \rightarrow K_S^0 K^-$, $D_s^- \rightarrow K^+ K^- \pi^-$, $D_s^- \rightarrow K^+ K^- \pi^- \pi^0$, $D_s^- \rightarrow K_S^0 K^+ \pi^- \pi^-$, $D_s^- \rightarrow \pi^- \eta_{\gamma\gamma}$, $D_s^- \rightarrow \pi^- \eta'_{\pi^+\pi^-\eta_{\gamma\gamma}}$, and $D_s^- \rightarrow K^- \pi^- \pi^+$. Here, the K_S^0 , π^0 , η , and η' mesons are

reconstructed from $K_S^0 \rightarrow \pi^+\pi^-$, $\pi^0 \rightarrow \gamma\gamma$, $\eta \rightarrow \gamma\gamma$, and $\eta' \rightarrow \pi^+\pi^-\eta$ decays, respectively. The selection criteria for charged and neutral particle candidates are identical to those used in Ref. [13]. For the decay mode $D_s^- \rightarrow K^+ K^- \pi^- \pi^0$, we reject events with $K^+ K^-$ invariant mass above $1.05 \text{ GeV}/c^2$ to suppress background. The DT candidates are selected by reconstructing the signal process $D_s^+ \rightarrow \pi^+\pi^+\pi^-\pi^0$ from the remaining particles that are not used in the ST reconstruction.

The invariant masses of the ST and DT $D_s^\pm$ candidates, denoted M_{tag} and M_{sig} , respectively, are required to be within the range $[1.87, 2.06] \text{ GeV}/c^2$. We calculate the recoiling mass $M_{\text{rec}} = \{[E_{\text{cm}} - (|\vec{p}_{D_s^-}|^2 + m_{D_s^-}^2)^{1/2}]^2 - |\vec{p}_{D_s^-}|^2\}^{1/2}$ in the e^+e^- center-of-mass system, where E_{cm} is the center-of-mass energy of the data sample, $\vec{p}_{D_s^-}$ is the momentum of the reconstructed D_s^- , and $m_{D_s^-}$ is the known mass of the D_s^- meson [1]. The value of M_{rec} is required to be in the range $[2.05, 2.18] \text{ GeV}/c^2$ for the data sample collected at 4.178 GeV to suppress the non- $D_s^{*\pm}D_s^\mp$ events. The M_{rec} ranges for the other data samples are the same as those in Ref. [13].

To suppress background from $D_s^+ \rightarrow K_S^0 \pi^+ \pi^0$ decays, events are rejected if any of the two $\pi^+\pi^-$ combinations of the candidate signal decay has an invariant mass lying within the range $[0.46, 0.52] \text{ GeV}/c^2$. The decay $D_s^+ \rightarrow \eta\pi^+$ is also considered as background because $\eta \rightarrow \pi^+\pi^-\pi^0$ lies at the boundary of the phase space and thus has little interference with other intermediate decays in the $D_s^+ \rightarrow \pi^+\pi^+\pi^-\pi^0$ process. Therefore, events are rejected if any of the two $\pi^+\pi^-\pi^0$ combinations in the final $\pi^+\pi^+\pi^-\pi^0$ state has an invariant mass within the η mass range of $[0.52, 0.58] \text{ GeV}/c^2$.

To reduce combinatorial background, a seven-constraint kinematic fit [42] is applied to the $e^+e^- \rightarrow D_s^{*+}D_s^\mp \rightarrow \gamma D_s^+D_s^-$ candidates, where D_s^- decays to one of the tag modes and D_s^+ decays to the signal mode. The constraints are: four-momentum conservation in the center-of-mass system, and imposing that the invariant mass of π^0 from the signal decay, the reconstructed D_s^- from the tag decays, and the D_s^{*+} candidate have their particle data group (PDG) values [1]. If there are multiple candidate combinations, the combination with the minimum χ^2 of the seven-constraint kinematic fit is retained.

An observable, $M_{\text{rec}0} = \{[E_{\text{cm}} - (\vec{p}_{D_s^+ \gamma}^2 + m_{D_s^{*\pm}}^2)^{1/2}]^2 - |\vec{p}_{D_s^+ \gamma}|^2\}^{1/2}$, is required to lie within the range $[1.958, 1.986] \text{ GeV}/c^2$. The energy of the radiative photon from the $D_s^{*\pm}$ is required to be less than 0.18 GeV . The invariant mass of the $D_s^{*\pm}$ candidate must be within $[2.066, 2.135] \text{ GeV}/c^2$. Finally, the mass of the signal D_s^+ candidate is required to be within the range $[1.930, 1.985] \text{ GeV}/c^2$.

