

Precision Measurement of the Branching Fraction of $D^+ \rightarrow \mu^+ \nu_\mu$

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 (Received 9 October 2024; revised 17 April 2025; accepted 30 June 2025; published 4 August 2025)

Using 20.3 fb^{-1} of e^+e^- collision data collected at a center-of-mass energy of $E_{\text{c.m.}} = 3.773 \text{ GeV}$ with the BESIII detector operating at the BEPCII collider, we determine the branching fraction of the leptonic decay $D^+ \rightarrow \mu^+ \nu_\mu$ to be $(4.034 \pm 0.080_{\text{stat}} \pm 0.040_{\text{syst}}) \times 10^{-4}$. Interpreting our measurement with knowledge of the Fermi coupling constant G_F , the masses of the D^+ and μ^+ as well as the lifetime of the D^+ , we determine $f_{D^+}|V_{cd}| = (48.02 \pm 0.48_{\text{stat}} \pm 0.24_{\text{syst}} \pm 0.12_{\text{input}} \pm 0.15_{\text{EM}}) \text{ MeV}$ after taking into account necessary radiative corrections. This result is a factor of 2.3 more precise than the previous best measurement. Using the value of the magnitude of the $c \rightarrow d$ Cabibbo-Kobayashi-Maskawa matrix element $|V_{cd}|$ given by the global standard model fit, we obtain the D^+ decay constant $f_{D^+} = (213.5 \pm 2.1_{\text{stat}} \pm 1.1_{\text{syst}} \pm 0.8_{\text{input}} \pm 0.7_{\text{EM}}) \text{ MeV}$. Alternatively, using the value of f_{D^+} from a precise lattice quantum chromodynamics calculation, we extract $|V_{cd}| = 0.2265 \pm 0.0023_{\text{stat}} \pm 0.0011_{\text{syst}} \pm 0.0009_{\text{input}} \pm 0.0007_{\text{EM}}$.

DOI: [10.1103/gb8v-4rnh](https://doi.org/10.1103/gb8v-4rnh)

The leptonic decays of charmed mesons offer an important testbed to access the quark mixing-matrix elements and test lepton flavor universality (LFU). In the standard model (SM) of particle physics, the fully radiative inclusive decay rate of $D^+ \rightarrow \ell^+ \nu_\ell$ ($\ell = e, \mu$, or τ) can be written as [1]

$$\Gamma_{D^+ \rightarrow \ell^+ \nu_\ell} = \Gamma_{D^+ \rightarrow \ell^+ \nu_\ell}^{(0)} \left[1 + \frac{\alpha}{\pi} C_p \right] = \frac{G_F^2 f_{D^+}^2 m_{D^+}^3}{8\pi} |V_{cd}|^2 \mu_\ell^2 (1 - \mu_\ell^2)^2 \left[1 + \frac{\alpha}{\pi} C_p \right], \quad (1)$$

where G_F is the Fermi coupling constant, f_{D^+} is the D^+ decay constant, $|V_{cd}|$ is the magnitude of the $c \rightarrow d$ Cabibbo-Kobayashi-Maskawa (CKM) matrix element, μ_ℓ is the ratio of the ℓ^+ lepton mass to the D^+ meson mass (m_{D^+}), and $[1 + (\alpha/\pi)C_p]$ represents the radiative correction term [2]. Previous measurements of $D^+ \rightarrow \mu^+ \nu_\mu$ have been performed by MARKIII [3], BES [4], BESII [5], CLEO [6–8], and BESIII [9–11] but with limited precision. In contrast, the value of f_{D^+} calculated by lattice quantum chromodynamics (LQCD) [12–18] has reached a precision of 0.3%. Precise measurements of f_{D^+} and $|V_{cd}|$ are key to testing the LQCD calculations of f_{D^+} and the CKM matrix unitarity at high precision.

