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The Quest for the Quark-Gluon Plasma From the Perspective of Dynamical Models of Relativistic Heavy-Ion Collisions

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ABSTRACT

The physics of heavy-ion collisions is one of the most exciting and challenging directions of science for the last four decades. On the theoretical side one deals with a non-abelian field theory, while on the experimental side today's largest accelerators are needed to enable these studies. The discovery of a new stage of matter—called the quark-gluon plasma (QGP) - and the study of its properties is one of the major achievements of modern physics. In this contribution we briefly review the history of theoretical descriptions of heavy-ion collisions based on dynamical models, focusing on the personal experiences in this inspiring field.

1 | Introduction to the History of Dynamical Modeling of Heavy-Ion Collisions

During the mid seventies and mid eighties of the last century, it was not a priori clear if high energetic heavy-ion reactions would allow to create super dense nuclear matter or if the reaction would become essentially transparent (McLerran 1982a, 1982b). Based on the parton picture (Andersson et al. 1983; Feynman et al. 1978) or Bjorken's estimates (Bjorken 1983) one assumed a large degree of transparency, while other groups using a hydrodynamic picture suggested the creation of shock waves that would lead to the creation of super dense matter (Baumgardt et al. 1975; Scheid et al. 1974). Later on, elaborate relativistic transport simulations became available but the controversy could not be settled based on theory alone. While parton-based approaches still favoured the idea of nuclear transparency, (see e.g., VENUS Werner (1989)), models based on the relativistic Boltzmann equation suggested a substantial stopping of the impinging nuclei, i.e., baryon number transport towards midrapidity even at high collision energies (Bleicher et al. 1996; von Keitz et al. 1991).

Nowadays, these questions have fortunately been resolved by experimental data. Luckily, the colliding nuclei are not transparent over a wide range of collision energies. In fact even at relativistic collision energies the impinging ions have been shown to loose a substantial amount of energy leading to the creation of not only a hot, but also a very baryon rich matter. Only this fact has allowed the field to thrive and to use heavy-ion collisions as a laboratory for the exploration of neutron star matter and the properties of binary neutron star mergers and the equation of state of compact stellar objects.

As a matter of fact, the dynamics of the early universe in terms of the 'Big Bang' may be experimentally studied by ultra-relativistic nucleus-nucleus collisions at the Large-Hadron-Collider (LHC) in terms of 'tiny bangs' at vanishing baryo-chemical potential. While at the GSI-Schwerionensynchrotron (SIS) or at the beam-energy scan with Relativistic Heavy-Ion Collider (RHIC's BES) program matter at moderate temperatures and high baryon densities can be explored and linked to the properties of stellar objects.

During the late 1980s to the mid 1990s both, hydrodynamic and transport theoretical descriptions of heavy-ion reactions where contenders for the best description of the Au + Au collision data taken at BNL's Alternating Gradient Synchrotron (AGS) ($\sqrt{s_{\rm NN}} \approx 3 - 5$ GeV) and for the upcoming data from CERN's Super-Proton Synchrotron (SPS) colliding light systems and later Pb nuclei at a center of mass energy of about $\sqrt{s_{\rm NN}} \approx 20$ GeV.

Unfortunately, at that time hydrodynamical or hydro-ispired models where not able to describe the upcoming data to sufficient detail (Mayer and Heinz 1997; Schlei 1997; Sollfrank et al. 1997) On the contrary, hadronic transport theory provided an unprecedented accuracy and detailed information of the collision systems evolution. The relativistic transport approaches could be grouped in two categories: (i) Relativistic Quantum Molecular Dynamics approaches, like RQMD, propagating the n-particle distribution (Aichelin and Stoecker 1986; Hartnack et al. 1989), and (ii) Relativistic Boltzmann(Valsov)-Ühling-Uhlenbeck (usually abbreviated as RBUU or RVUU) approaches (Uehling and Uhlenbeck 1933), propagating one-body distribution functions in averaged mean-fields (Aichelin and Bertsch 1985; Bertsch and Das Gupta 1988; Bertsch et al. 1984; Cassing et al. 1990).

During the early 1990s essentially a few active groups where spearheading the transport model era:

- The RQMD group in Frankfurt, using the 8-dimensional phase space approach developed by Heinz Sorge and the group of Horst Stöcker, who had merged the non-relativistic IQMD (Aichelin and Stoecker 1986; Hartnack et al. 1989) with the LUND string model and implemented a large body of baryonic and mesonic resonances.
- The RBUU group at Giessen, lead by Wolfgang Cassing and Ulrich Mosel. Later they extended the semi-classical BUU (Cassing et al. 1990) to covariant formulations and incorporated the LUND string model for the description of multi-particle interactions in the expanding hadronic phase, which became the basis for the Hadron-String-Dynamics (HSD) transport approach (Cassing and Bratkovskaya 1999; Ehehalt and Cassing 1996).
- The group by Che-Ming Ko developing the extended relativistic transport (ART—A Relativistic Transport) model (Li and Ko 1995) used for describing interactions among hadrons in the final hadronic phase.

2 | The Need for Partons and the Deconfinement Transition

We recall that in the beginning microscopic transport models have been developed to explore the dynamics of hadrons using a nuclear matter EoS (where nuclear matter EoS usually meant some type of Skyrme potential and electric potential) and cross sections based on measured experimental data or effective hadronic Lagrangians. Later on, the models started to include the excitation and decay of color strings (essentially either based on the LUND picture (momentum exchange) or the parton picture based the idea of color exchange). Such improvements became necessary with the increase in collision energy which becomes essential at AGS and SPS energies.

Already at that time, first studies on the production of partons had been performed within these hadron-string models. E.g. in Weber et al. (1998) it was shown that quark degrees of freedom—hidden inside strings—carry a substantial part of the energy density at collision energies above 30 A GeV. In Figure 1 we show the fraction of energy density in the quark state as a function of collision energy in central Pb + Pb collisions at mid-rapidity as calculated within the UrQMD model. Here quark degrees of freedom mean those quarks that are produced during the string fragmentation process. Let us point out as a side remark: A precursor to the inclusion of full parton dynamics in the early stage of the reaction was the development of string fusion models, also called color