In particular for amplitude analysis, to achieve a better resolution for the reconstructed momentum, an additional

TABLE I. Physical parameters for the resonances involved in amplitude analysis.

Resonance	Mass (MeV/ c^2)	Width (MeV/ c^2)
$f_0(1370)$	1350	265
$f_0(980)$	960	$g_1 = 165, g_2 = 4.21g_1$ [50,51]
$f_2(1270)$	1275	190
$\rho(770)$	775	149
$\rho(1450)$	1465	400
$\phi(1020)$	1019	4
$\omega(782)$	783	9
$a_1(1260)$	1230	420
$\pi(1300)$	1371	314

constraint is added, imposing that the reconstructed signal D_s^+ mass has the PDG value. The four momenta of candidate events are updated following this eight-constraint kinematic fit for the amplitude analysis.

The datasets are divided into four categories according to the center-of-mass energy range: 4.13–4.16, 4.178, 4.189–4.219, and 4.226 GeV. We fit the D_s^+ peaks in these samples with a signal shape taken from MC simulation, convolved with a Gaussian function, and a shape for the background distribution also taken from simulation. The purities are determined to be $(83.8 \pm 1.1)\%$, $(81.0 \pm 0.7)\%$, $(80.2 \pm 1.0)\%$, and $(75.7 \pm 2.2)\%$, with corresponding signal yields of 189 ± 17 , 778 ± 35 , 448 ± 26 , and 137 ± 15 , respectively.

A simultaneous unbinned maximum-likelihood fit is performed on the four categories of data. The probability density function (PDF) is constructed depending on the momenta of the four final-state particles, using a signal-background model: $\text{PDF}(\mathbf{x}) = \xi f_S(\mathbf{x}) + (1 - \xi)f_B(\mathbf{x})$, where ξ is the purity of dataset, $\mathbf{x}$ is the location in phase space of the decay (determined by the momenta of the four final particles), f_S is the normalized signal-process distribution function, and f_B is the normalized background-distribution function. The signal model is constructed as a coherent sum of intermediate processes $M(\mathbf{x}) = \sum \rho_n e^{i\phi_n} A_n(\mathbf{x})$, where $\rho_n e^{i\phi_n}$ is the complex coefficient of the n th amplitude. The component amplitude $A_n(\mathbf{x})$ is given by $A_n = P_n^1 P_n^2 S_n F_n^1 F_n^2 F_n^3$, where the indices 1, 2, and 3 correspond to the two subsequent intermediate resonances and the D_s^+ meson, F_n^i the Blatt-Weisskopf barrier factor [43,44], and P_n^i the propagator of the intermediate resonance. The function S_n is the spin factor describing the L - S coupling in the amplitude and is constructed using the covariant-tensor formalism [44–46]. The propagators employed in this analysis are as follows: a relativistic Breit-Wigner [47] function for the $f_0(1370)$, $f_2(1270)$, $\pi(1300)$, a_1 , $\rho(1450)$, ϕ , and ω resonance; a Gounaris-Sakurai [48] line shape for the ρ resonance; and a coupled Flatté [49] formula for the $f_0(980)$ resonance, whose parameters are taken from Refs. [50,51]. The other physical

parameters for involving resonances in amplitude analysis are listed in Table I.

The background model $B(\mathbf{x})$ is constructed from inclusive MC samples by using a multidimensional kernel density estimator [52] with five independent Lorentz invariant variables ($M_{\pi^+\pi^+}$, $M_{\pi^+\pi^-}$, $M_{\pi^+\pi^0}$, $M_{\pi^-\pi^0}$, and $M_{\pi^+\pi^-\pi^0}$). The extracted shape shows good consistence with data sideband. As a consequence, the combined PDF can be written as

$$\epsilon R_4 \left[\xi \frac{|M(\mathbf{x})|^2}{\int \epsilon |M(\mathbf{x})|^2 R_4 d\mathbf{x}} + (1 - \xi) \frac{B_\epsilon(\mathbf{x})}{\int \epsilon B_\epsilon(\mathbf{x}) R_4 d\mathbf{x}} \right], \quad (1)$$

where ϵ is the acceptance function determined with phase-space (PHSP) MC samples generated with a uniform distribution over final particles' momentum of $D_s^+ \rightarrow \pi^+\pi^+\pi^-\pi^0$ decays, $B_\epsilon(\mathbf{x})$ is defined as $B(\mathbf{x})/\epsilon$, and $R_4 d\mathbf{x}$ is an element of four-body PHSP. The normalization integral in the denominator is calculated by the MC technique described in Ref. [53].