By inputting $m_\mu = 105.6583755 \text{ MeV}/c^2$, $m_\tau = 1776.93 \text{ MeV}/c^2$, and $m_{D^+} = 1869.66 \text{ MeV}/c^2$ [2] into $\Gamma_{D^+ \rightarrow \ell^+ \nu_\ell}^{(0)}$, the ratio of the branching fraction (BF) of $D^+ \rightarrow \tau^+ \nu_\tau$ to that of $D^+ \rightarrow \mu^+ \nu_\mu$ is expected to be $R_{\tau/\mu} = 2.66 \pm 0.01$. Current experimental measurement [10] is consistent with the SM prediction within the experimental uncertainty. However, there have been reports indication of potential LFU violation in semileptonic decays of B mesons from the BABAR, LHCb, and Belle [19–23] experiments. Potential violation in (semi)leptonic D decays may occur due to the interference between different amplitudes [24] or the interactions with scalar operators [25]. Precision measurements of the BF of $D^+ \rightarrow \ell^+ \nu_\ell$ offer an important test of LFU in the charm sector.

This Letter reports a precise measurement of the BF of $D^+ \rightarrow \mu^+ \nu_\mu$ obtained from the analysis of 20.3 fb^{-1} of e^+e^- collision data collected in 2010, 2011, 2022, 2023, and 2024 with the BESIII detector at the center-of-mass energy of $E_{\text{c.m.}} = 3.773 \text{ GeV}$ [26]. Charge-conjugate modes are always implied throughout this Letter unless stated specifically. The achieved precision is improved by a factor of 2.3 compared to the previous best measurement [9], which used 2.93 fb^{-1} of data taken in 2010 and 2011.

The BESIII detector is a magnetic spectrometer [27] located at the Beijing Electron Positron Collider [28]. The cylindrical core of the BESIII detector consists of a helium-based multilayer drift chamber (MDC), a plastic scintillator time-of-flight system (TOF), and a CsI (Tl) electromagnetic calorimeter (EMC), which are all enclosed in a superconducting solenoidal magnet providing a 1.0 T magnetic field. The solenoid is supported by an octagonal flux-return

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yoke with resistive plate counter modules for muon identification interleaved with steel. More details of the design and performance of the BESIII detector can be found in Ref. [27]. For 86% of the data used in this Letter, the end-cap TOF was upgraded with multigap resistive plate chambers with a time resolution of 60 ps [29,30]. Simulated data samples produced with a Geant4-based [31] Monte Carlo (MC) package, which includes the geometric description of the BESIII detector and the detector response, are used to determine detection efficiencies and to estimate backgrounds. The beam-energy spread and initial-state radiation (ISR) in the e^+e^- annihilations are simulated with the generator KKMC [32]. The inclusive MC sample includes the production of $D\bar{D}$ pairs (including quantum coherence for the neutral D channels), the non- $D\bar{D}$ decays of the $\psi(3770)$, the ISR production of the J/ψ and $\psi(3686)$ states, and the continuum processes incorporated in KKMC [32,33]. All particle decays are modeled with EvtGen [34,35] using BF's either taken from the Particle Data Group [2], when available, or otherwise estimated with LUNDCHARM [36,37]. Final-state radiation from charged final-state particles is incorporated using the PHOTOS package [38]. The leptonic decay $D^+ \rightarrow \mu^+\nu_\mu$ is simulated with the PHOTOS SLN model [34,35].

At $E_{\text{c.m.}} = 3.773$ GeV, the D^+D^- meson pairs are produced from $\psi(3770)$ decays without accompanying hadrons. This favorable environment provides an ideal opportunity to study leptonic D^+ decays with the double-tag (DT) method [39,40]. Initially, single-tag (ST) D^- mesons are reconstructed via the eight hadronic decay modes $K^+\pi^-\pi^-$, $K_S^0\pi^-\pi^-$, $K^+\pi^-\pi^-\pi^0$, $K_S^0\pi^-\pi^0$, $K_S^0\pi^+\pi^-\pi^-$, $K^+K^-\pi^-$, $\pi^+\pi^-\pi^-$, and $K^+\pi^-\pi^-\pi^+\pi^+$. Then the $D^+ \rightarrow \mu^+\nu_\mu$ candidates are selected by using the remaining tracks which have not been used in the selection of the tag side. The event, in which the $D^+ \rightarrow \mu^+\nu_\mu$ signal and the ST D^- are simultaneously reconstructed, is called a DT event. The BF of the $D^+ \rightarrow \mu^+\nu_\mu$ decay is determined by