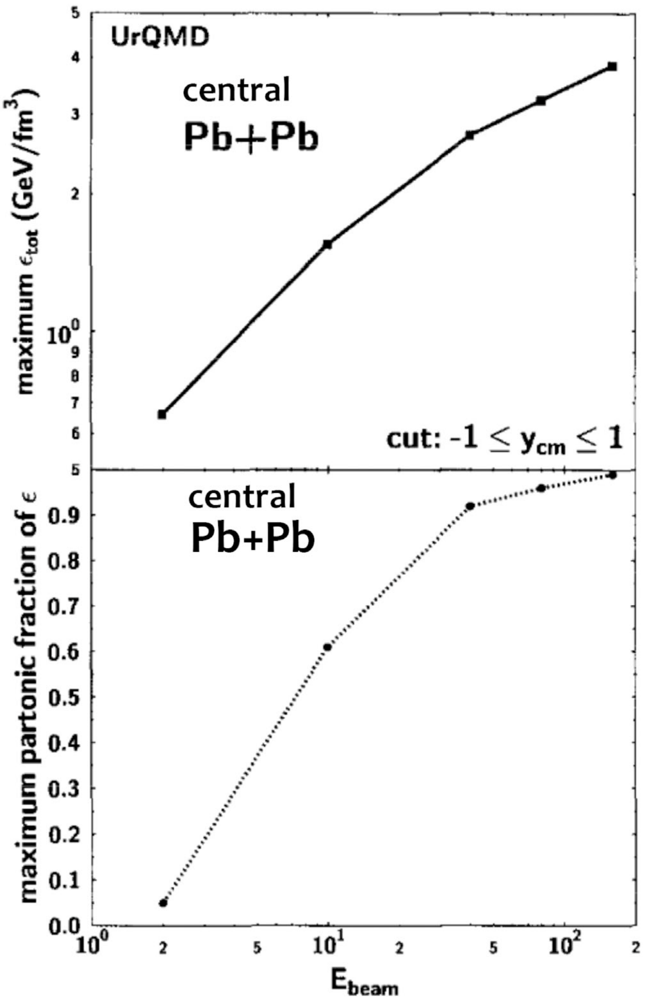

FIGURE 1 | Top: Maximum energy density achieved in central Pb + Pb collisions at mid-rapidity. Bottom: Fraction of energy density in the 'quark state' (inside strings) as a function of collision energy from UrQMD. The figures are adopted from Weber et al. (1998).

ropes. Here, one recombines color charges from different string ends to higher color charges, which in turn modify the fragmentation of the color field (prime examples are Amelin et al. (1993), Armesto et al. (1995), Merino et al. (1992), Sorge (1995)).

The observation of high parton densities in the UrQMD model (and also similar observations made in other models as discussed above) did already hint towards the need to implement partonic dynamics into transport simulations. However, it seemed that the influence of the partonic stage on observables in the SPS energy regime was rather moderate. Therefore, the immediate need to tackle this problem was not as pressing as it might look from todays perspective. This situation changed drastically when the first RHIC data emerged.

Among the first persons to implement an equation of state mimicking a phase transition between partonic and hadronic degrees of freedom could be implemented into transport simulation was Sorge (1999). In contrast to todays implementation of the parton-hadron transition into simulations using mostly the real part of the potential to modify the EoS (Omana Kuttan et al. 2022; Steinheimer et al. 2022), Sorge used the virial theorem and implemented the QCD-EoS in the collision term. To show the effects emerging from the EoS with a phase transition in comparison to the hadron gas EoS, the elliptic flow of hadrons was suggested. As a matter of fact, elliptic flow has now become a central observable to map out the properties of the produced QCD matter.

However, let us return to the timeline. When hadronic models where applied to heavy-ion collisions at RHIC energies a couple of problems emerged since a number of observables (elliptic flow of charged hadrons, transverse mass spectra of hadrons, intermediate mass dileptons etc.) could no longer be properly described by hadron-string degrees of freedom alone (Bratkovskaya, Bleicher, et al. 2004; Bratkovskaya, Soff, et al. 2004).

One of the observables which played an essential role in heavy-ion physics was the ratio of strange to non-strange hadrons. As has been proposed in 1982 by Rafelski and Muller (1982) the strangeness degree of freedom might play an important role in distinguishing hadronic and partonic dynamics. In 1999 Gaździcki and Gorenstein (Gazdzicki and Gorenstein 1999)—within the statistical model—have predicted experimental observables which should show an anomalous behaviour at the phase transition: the 'kink' – an enhancement of pion production in central Au + Au (Pb + Pb) collisions relative to scaled *pp* collisions; the 'horn' – a sharp maximum in the K^+/π^+ ratio at 20–30 A GeV; the 'step' – an approximately constant slope of $K^\pm$ spectra starting from 20 to 30 A GeV. Indeed, such "anomalies" have been observed experimentally by the NA49 Collaboration (Afanasiev et al. 2002; Friese et al. 2004).

On the theoretical side we have investigated the hadron production as well as transverse hadron spectra in nucleus-nucleus collisions from 2 A GeV to 21.3 A TeV within the independent transport approaches UrQMD and HSD (Bratkovskaya, Bleicher, et al. 2004; Bratkovskaya, Soff, et al. 2004; Weber et al. 2003). The comparison to experimental data demonstrates that both approaches agree quite well with each other and with the experimental data on hadron production. The enhancement of pion production in central Au + Au (Pb + Pb) collisions relative to scaled *pp* collisions (the 'kink') is well described by both approaches without involving any phase transition. However, the maximum in the K^+/π^+ ratio at 20–30 A GeV (the 'horn') is missed by ∼ 40% (Bratkovskaya, Bleicher, et al. 2004; Weber et al. 2003 – cf. Figure 2) (l.h.s.). A comparison to the transverse mass spectra from *pp* and C + C (or Si + Si) reactions shows the reliability of the transport models for light systems (Bratkovskaya, Soff, et al. 2004). For central Au + Au (Pb + Pb) collisions at bombarding energies above ∼ 5 A GeV, however, the measured $K^\pm$ m_T-spectra have a larger inverse slope parameter than expected from the calculations. The approximately constant slope of $K^\pm$ spectra at SPS (the 'step') is not reproduced either (Bratkovskaya, Bleicher, et al. 2004; Bratkovskaya, Soff, et al. 2004 – cf. Figure 2) (r.h.s.). The slope parameters from *pp* collisions (r.h.s. in Figure 2) are seen to increase smoothly with energy both in the experiment (full squares) and in the transport calculations (full lines with open circles) and are significantly