In the amplitude analysis, the initial model is constructed from those significant components known to be present, namely $\phi\pi^+$, $\omega\pi^+$, $f_0(980)\rho^+$, and $f_0(1370)\rho^+$. Then, further components are added, one at a time, to the fit. The statistical significance of a component is calculated from the resulting change of likelihood and number of degrees of freedom. Only those components with significance larger than 5σ are retained for the optimal model. Some components, such as nonresonance, $f_0(1500)$ and $a_2(1320)$, because of limited significance, are not included in the model. The dominant Cabibbo-favored process $D_s^+ \rightarrow f_0(1370)\rho^+$ is selected as the reference component, with its phase fixed to zero and magnitude to unity. The coefficients of the isospin-related subdecays of the ϕ , ω , and a_1 are related by Clebsch-Gordan coefficients. The final model contains 11 components, as listed in Table II. The mass projections of the fit are shown in Fig. 1, and the angular distribution of final-state π mesons can be found in Ref. [54]. The contribution of the n th component relative to the total BF is quantified by the fit fraction (FF) defined as $\text{FF}_n = \int |\rho_n A_n(\mathbf{x})|^2 R_4 d\mathbf{x} / \int |M(\mathbf{x})|^2 R_4 d\mathbf{x}$. The measured phases and FFs for the different components in the optimal fit are listed in Table II. Dominant interference is found to be from $f_0(980)$. The mass projections for contributions from different components, and the interference from $f_0(980)$ can be found at Ref. [54].

We determine the systematic uncertainties by taking the differences between the values of ϕ_n and FF_n found by the optimal fit and those found from fit variations. The masses and widths of intermediate states, such as $f_0(1370)$, $\rho(1450)$, and $\pi(1300)$, are varied by $\pm 1\sigma$ [1]. The masses and coupling constants of the $f_0(980)$ are varied within the uncertainties reported in Refs. [50,51]. The barrier radii for D_s^+ and the other intermediate states are varied by $\pm 1 \text{ GeV}^{-1}$. The uncertainties from detector effects are

TABLE II. Phases, FFs, and BFs for various intermediate processes in $D_s^+ \rightarrow \pi^+\pi^+\pi^-\pi^0$ decay. The first and the second uncertainties are statistical and systematic, respectively. The subsequent decay is given in parentheses, with the subscripts S and P indicating the spatial wave mode.

Component	Phase (rad)	FF (%)	BF (10^{-3})
$f_0(1370)\rho^+$	0.0(fixed)	$24.9 \pm 3.8 \pm 2.1$	$5.08 \pm 0.80 \pm 0.43$
$f_0(980)\rho^+$	$3.99 \pm 0.13 \pm 0.07$	$12.6 \pm 2.1 \pm 1.0$	$2.57 \pm 0.44 \pm 0.20$
$f_2(1270)\rho^+$	$1.11 \pm 0.10 \pm 0.10$	$9.5 \pm 1.7 \pm 0.6$	$1.94 \pm 0.36 \pm 0.12$
$(\rho^+\rho^0)_S$	$1.10 \pm 0.18 \pm 0.10$	$3.5 \pm 1.2 \pm 0.6$	$0.71 \pm 0.25 \pm 0.12$
$[\rho(1450)^+\rho^0]_S$	$0.43 \pm 0.18 \pm 0.17$	$4.6 \pm 1.3 \pm 0.8$	$0.94 \pm 0.27 \pm 0.16$
$[\rho^+\rho(1450)^0]_P$	$4.58 \pm 0.16 \pm 0.09$	$8.6 \pm 1.3 \pm 0.4$	$1.75 \pm 0.27 \pm 0.08$
$\phi[(\rho\pi) \rightarrow \pi^+\pi^-\pi^0]\pi^+$	$2.90 \pm 0.15 \pm 0.18$	$24.9 \pm 1.2 \pm 0.4$	$5.08 \pm 0.32 \pm 0.10$
$\omega[(\rho\pi) \rightarrow \pi^+\pi^-\pi^0]\pi^+$	$3.22 \pm 0.21 \pm 0.09$	$6.9 \pm 0.8 \pm 0.3$	$1.41 \pm 0.17 \pm 0.06$
$a_1^+[(\rho^0\pi^+)_S\pi^0]$	$3.78 \pm 0.16 \pm 0.12$	$12.5 \pm 1.6 \pm 1.0$	$2.55 \pm 0.34 \pm 0.20$
$a_1^+[(\rho\pi)_S \rightarrow \pi^+\pi^-\pi^0]\pi^+$	$4.82 \pm 0.15 \pm 0.12$	$6.3 \pm 1.9 \pm 1.2$	$1.29 \pm 0.39 \pm 0.24$
$\pi(1300)^0[(\rho\pi)_P \rightarrow \pi^+\pi^-\pi^0]\pi^+$	$2.22 \pm 0.14 \pm 0.08$	$11.7 \pm 2.3 \pm 2.2$	$2.39 \pm 0.48 \pm 0.45$