$$\mathcal{B}_{D^+ \rightarrow \mu^+\nu_\mu} = \frac{N_{\text{DT}}}{N_{\text{ST}}^{\text{tot}} \cdot \bar{\epsilon}_{\text{sig}}}, \quad (2)$$

where $N_{\text{ST}}^{\text{tot}}$ is the total yield of ST D^- mesons, N_{DT} is the DT signal yield; and $\bar{\epsilon}_{\text{sig}}$ is the average signal efficiency weighted by the ST yields of the i th tag mode in data,

$$\bar{\epsilon}_{\text{sig}} = \frac{\sum_i (N_{\text{ST}}^i \cdot \epsilon_{\text{sig}}^i)}{N_{\text{ST}}^{\text{tot}}} = \frac{\sum_i (N_{\text{ST}}^i \cdot \epsilon_{\text{DT}}^i / \epsilon_{\text{ST}}^i)}{N_{\text{ST}}^{\text{tot}}}, \quad (3)$$

where N_{ST}^i is the number of ST D^- mesons for the i th tag mode in data, ϵ_{sig}^i is the signal efficiency of the i th tag mode, ϵ_{ST}^i is the efficiency of reconstructing the ST mode i (called the ST efficiency), and ϵ_{DT}^i is the efficiency of finding the tag mode i and the $D^+ \rightarrow \mu^+\nu_\mu$ decay simultaneously (called the DT efficiency).

For charged tracks, not originating from K_S^0 decays, the polar angles with respect to the MDC z axis (θ) are required to satisfy $|\cos \theta| < 0.93$. In addition, the distance of the closest approach to the interaction point (IP) must be less than 1 cm in the transverse plane, $|V_{xy}|$ and less than 10 cm along the z axis, $|V_z|$. The particle identification (PID) for charged particles combines measurements of the energy deposition in the MDC and the time of flight in the TOF to form likelihoods $\mathcal{L}(h)(h = K, \pi)$ for each hadron h hypothesis. The charged particles are assigned a particle type based on the hypothesis with the higher likelihood.

Each K_S^0 candidate is reconstructed from two oppositely charged tracks satisfying $|V_z| < 20$ cm. The two charged tracks are assigned to be $\pi^+\pi^-$ without requiring any further PID criteria. They are constrained to originate from a common vertex, which is required to be displaced from the IP by a flight distance of at least twice the vertex resolution. The χ^2 of the vertex fits (primary vertex fit and second vertex fit), where the number of degrees of freedom is 3, is required to be less than 100. This requirement retains 99% of signals. The invariant mass of the $\pi^+\pi^-$ pair is required to be within $(0.487, 0.511)$ GeV/ c^2 [41] corresponding to $\pm 4\sigma$ around the K_S^0 nominal mass.

The π^0 candidates are reconstructed via the dominant decay $\pi^0 \rightarrow \gamma\gamma$. Candidates with both photons detected in the end-cap EMC are rejected because of poor resolution. The photon candidates are identified using isolated showers in the EMC. The EMC time deviation from the event start time is required to be within $[0, 700]$ ns. The selection retains 99% of reconstructed signal photons and removes 75% of background energy depositions in the EMC. The energy deposition in the EMC is required to be greater than 25 MeV in the barrel region ($|\cos \theta| < 0.80$) and 50 MeV in the end-cap region ($0.86 < |\cos \theta| < 0.92$). To exclude showers that originate from charged tracks, the angle subtended by the EMC shower and the position of the closest charged track at the EMC must be greater than 10° . The π^0 candidates are required to have the invariant mass of the $\gamma\gamma$ lying within $(0.115, 0.150)$ GeV/ c^2 , corresponding to $\pm 3\sigma$ around the π^0 nominal mass. A mass-constrained (1C) fit, where the number of degrees of freedom is 1, to the nominal π^0 mass [2] is imposed on the photon pair, to improve the momentum resolution. The χ^2 of the 1C kinematic fit is required to be less than 50. The four momentum of the π^0 candidate updated by this kinematic fit is retained for the subsequent analysis.