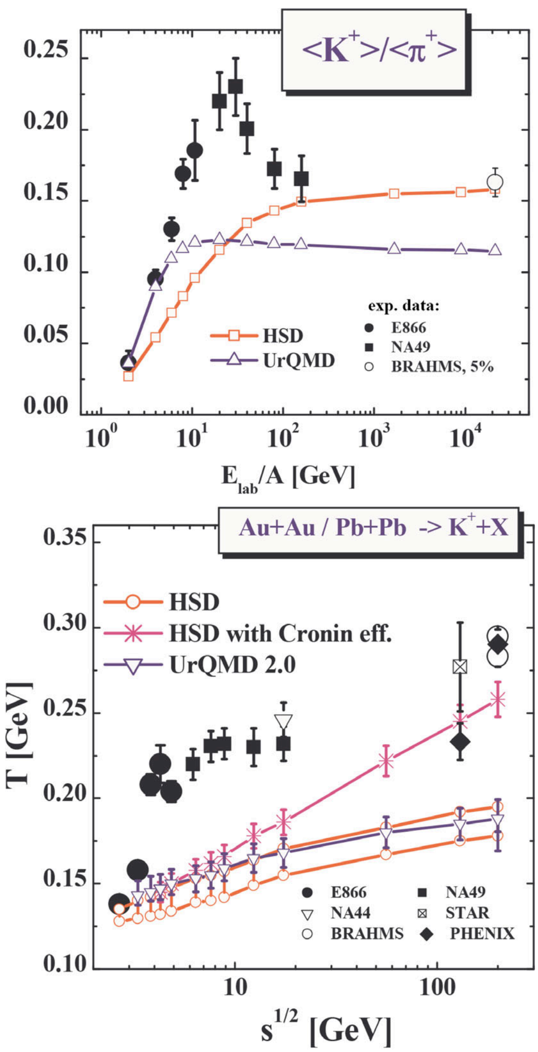

FIGURE 2 | Excitation function of the K^+/π^+ ratio (l.h.s.) and inverse slope parameter for K^+ (r.h.s.) from central Au + Au (AGS and RHIC) or Pb + Pb (SPS) collisions. The solid lines with open squares show the results from HSD whereas the dashed lines with open triangles indicate the UrQMD calculations. The solid lines with stars correspond to HSD calculations including 'Cronin' initial state enhancement. The figures are adopted from Bratkovskaya, Bleicher, et al. (2004).

lower than those from central Au + Au reactions for $\sqrt{s} > 3.5$ GeV. Thus, the pressure generated by hadronic interactions in the transport models above ~ 5 A GeV is lower than observed in the experimental data. This finding suggests that the additional pressure—as expected from lattice QCD calculations at finite quark chemical potential and temperature—might be generated by strong interactions in the early pre-hadronic/partonic phase of central Au + Au (Pb + Pb) collisions.

Another prime example showing the effect of the early partonic pressure was advanced by the competition between the groups performing hydrodynamic calculations (Huovinen et al. 2001; Kolb et al. 2001) and transport simulations (Bleicher and Stoecker 2002; Molnar and Gyulassy 2002) on the description of elliptic flow. The main result was that the description of the elliptic flow data needs an extremely low viscosity of the matter created at RHIC. This was first recognized in Bleicher and Stoecker (2002) and later confirmed in more detail in Molnar and Gyulassy (2002). Of course the ideal hydrodynamic simulation done at that time used $\eta/s = 0$ by definition, allowing them to obtain an excellent description of the flow data at RHIC.

Thus, the discrepancies between the experimental data and the results of existing (at those time) hadron-string based microscopic transport models stimulated an outstanding development of dynamical models. The need to reformulate the transport models for partonic degrees of freedom became imminent. This included purely perturbative QCD (pQCD) based models, like Boltzmann-type simulations employing perturbative QCD (pQCD) derived two- and three-particle cross sections: Especially the pioneering work by Geiger (realized in the VNI model Geiger and Muller (1992)) popularized this approach, later followed by Zhangs Parton Cascade Zhang (1998) and Molnar's Parton Cascade Molnar and Gyulassy (2000). The pinnacle of parton cascades was reached with the realization of BAMPS Z. Xu and Greiner (2005) that included three gluon interactions in a consistent fashion and was extensively used to study the approach to thermalization of super hot gluon matter. However, all of these approaches did only employ an ideal gas EoS for the partonic phase and could not describe any phase transition. Later on, some of these models where actually merged together to increase the region of applicability. Here the main representative of the modularized approaches is known as the "A Multi Phase Transport Model" (AMPT) (Lin et al. 2005), which merged Zhang's Parton Cascade and the ART transport model.

Unfortunately, pQCD scattering cross sections between massless partons turned out too low in order to describe the elliptic flow of hadrons measured experimentally (Molnar and Gyulassy 2002; Xu and Greiner 2009), either effective (enhanced) two-body cross sections have been used (Ko et al. 2014) or additional $2 \leftrightarrow 3$ channels needed to be added as in BAMPS Xu and Greiner (2005). The formation of hadrons (if implemented at all) was usually performed by coalescence either in momentum space or—more recently—in phase space (Ellis and Geiger 1995a, 1995b). Another branch of transport models is based on NJL-like approaches including a coupling to a scalar mean field and/or a vector mean field (Florkowski et al. 1996; Marty and Aichelin 2013; Ruggieri et al. 2013). In these models the partons have a finite dynamical mass and the binary cross sections are either extracted from the NJL Lagrangian (Marty and Aichelin 2013) or parameterized to simulate a finite η/s (as in hydro models) (Ruggieri et al. 2014). All these approaches provide a reasonable description of experimental data at RHIC energies as well as for LHC energies.

2.1 | Inclusion of a Partonic Phase in Transport Models: The PHSD Way

In order to achieve a fully microscopic description of the hadronic and partonic phase, the Parton-Hadron-String Dynamics (PHSD) transport approach has been developed (Cassing 2009; Cassing and Bratkovskaya 2008). The PHSD is a microscopic covariant off-shell dynamical model for strongly interacting systems formulated on the basis of Kadanoff-Baym equations (Cassing 2009) for Green's functions in phase-space representation (in first order gradient expansion beyond the quasiparticle approximation). The approach consistently provides the full evolution of a relativistic heavy-ion collision from the initial hard scatterings and string formation through the dynamical deconfinement phase transition to the strongly-interacting quark-gluon plasma (sQGP) as well as hadronization and the subsequent interactions in the expanding hadronic phase as in the Hadron-String Dynamics (HSD) transport approach (Cassing and Bratkovskaya 1999; Ehehalt and Cassing 1996).

The transport theoretical description of the QGP phase in the PHSD is based on the Dynamical Quasi-Particle Model (DQPM) (Berrehrah et al. 2014; Cassing 2007a, 2007b; Peshier and Cassing 2005) which is an effective model constructed to describe the strongly interacting non-perturbative nature of the QCD matter and to reproduce lattice-QCD results (Aoki et al. 2009; Borsanyi et al. 2015; Cheng et al. 2008) for a quark-gluon plasma in thermodynamic equilibrium. The degrees-of-freedom in the DQPM are strongly interacting massive quarks and gluons (contrary to the massless pQCD partons). They are described in terms of effective propagators with complex self-energies: the real part of self-energies corresponds to the effective finite masses (scalar mean-fields) for gluons/quarks while the imaginary part—for the finite widths related to the medium dependent reaction rate. For fixed thermodynamic temperature T and baryon chemical potential μ_B the partonic width's $\Gamma_i(T, \mu_B)$ fix the effective two-body interactions represented in terms of quasi-elastic and inelastic parton cross sections that are implemented in the PHSD (Ozvenchuk et al. 2013a).