investigated by weighting PHSP MC samples according to data-MC difference. The same method is also employed in Ref. [19]. The uncertainty related to background is estimated by varying the estimated purity within its statistical uncertainty. The total uncertainties are obtained by adding

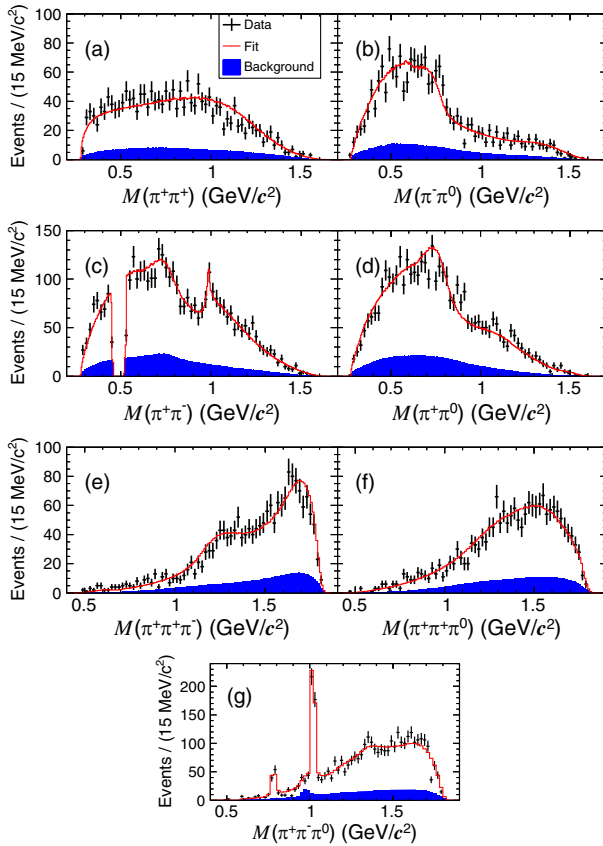


FIG. 1. Projections on (a) $M_{\pi^+\pi^+}$, (b) $M_{\pi^-\pi^0}$, (c) $M_{\pi^+\pi^-}$, (d) $M_{\pi^+\pi^0}$, (e) $M_{\pi^+\pi^+\pi^-}$, (f) $M_{\pi^+\pi^+\pi^0}$, (g) $M_{\pi^+\pi^-\pi^0}$ of the amplitude analysis. The combinations of two identical π^+ are added in (c), (d), and (g) because of the exchange symmetry.

the separate contributions in quadrature, as listed in Table II.

The measurement of the $D_s^+ \rightarrow \pi^+\pi^+\pi^-\pi^0$ BF is performed using a DT technique based on seven ST modes, the same as for the amplitude analysis. It is performed separately for “non- η ” and “ $\eta\pi$ ” contributions. The “ $\eta\pi$ ” events are defined as those with the invariant mass of any of the two $\pi^+\pi^-\pi^0$ combinations in the final state of $\pi^+\pi^+\pi^-\pi^0$, within the η mass range of $[0.52, 0.58]$ GeV/c^2 , with all other events classified in the “non- η ” category. If there are multiple tag D_s^- candidates for each tag mode, then the one with M_{rec} closest to the known mass of $D_s^{\pm}$ [1] is retained. A DT candidate with average mass $(M_{\text{sig}} + M_{\text{tag}})/2$ closest to the known mass of D_s^+ [1] is retained for each tag mode. The ST yields (Y_{tag}) and DT yield (Y_{sig}) in data are determined from fits to the M_{tag} and M_{sig} distributions, respectively. The ST fit results are the same as Refs. [13,19]. The DT fits are shown in Fig. 2. The signal shape is modeled with the shape from MC simulation convolved with a Gaussian resolution function, and the background is estimated from the inclusive MC sample.