To separate the ST D^- mesons from the combinatorial background, we define the energy difference $\Delta E \equiv E_{D^-} - E_{\text{beam}}$ and the beam-constrained mass $M_{\text{BC}} \equiv \sqrt{E_{\text{beam}}^2 - |\vec{p}_{D^-}|^2}$, where E_{beam} is the beam energy, and E_{D^-} and $\vec{p}_{D^-}$ are the total energy and momentum of the ST D^- meson in the e^+e^- center-of-mass frame. If there is more than one D^- candidate in a given ST mode, the one with the smallest $|\Delta E|$ value is kept for the subsequent

TABLE I. Requirements of ΔE , ST D^- yields in data, ST efficiencies (ϵ_{ST}^i), and DT efficiencies (ϵ_{DT}^i). The numbers in parentheses are the last two significant digits of the statistical uncertainties. The $\epsilon_{\text{ST}}^i/\epsilon_{\text{DT}}^i$ varies within 8% for different tag modes, which are mainly caused by the significantly different signal environments for some tag modes containing low momentum photon and pions in the signal and inclusive MC samples.

Tag mode	ΔE (MeV)	$N_{\text{ST}}^i (\times 10^3)$	ϵ_{ST}^i (%)	ϵ_{DT}^i (%)
$K^+\pi^-\pi^-$	$[-25, 24]$	5449.1(25)	50.1	35.81(11)
$K_S^0\pi^-$	$[-25, 26]$	656.7(08)	50.6	36.25(11)
$K^+\pi^-\pi^-\pi^0$	$[-57, 46]$	1644.4(18)	23.3	18.24(09)
$K_S^0\pi^-\pi^0$	$[-62, 49]$	1384.8(15)	25.4	19.33(09)
$K_S^0\pi^-\pi^-\pi^+$	$[-28, 27]$	771.6(11)	28.9	21.37(09)
$K^+K^-\pi^-$	$[-24, 23]$	472.6(09)	40.0	28.88(10)
$\pi^+\pi^-\pi^-$	$[-30, 29]$	204.7(08)	52.4	37.77(11)
$K^+\pi^-\pi^-\pi^+\pi^+$	$[-29, 27]$	215.1(07)	22.5	16.69(08)

analysis. The ΔE requirements and ST efficiencies are summarized in Table I.

For each tag mode, the yield of ST D^- mesons is extracted by fitting the corresponding M_{BC} distribution. In the fit, the signal shape is described as the sum of a simulated signal shape convolved with a double-normal distribution plus a single-normal distribution with free parameters. The double-normal and single-normal distributions account for different resolution and ISR effects between the data and MC simulation, respectively. The background shape is described by an ARGUS function [40], with the endpoint fixed at 1.8865 GeV/c^2 corresponding to E_{beam} . Figure 1 shows the results of the fits to the M_{BC} distributions of the accepted ST candidates for different tag modes in the data. The candidates with M_{BC} lying within $(1.865, 1.875) \text{ GeV}/c^2$ are retained. We veto $D^- \rightarrow K_S^0\pi^-$ in $D^- \rightarrow \pi^+\pi^-\pi^-$ by requiring $|m_{\pi^+\pi^-} - 0.4977| > 0.03 \text{ GeV}/c^2$. The contributions from the peaking backgrounds $D^- \rightarrow \pi^+\pi^-\pi^-$, $K_S^0e^-\bar{\nu}_e$, $K_S^0\mu^-\bar{\nu}_\mu$ in $D^- \rightarrow K_S^0\pi^-$, $D^- \rightarrow \pi^+\pi^+\pi^-\pi^-\pi^-$ in $D^- \rightarrow K_S^0\pi^-\pi^-\pi^+$, and $D^- \rightarrow K_S^0\pi^-$ in $D^- \rightarrow \pi^+\pi^-\pi^-$ are estimated by analyzing the inclusive MC sample and then are subtracted from the ST yields. These background fractions in the ST yields of $D^- \rightarrow K_S^0\pi^-$, $D^- \rightarrow K_S^0\pi^-\pi^-\pi^+$, and $D^- \rightarrow \pi^+\pi^-\pi^-$ are 0.2%, 0.1%, and 2.8%, respectively. Summing all tag modes, we obtain a total yield of ST D^- mesons of $(10789.0 \pm 3.9_{\text{stat}}) \times 10^3$.