The parton scattering cross sections are probed by transport coefficients (correlators) in thermodynamic equilibrium by performing the PHSD calculations in a finite box with periodic boundary conditions (shear- and bulk viscosity, electric conductivity, magnetic susceptibility etc. Cassing et al. (2013), Ozvenchuk et al. (2013b), Steinert and Cassing (2014)). The transport coefficients are of great importance for a 'viscous' hydrodynamical description of heavy-ion collisions since they are input parameters which define the 'deviation' of the QGP or hadronic 'fluid' from an ideal fluid (Huovinen et al. 2001; Luzum and Romatschke 2008; Romatschke and Romatschke 2007; Song and Heinz 2008).

We note that in the recent versions of the PHSD 5.0 (and extensions) the off-shell partonic interaction cross sections have been evaluated based on the leading order scattering diagrams and depend on T, μ_B, the invariant energy of the colliding partons $\sqrt{s}$ as well as the scattering angle (Moreau et al. 2019). In Refs. Fotakis et al. (2021), Moreau et al. (2019), Soloveva et al. (2022), Soloveva et al. (2020) the transport coefficients (such as specific shear and bulk viscosities to entropy ratios $\eta/s, \zeta/s$, electric conductivities σ_{QQ}/T, diffusion coefficients etc.) have been explore in the T, μ_B plane based on relaxation-time approach and Kubo formalism and a good agreement with available lattice-QCD data has been found.

The PHSD successfully describes the many observables from SIS to LHC energies for the $p + A$ and $A + A$ collisions (Bratkovskaya et al. 2011; Cassing 2009; Cassing and Bratkovskaya 2008, 2009; Linnyk et al. 2016; Moreau et al. 2019). For the theoretical foundation of the PHSD model we refer the reader to Ref. Cassing (2021).

In order to illustrate how the microscopic description of heavy-ion collisions proceeds within the PHSD, we show in Figure 3 the time evolution of central Au + Au collisions (upper row, section view) at a collisional energy of $\sqrt{s_{NN}} = 19.6$ GeV within the PHSD (Moreau 2019). The snapshots are taken at times $t = 0.005, 1, 2, 4$ and 8 fm/c. The baryons, antibaryons, mesons, quarks and gluons are shown as colored dots. The middle row of Figure 3 shows the local temperature T and the lower row displays the baryon chemical potential μ_B, as extracted from the PHSD in the region with $y \approx 0$. The black lines (middle row) indicate the critical temperature $T_c = 0.158$ GeV. As follows from the upper part of Figure 3, the QGP is created in the early phase of the collisions and when the system expands, a hadronization occurs. One can see that during the overlap phase the temperature T and chemical potential μ_B are very large and then decrease with time. However, even at 8 fm/c there are "hot spots" of QGP at the front surfaces of high rapidity.

2.2 | Inclusion of Partonic Phase in Transport Models: The UrQMD Hybrid Way

An alternative way to include a phase transition into a transport simulation is to couple a relativistic Boltzmann-equation to the hydrodynamic equations. By this, one is able to describe the initial non-equilibrium dynamics of the QCD matter and its thermalization, while this state then provides a source term for the hydrodynamic evolution in which a QCD equation of state with a (phase) transition between a hadron gas and a QGP can