These fits result in a total ST yield of $Y_{\text{tag}} = 471\,617 \pm 1733$. For the “non- η ” part, the signal yield is $Y_{\text{sig,non-}\eta} = 2489 \pm 91$, and for the “ $\eta\pi$ ” part the signal yield is

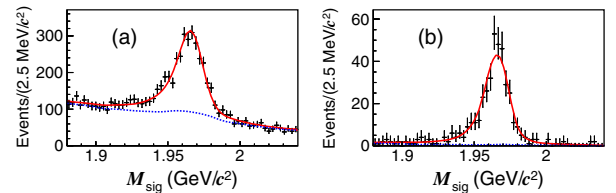


FIG. 2. Fits to the M_{sig} distributions of the DT candidates for (a) “non- η ” and (b) “ $\eta\pi$ ” contributions. The data with error bars represent data from all samples, while the red solid lines are the total fits to the data. The dashed blue lines indicate the fitted background shapes.

$Y_{\eta\pi^+} = 392 \pm 22$. An updated inclusive MC sample based on our amplitude analysis results is used to determine the ST efficiencies (ϵ_{ST}^i) and DT efficiencies (ϵ_{DT}^i). Substituting these results into $\mathcal{B}(D_s^+ \rightarrow \pi^+\pi^+\pi^-\pi^0|_{\text{non-}\eta}) = Y_{\text{sig,non-}\eta}/(\mathcal{B}(\pi^0 \rightarrow \gamma\gamma) \times \sum_{i,\alpha} Y_{\text{tag}}^{i,\alpha}/\epsilon_{\text{ST}}^{i,\alpha})$ and $\mathcal{B}(D_s^+ \rightarrow \eta(\rightarrow \pi^+\pi^-\pi^0)\pi^+) = Y_{\text{sig,}\eta\pi^+}/[\mathcal{B}(\pi^0 \rightarrow \gamma\gamma) \times \sum_{i,\alpha} Y_{\text{tag}}^{i,\alpha}/\epsilon_{\text{ST}}^{i,\alpha}]$, where i denotes the i th tag mode and α denotes the α th center-of-mass energy point, we obtain $\mathcal{B}(D_s^+ \rightarrow \pi^+\pi^+\pi^-\pi^0|_{\text{non-}\eta}) = (2.04 \pm 0.08)\%$ and $\mathcal{B}[D_s^+ \rightarrow \eta(\rightarrow \pi^+\pi^-\pi^0)\pi^+] = (3.58 \pm 0.21) \times 10^{-3}$, where the uncertainties are statistical only.

The systematic uncertainties for the BF measurement are categorized in five sources: (a) uncertainty from the number of ST D_s^- mesons, estimated by considering the statistical effect related to the ST background; (b) the DT background shape, estimated by changing to alternative background shapes; (c) the $\pi^\pm$ tracking (particle identification) efficiency and π^0 reconstruction, estimated by studying related control samples of $D_s^+ \rightarrow K^+K^-K^+K^-$ and $D_s^+ \rightarrow K^+K^-K^+K^-\pi^0$ decays; (d) MC sample size and model, estimated by studying the change in result when varying the signal-model parameters; and (e) the knowledge of the BFs of $\mathcal{B}(\pi^0 \rightarrow \gamma\gamma)$ and $\mathcal{B}(\eta \rightarrow \pi^+\pi^-\pi^0)$ [1]. Adding all sources of uncertainties in quadrature gives a total of 2.4% systematic uncertainty for $\mathcal{B}(D_s^+ \rightarrow \pi^+\pi^+\pi^-\pi^0|_{\text{non-}\eta})$, and 1.6% for $\mathcal{B}(D_s^+ \rightarrow \eta(\rightarrow \pi^+\pi^-\pi^0)\pi^+)$.