The $D^+ \rightarrow \mu^+\nu_\mu$ candidates are selected in the presence of the ST D^- using the remaining neutral clusters and charged tracks. The muon candidate must have an opposite charge to the ST D^- meson and deposited energy within $(0.00, 0.35) \text{ GeV}$ in the EMC. To separate muons from hadrons, requirements based on the muon hit depth (d_{μ^+}) in the muon identifier modules are applied, taking into account the expected dependence on momentum (p_{μ^+}) and flight direction $\cos \theta$. These criteria are established from the

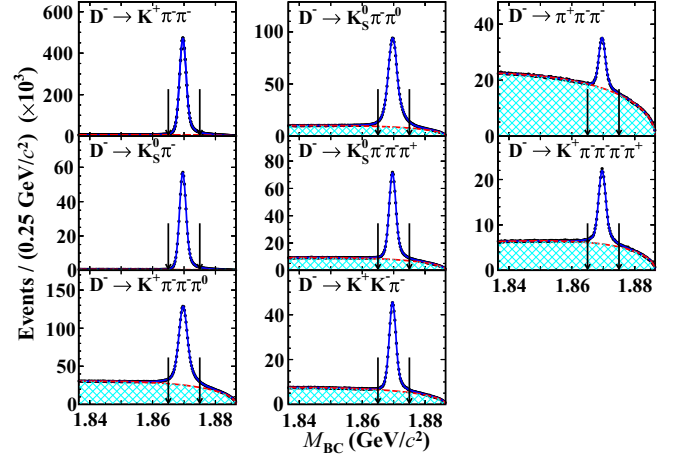


FIG. 1. Fits to the M_{BC} distributions of the ST D^- candidates. The dots with error bars are data. The blue solid curves are the fit results. The red dashed curves are the fitted combinatorial backgrounds. The pairs of arrows denote the M_{BC} signal regions. The cyan hatched histograms are background events from the inclusive MC sample.

distributions of d_{μ^+} versus p_{μ^+} using $e^+e^- \rightarrow (\gamma)\mu^+\mu^-$ candidates selected from data. The $|\cos \theta_{\mu^+}|$ and p_{μ^+} dependent requirements on d_{μ^+} follow those adopted in our previous measurements [42].

To suppress backgrounds with extra photon(s), the maximum energy of the unused showers in the DT selection ($E_{\text{max}}^{\text{extra}\gamma}$) is required to be less than 0.3 GeV . No additional charged track is allowed in the event. The yield of $D^+ \rightarrow \mu^+\nu_\mu$ is determined by fitting the distribution of the missing-mass squared of the undetected neutrino

$$M_{\text{miss}}^2 \equiv E_\nu^2 - |\vec{p}_\nu|^2. \quad (4)$$

Here $E_\nu \equiv E_{\text{c.m.}} - E_{D^-} - E_\mu$ and $\vec{p}_\nu \equiv -\vec{p}_{D^-} - \vec{p}_\mu$, where E_μ and $\vec{p}_\mu$ denote the energy and momentum of the muon, respectively.

The efficiencies of the DT reconstruction are determined with the signal MC samples, with D^- decaying to tag modes and D^+ decaying to the signal mode. Dividing these efficiencies by the ST efficiencies determined with the inclusive MC sample gives the corresponding efficiencies of the $\mu^+\nu_\mu$ reconstruction. The average efficiency over all tag modes is determined to be $\bar{\epsilon}_{\text{sig}} = (65.08 \pm 0.12)\%$. This efficiency has been corrected by a factor of

$$f_{\mu\text{PID}}^{\text{cor}} = (88.7 \pm 0.1)\%,$$

to account for the differences of μ^+ identification efficiencies between the data and MC simulation, mainly due to the imperfect simulation of the d_{μ^+} variable [43]. $f_{\mu\text{PID}}^{\text{cor}}$ is determined by using $e^+e^- \rightarrow \gamma\mu^+\mu^-$ events and reweighting by the μ^+ two-dimensional distribution in $|\cos \theta_{\mu^+}|$ and p_{μ^+} of $D^+ \rightarrow \mu^+\nu_\mu$ decays.