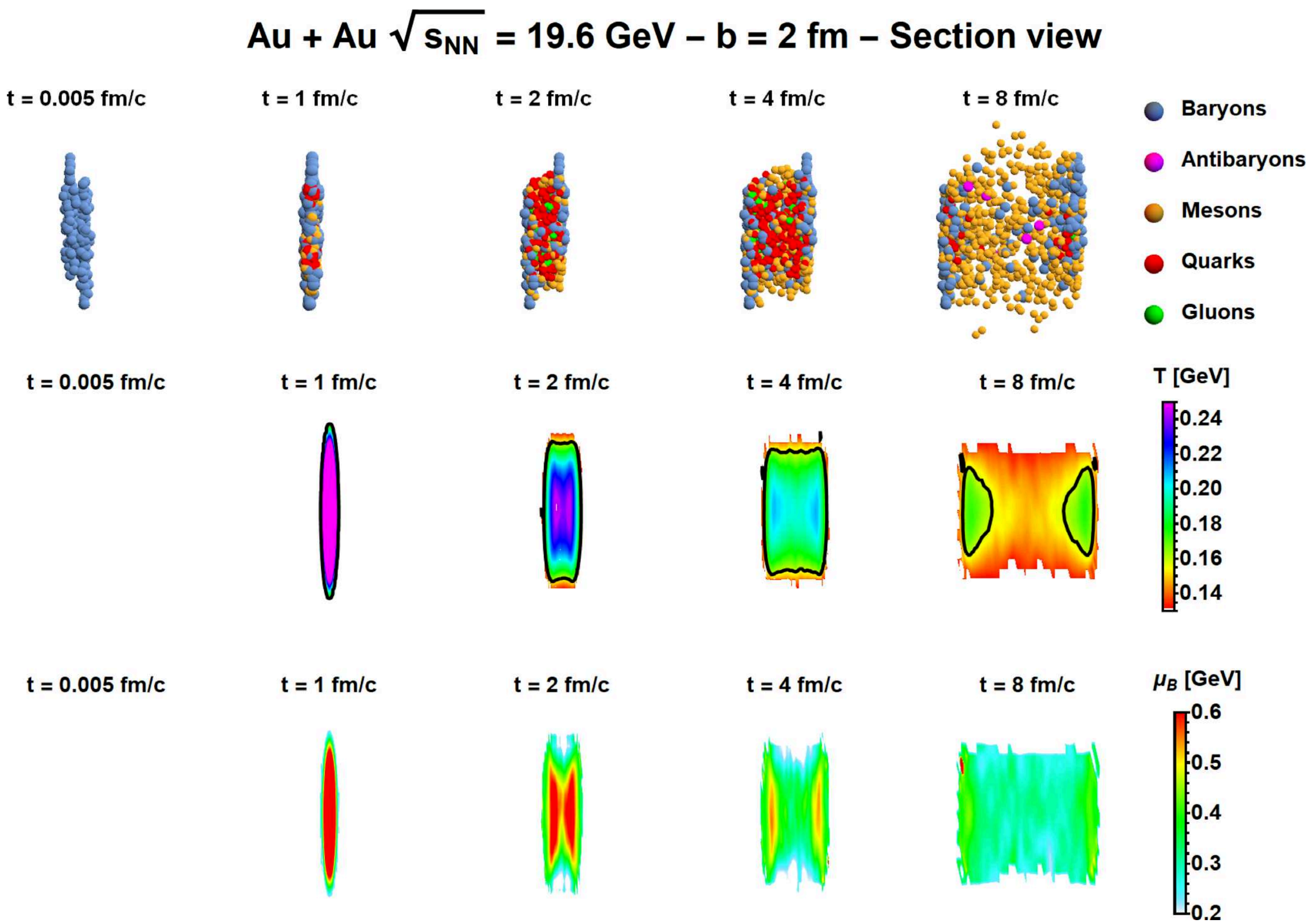

FIGURE 3 | Illustration of the time evolution of central Au + Au collisions (upper row, section view) at a collisional energy of $\sqrt{s_{NN}} = 19.6$ GeV within the PHSD (Moreau 2019). The local temperature T (middle row), baryon chemical potential μ_B (lower row), as extracted from the PHSD for $y \approx 0$. The black lines (middle row) indicate the critical temperature $T_c \simeq 0.158$ GeV. The figures are adopted from Moreau et al. (2021).

be included. After the phase transition, when local equilibrium cannot be maintained any longer, one uses a loss term in the hydrodynamic equations and transfers the matter back into a relativistic Boltzmann simulation. This allows to model the freeze-out/decoupling stage using resonant hadronic scattering from the loss of local equilibrium (chemical freeze-out) to kinetic freeze-out (Steinheimer et al. 2017). In this way one can merge the advantages of hydrodynamical and Boltzmann-type simulations. Physically, such an approach is possible due to the sufficiently strong parton interactions, leading to rather fast equilibration in the central collision zone and allowing to keep local equilibrium after some initial delay. It is further supported by the success of plain (ideal) hydrodynamical models (Heinz and Kolb 2002; Ollitrault 1992; Shuryak 2009) in description of the first experimental data on the elliptic flow $v_2(p_T)$ at RHIC.

This lead to the development of hybrid models which incorporate three different type of model components: (i) the initial nonequilibrium phase to specify the initial state fluctuations or initial flow, (ii) viscous or ideal hydrodynamic for the hadronic and partonic (fluid) phase including a possible phase transition, and (iii) hadronic 'afterburner' for resonant interactions in the hadronic phase to model the freeze-out after the loss of local equilibrium.

Due to the matching of the different phases a couple of new parameters enter such models that define the matching conditions. Accordingly, a multi-parameter approach (on the scale of ∼ 15 independent parameters) emerges that has to be optimized in comparison to a multitude of experimental data in order to extract physical information on the transport coefficients. This has been done within a Bayesian analysis by a couple of authors and some proper information could be extracted so far on $\eta/s(T)$ as well as for the charm diffusion coefficient $D_s(T)$ (Auvinen et al. 2018; Bass et al. 2017; Xu et al. 2018). For explicit results we refer the reader to Refs. Auvinen et al. (2018), Bass et al. (2017), Bernhard et al. (2016), Pratt et al. (2015), Pratt and Young (2017), Sangaline and Pratt (2016), Xu et al. (2018). We note in passing that within such approaches semi-central and central nucleus-nucleus collisions at ultra-relativistic energies can well be described (Song et al. 2014) but an application to elementary high-energy $p + p$ or $\pi + p$ reactions is restricted to very high energies and high multiplicity triggers (Werner et al. 2014).

A pioneering framework in this class of models is the ultra-relativistic quantum molecular dynamics (UrQMD) hybrid approach which starts with UrQMD (Bass et al. 1998; Bleicher et al. 1999) for the initial nonequilibrium phase on an event by event basis, switches to hydro after the baryon currents have separated from each other in phase space and approximate equilibration in the local cells is reached. The evolution continues with a hydrodynamic expansion until a critical energy density is reached in each cell (individually) and merges, after a Cooper-Fry particlization, into the UrQMD cascade to describe the final hadronic rescatterings (Petersen et al. 2008; Steinheimer et al. 2008). By construction such hybrid models may be used for lower (AGS) energies as well as for ultra-relativistic (LHC) energies. Further improvements by incorporating e.g., a color glass condensate (CGC), IP-glasma or EPOS2 initial conditions (Gelis and Schenke 2016; McDonald et al. 2019, 2021; Nahrgang et al. 2014; Schenke et al. 2014; Werner et al. 2012; Werner et al. 2010) provide also a very good description of the collective flows as a function of bombarding energy and collision centrality at RHIC and LHC energies.

2.3 | QGP in Theoretical Laboratories—UrQMD(−Hybrid) and PHSD

Looking in the past and having the present experience we believe that it was a fortunate decision by our transport groups to follow alternative ways in incorporating a quark-gluon plasma in our transport approaches UrQMD and PHSD:

i. To follow the fully microscopic description of QGP by the inclusion of partonic degrees of freedom and their interaction in the PHSD explicitly and

ii. To follow a hybrid description by switching from the microscopic hadronic phase to a macroscopic description of the QGP phase in terms of a propagating 'fluid'.

Each of these approaches has advantages and disadvantages. The basic differences are the following: the advantage of the PHSD is a fully microscopic description of all stages of heavy-ion collisions—partonic and hadronic—with full conservation of energy-momentum and all quantum numbers in each elementary collision, which allows to analyze the history of each individual parton or hadron created during the expansion of the system. In the UrQMD it is not possible to do so due to the switch to a 'fluid' description for the partonic phase by converting the energy density and flow velocity of individual cells to the hydro evolution, which leads to a discontinuity in entropy density etc. However, in a hydrodynamical description it is much more straightforward to incorporate and study different equations of state, e.g., a crossover lattice-EoS at zero μ_B or some chiral model EoS with a 1st order phase transition. Moreover, one avoids an explicit solution of the hadronization problem and replaces it by the 'particalarization' problem which is much easier in realization using e.g., the Cooper-Frye procedure. In the PHSD the incorporation of different EoS requires first it's interpretation in terms of degrees of freedom and their interactions. Since lattice-QCD cannot provide this information and delivers only averaged thermodynamic quantities, one needs to develop some effective models—as done by the PHSD group when the DQPM model has been introduced. Moreover, since the QGP is a strongly interacting matter, one cannot apply semi-classical BUU-type of kinetic approaches which are valid for the description of systems where the mean-free pass is larger then the range of interactions. The description of strongly interacting quasi-particles with dynamical broad spectral functions and complex self-energies required a substantial step in the development of the kinetic transport approach based on a field-theoretical description within Kadanoff-Baym theory and a derivation of new transport equations—i.e. the Cassing-Juchem equations for the test particles in first-order gradient expansion (Cassing 2009; Juchem et al. 2004)—to be solved numerically.

Finally, two comprehensive transport approaches—the UrQMD(−hybrid) and PHSD—have been developed and they became real theoretical 'laboratories' to study the dynamics of heavy-ion collisons on computers.