In summary, we measure the absolute BFs $\mathcal{B}(D_s^+ \rightarrow \pi^+\pi^+\pi^-\pi^0|_{\text{non-}\eta}) = (2.04 \pm 0.08_{\text{stat}} \pm 0.05_{\text{syst}})\%$ for the first time, and $\mathcal{B}[D_s^+ \rightarrow \eta(\rightarrow \pi^+\pi^-\pi^0)\pi^+] = (3.58 \pm 0.21_{\text{stat}} \pm 0.06_{\text{syst}}) \times 10^{-3}$. Utilizing $\mathcal{B}(\eta \rightarrow \pi^+\pi^-\pi^0)$ quoted from the PDG [1], the BF $\mathcal{B}(D_s^+ \rightarrow \eta\pi^+)$ is determined to be $(1.56 \pm 0.09_{\text{stat}} \pm 0.04_{\text{syst}})\%$. Moreover, we perform the first amplitude analysis of $D_s^+ \rightarrow \pi^+\pi^+\pi^-\pi^0|_{\text{non-}\eta}$ and report the observation of $D_s^+ \rightarrow f_0(980)\rho^+$. The phases and FFs of the significant intermediate processes are summarized in Table II. The BFs for the intermediate processes are calculated as $B_n = \text{FF}_n \times \mathcal{B}(D_s^+ \rightarrow \pi^+\pi^+\pi^-\pi^0|_{\text{non-}\eta})$, which are summarized in Table II. The $D_s^+ \rightarrow f_0(1370)(\rightarrow \pi^+\pi^-) \times \rho^+(\rightarrow \pi^+\pi^0)$ and $D_s^+ \rightarrow \phi(\rightarrow \pi^+\pi^-\pi^0)\pi^+$ contributions dominate with BFs of $(5.08 \pm 0.80_{\text{stat}} \pm 0.43_{\text{syst}}) \times 10^{-3}$ and $(5.08 \pm 0.32_{\text{stat}} \pm 0.10_{\text{syst}}) \times 10^{-3}$, respectively. The BF of $D_s^+ \rightarrow f_0(980)(\rightarrow \pi^+\pi^-)\rho^+(\rightarrow \pi^+\pi^0)$ is found to be $(2.57 \pm 0.44_{\text{stat}} \pm 0.20_{\text{syst}}) \times 10^{-3}$, which is valuable input for improving understanding of the nature of the $f_0(980)$ meson. Taking the BF of $D_s^+ \rightarrow \phi(\rightarrow K^+K^-)\pi^+$ from Ref. [19], enables the relative BF between ϕ decays into $\pi^+\pi^-\pi^0$ and K^+K^- to be calculated. The result of $R_\phi = \{\mathcal{B}[\phi(1020) \rightarrow \pi^+\pi^-\pi^0]/\mathcal{B}[\phi(1020) \rightarrow K^+K^-]\} = 0.230 \pm 0.014_{\text{stat}} \pm 0.010_{\text{syst}}$, significantly deviates from the PDG value $R_\phi^{\text{PDG}} = \{\mathcal{B}[\phi(1020) \rightarrow \pi^+\pi^-\pi^0]/\mathcal{B}[\phi(1020) \rightarrow K^+K^-]\} = 0.313 \pm 0.010$ by more than 4σ [1]. Further

checks on the $\phi\pi^+$ BF can be accessed through Ref. [54]. The data in the PDG on the Okubo-Zweig-Iizuka suppressed $\phi \rightarrow \rho\pi$ decays are mostly from e^+e^- collisions on the ϕ peak [1]. The possible interference effect between $e^+e^- \rightarrow \gamma^* \rightarrow \rho\pi$ and $e^+e^- \rightarrow \phi \rightarrow \rho\pi$ may not be well considered, and therefore, BESIII D_s decay provides another test of this ratio, in which none such interference arises. This is the first measurement of R_ϕ in hadronic decays of charmed mesons, and the lower than expected value motivates further studies. The BF of the W -annihilation decay $D_s^+ \rightarrow \omega(\rightarrow \pi^+\pi^-\pi^0)\pi^+$ is determined to be $(1.41 \pm 0.17_{\text{stat}} \pm 0.06_{\text{syst}}) \times 10^{-3}$. This result is a factor 2 more precise than previous measurements, and obtained in a manner that takes full account of interference with other intermediate processes decaying into the same final state. The significantly improved precision will benefit investigations of the underlying dynamics for nonperturbative W -annihilation amplitudes and allow for better predictions of the BFs and direct CP violation of decays involving W annihilation [26–29].

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