3 | Success of Transport Models With the QGP

In this section we recall a few most prominent examples how the explicit inclusion of the QGP dynamics—on a fully microscopic level in transport approaches or via the hydrodynamic phase in the hybrid approach—substantially improves the description of experimental observables at relativistic energies. For a more detailed comparison we refer the reader to the review Bleicher and Bratkovskaya (2022).

It is important to stress that the influence of the QGP increases with increasing bombarding energy. Although QGP droplets can be created already at relatively low collision energies of 3 – 5 GeV due to local fluctuations in energy density, their size is tiny and their traces in observables are hardly visible since the dynamics is dominated by hadronic interactions. Oppositely, with increasing collision energy of heavy-ions the QGP fraction grows such that partonic interactions become dominant for about 10 fm/c in Au + Au collisions and have visible consequences on observables.

In Figure 4 we show the QGP energy fraction versus the total energy for Au + Au at different collisional energies $\sqrt{s_{NN}}$ accounting only for the midrapidity region $|y| < 0.5$. One can see that for high energies the QGP fraction is large compared to lower collisional energies where the QGP volume is small. While in high energy heavy-ion collisions the QGP phase appears suddenly after the initial primary NN collisions, at low energies its appearance is smoother since the passing time of the nuclei (inversely proportional to the Lorentz γ-factor of A + A collision) is much longer. Correspondingly, at low energies the QGP lifetime is large, however, the total QGP volume is very small; thus its influence on the dynamics is much reduced compared to high energy collisions where practically 90% of matter at midrapidity is in the QGP phase (at least for a short time). We note that the PHSD calculations for the partonic fraction in Figure 4 are qualitatively consistent with the early UrQMD estimates shown in Figure 1.

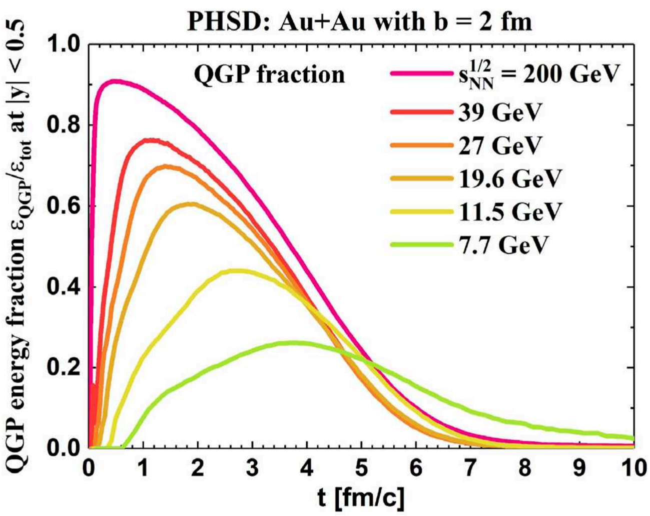

FIGURE 4 | The QGP energy fraction from PHSD as a function of time t in central (impact parameter b = 2 fm) Au + Au collisions for different collisional energies $\sqrt{s_{NN}}$, taking into account only the midrapidity region $|y| < 0.5$. The figures are adopted from Moreau et al. (2021).

We recall that, one of the main problems with the hadron-string dynamical description of heavy-ion collisions is related to a 'missing pressure' in transverse direction reflected in such observables as p_T- (or m_T-) spectra and elliptic flow v_2. This has been illustrated in Figure 2 (r.h.s.) which shows a substantial deviation of the inverse slope parameter T of the transverse spectra in comparison to experimental data. Since in heavy-ion collisions a pressure generation can occur only by interactions, this clearly demonstrates that hadronic interactions are not sufficient to push the matter in transverse direction, i.e., partonic interactions are needed. Now we step on to the same observables but include a QGP phase in the microscopic transport approach PHSD.

In Figure 5 we show the π^-, K^+ and K^- transverse mass spectra for central Pb + Pb collisions at 40, 80 and 158 A GeV from PHSD (thick solid lines) in comparison to the distributions from HSD (thin solid lines) and the experimental data from the NA49 Collaboration (Afanasiev et al. 2002; Alt et al. 2008). One can see that the calculations in the PHSD mode (i.e., including the formation of the sQGP) provide a better description of the experimental data compared to the HSD one (i.e., in the string-hadron mode).

Now we continue with the next example for partonic interactions in the QGP phase that play an important role in the building of pressure in the system. This can be probed by the elliptic flow coefficient v_2 which is a widely used quantity characterizing the azimuthal anisotropy of emitted particles,

$$v_2 =< cos(2\psi - 2\Psi) >=< \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} >, \quad (1)$$

where Ψ_{RP} is the azimuth of the reaction plane, p_x and p_y are the x and y component of the particle momenta and the brackets denote averaging over particles and events. The azimuthal anisotropies have been studied within the PHSD in Ref. Konchakovski et al. (2012) and here we show one of the prominent examples from this study. In Figure 6 we show the average elliptic flow v_2 of charged particles at mid-pseudorapidity for two centrality selections calculated for two cases: (i) the PHSD model (solid curves), i.e. including the QGP formation and partonic interactions in terms of the DQPM, and (ii) the HSD model (dashed lines) – without formation of the QGP, i.e., within the hadron-string dynamics. One can see that the pure hadronic scenario is not able to describe the experimental data for the excitation function of v_2. The errorbars indicate the statistical fluctuations in the PHSD results since the calculation of v_2 is a computer time consuming task, because the signal is of the order of few percent only.

Similar finding has been done with the UrQMD model (Petersen et al. 2015: in Figure 7) we show the transverse momentum dependence of the elliptic flow v_2 of pions from Au + Au collisions at mid-rapidity for two beam energies (40 and 160 A GeV) from UrQMD. Full and dashed lines show the results of the hybrid simulations, which include the QGP phase in EoS, while the dotted lines show the result of the standard hadronic cascade mode (i.e., without the QGP interactions).

We note that the deviation of hadron-string models (HSD and UrQMD) from the data grows with the energy, which clearly indicates that the partonic interactions (as implemented in a

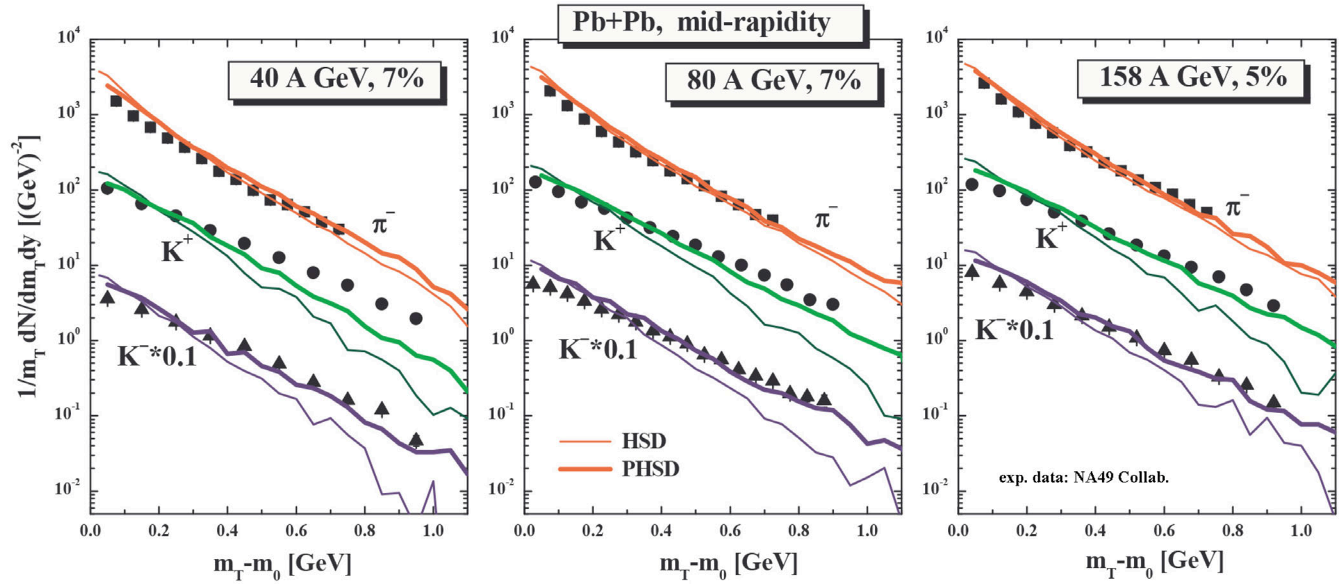

FIGURE 5 | The π^-, K^+ and K^- transverse mass spectra for central Pb + Pb collisions at 40, 80 and 158 A GeV from PHSD (thick solid lines) in comparison to the distributions from HSD (thin solid lines) and the experimental data from the NA49 Collaboration Afanasiev et al. (2002); Alt et al. (2008). The figures are adopted from Cassing and Bratkovskaya (2009).

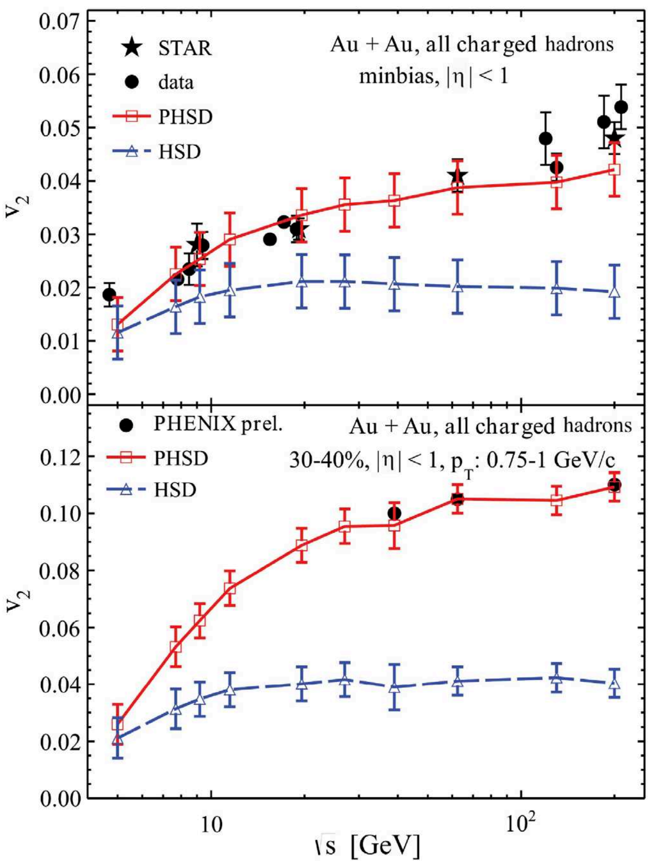

FIGURE 6 | Average elliptic flow v_2 of charged particles at mid-pseudorapidity for two centrality selections calculated within the PHSD (solid curves) and HSD (dashed lines) models. The experimental data for minimal bias are from the STAR Collaboration (stars) Nasim et al. (2010), from the PHENIX Collaboration (full circles) Gong (2011) and other data are taken from the compilation in Ref. Abelev et al. (2010). The figures are adopted from Konchakovski et al. (2012).

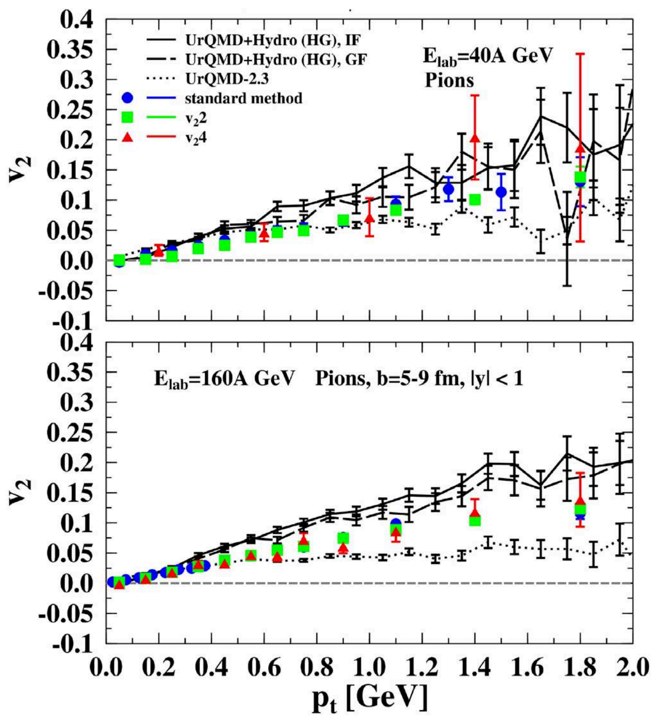

FIGURE 7 | Transverse momentum dependence of the elliptic flow v_2 of pions from Au + Au collisions at mid-rapidity for two beam energies (40 and 160 A GeV) from UrQMD. Full and dashed lines show the results of the hybrid simulation, while the dotted lines show the result of the standard hadronic cascade mode. The data by NA49 is shown by symbols. The figures are adopted from Petersen et al. (2015).

microscopic way in the PHSD or in the hybrid UrQMD) are getting more and more important with increasing energy. Indeed, the volume of the QGP grows with increasing energy and the

QGP phase becomes dominant at top RHIC energies as illustrated in Figure 4.

Figures 5 and 6 provide an example for the importance of the partonic degrees of freedom in the dynamical description of the heavy-ion collisions. We note that the incorporation of the QGP in the PHSD left the K^+/π^+ ratio puzzle unsolved (Cassing et al. 2016)—the PHSD was still missing the 'horn' structure at ~ 20 AGeV. Indeed, the QGP volume is rather low at this energies—cf. Figure 4 and cannot substantially change the 'chemistry' of the reactions, i.e., enhance the strangeness production. On the other hand, the hadron density in the early 'cold' stage of heavy-ion collisions—when the hadron production occurs via string formation and decay—is very high. Thus, one should expect such phenomena as a signal of partial chiral symmetry restoration (CSR) on the quark level inside strings. In Refs. Cassing et al. (2016), Palmese et al. (2016) the description of chiral symmetry restoration via the Schwinger mechanism for the string decay in a dense medium has been incorporated in the PHSD. This leads to a dropping of the 'dressed' quark masses—coupled to the scalar quark condensate which changes the final chemistry of produced hadrons during the string breaking. In the transport model the scalar quark condensate can be estimated via the scalar density of baryons and mesons based on the non-linear σ-ω-model (Friman et al. 1998).

Figure 8 shows the ratios K^+/π^+ and $(\Lambda + \Sigma^0)/\pi$ at midrapidity from 5% central Au + Au collisions as a function of the invariant energy $\sqrt{s_{NN}}$ up to the top SPS energy in comparison to the experimental data. The grey shaded area represents the results from PHSD including chiral symmetry restoration taking into account the uncertainty from the parameters of the $\sigma - \omega$-model for the nuclear EoS. As compared to the blue dashed lines, which show the 'old' HSD results without CSR and QGP (as shown in the left part of Figure 2), one sees a clear enhancement of the ratios due to the enhanced strangeness production in the initial phase of the collisions. Thus, the inclusion of chiral symmetry restoration together with a partonic phase allows to describe the maximum in the K^+/π^+ ratio as an interplay between the dense hadronic medium and the QGP transition.

In Figure 9 we show the energy dependence of Ξ^-/π ratios in Au + Au/Pb + Pb collisions. Black lines show results of the hybrid model with phase transition, the gray line shows the UrQMD result without phase transition.

For a variety of further observables, which signal the formation of the QGP, we refer the reader to the recent review (Bleicher and Bratkovskaya 2022).

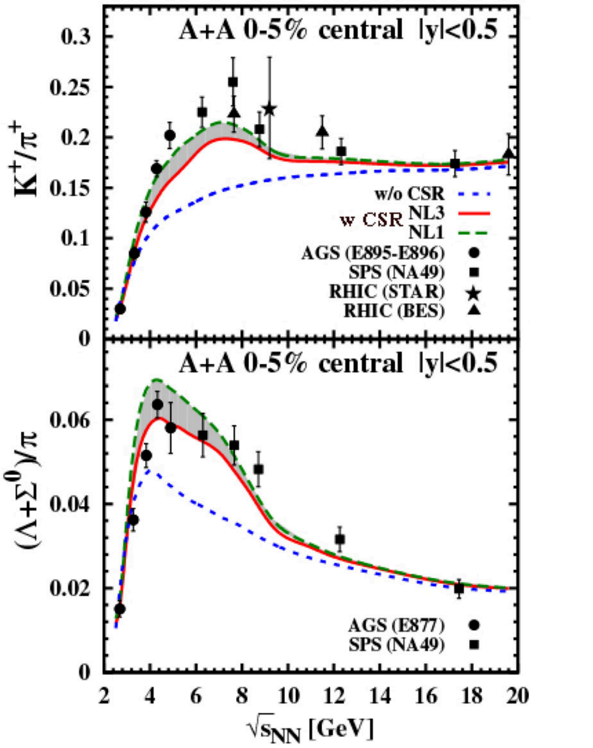

FIGURE 8 | The ratios K^+/π^+ and $(\Lambda + \Sigma^0)/\pi$ at midrapidity from 5% central Au + Au collisions as a function of the invariant energy $\sqrt{s_{NN}}$ up to the top SPS energy in comparison to the experimental data. The grey shaded area represents the results from PHSD including chiral symmetry restoration (CSR) taking into account the uncertainty from the parameters of the $\sigma - \omega$-model for the nuclear EoS. The figure is adopted from Ref. Palmese et al. (2016).

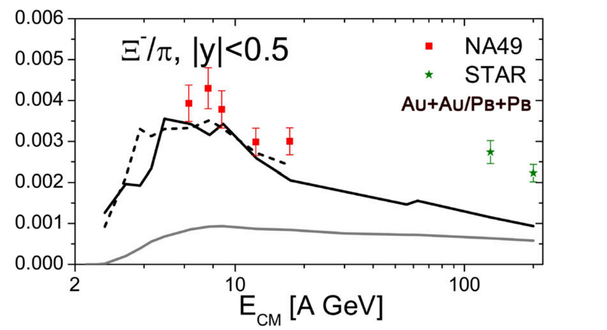

FIGURE 9 | Energy dependence of Ξ^-/π ratios in Au + Au/Pb + Pb collisions. Black lines show results of the hybrid model with a phase transition, the gray line shows the UrQMD result without phase transition. The figure is adopted from Ref. Steinheimer et al. (2010).

4 | Final Remarks

In this review we have recalled the historical progress in the dynamical modeling of heavy-ion collisions which rapidly developed on the boarder of the new millennium and was strongly driven by experimental observations. Since the QGP cannot be observed directly in experiments, a proper theoretical interpretation of the experimental measurements is mandatory. This can be done in a consistent way only by transport approaches—derived from ab initio kinetic and many-body theories and providing a full description of the time evolution of heavy-ion collisions, following all stages of the expanding system. An important step in our understanding of experimentally measured results has been achieved due to the theoretical development of transport approaches by inclusion of the quark-gluon plasma phase: (i) on a fully microscopic level by means of partonic degrees of freedom and their explicit interactions as in the PHSD or (ii) on a macroscopic level by means of a hydrodynamical description of the QGP fluid based on a partonic equation-of state for the QGP phase.

We have demonstrated for some prominent examples that the partonic phase is mandatory for a proper description of the experimental observables on particle yields, ratios, spectra as well as on the flow harmonics characterizing the dynamical expansion during the relativistic heavy-ion collisions. Furthermore, we have shown that the strangeness degrees of freedom show an exceptional sensitivity to the dynamical description of the heavy-ion collisions.

Our historical examples indicate the importance of comprehensive efforts of theory and experiments in obtaining progress in our understanding of many physical phenomena happening in nature.

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Conflicts of Interest

The authors declare no conflicts of interest.

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