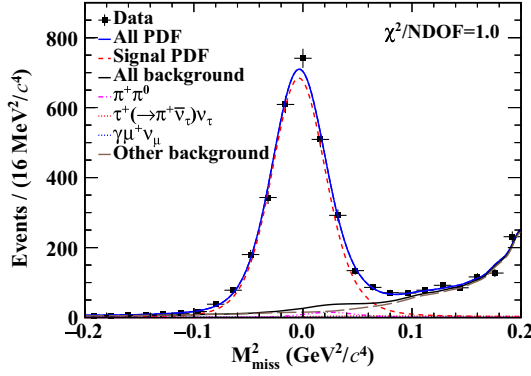


FIG. 2. Fit to the M_{miss}^2 distribution of the accepted candidates for $D^+ \rightarrow \mu^+ \nu_\mu$.

The background includes two components. One consists of events with wrongly tagged D^- decays (18.9%), and the other contains correctly tagged D^- decays but incorporating particle misidentifications, which is mainly from the decays of $D^+ \rightarrow \tau^+(\rightarrow \pi^+ \bar{\nu}_\tau) \nu_\tau$ (4.8%), $D^+ \rightarrow \pi^+ \pi^0$ (6.8%), and $D^+ \rightarrow \bar{K}^0 \pi^+$ (30.2%). These background fractions are counted over all backgrounds. Analysis of the inclusive MC sample shows that these two components make comparable contributions and the main peaking backgrounds in the resulting M_{miss}^2 distribution are $D^+ \rightarrow \tau^+(\rightarrow \pi^+ \bar{\nu}_\tau) \nu_\tau$ and $D^+ \rightarrow \pi^+ \pi^0$. Furthermore, the radiative decay $D^+ \rightarrow \gamma \mu^+ \nu_\mu$ from structure-dependent bremsstrahlung contribution, where the D^+ meson decays into a real photon and an off shell vector meson (the meson subsequently decays weakly to μ^+ and ν_μ), can also contribute a peaking structure in the resulting M_{miss}^2 distribution. The yield of $D^+ \rightarrow \gamma \mu^+ \nu_\mu$ is estimated to be 6.6 ± 4.6 with the BF of $D^+ \rightarrow \gamma e^+ \nu_e$, $(0.54 \pm 0.38) \times 10^{-5}$ [44].

To obtain the signal yield of $D^+ \rightarrow \mu^+ \nu_\mu$, we perform a fit to the M_{miss}^2 distribution of the $D^+ \rightarrow \mu^+ \nu_\mu$ candidates in data. In the fit, the signal shape is modeled by the MC simulated shape convolved with a normal distribution with free parameters. The shapes of the peaking backgrounds from $D^+ \rightarrow \tau^+(\rightarrow \pi^+ \bar{\nu}_\tau) \nu_\tau$, $D^+ \rightarrow \pi^+ \pi^0$, $D^+ \rightarrow \gamma \mu^+ \nu_\mu$ and the remaining background are modeled by individual MC simulated events. The corresponding probability density functions (PDFs) are derived from individual simulated shapes with the kernel estimation method [45]. The yields of $D^+ \rightarrow \tau^+(\rightarrow \pi^+ \bar{\nu}_\tau) \nu_\tau$ and $D^+ \rightarrow \pi^+ \pi^0$, corrected by the differences in misidentifying π^+ as μ^+ between the data and MC simulation, as well as $D^+ \rightarrow \gamma \mu^+ \nu_\mu$ are fixed in the fit, while the yields of the signal and the remaining background are floated. The fit result is shown in Fig. 2. The mean and resolution of the smeared normal function are $-2 \text{ MeV}^2/c^4$ and $11 \text{ MeV}^2/c^4$, respectively. From this fit, we obtain the signal yield of $D^+ \rightarrow \mu^+ \nu_\mu$ to be $N_{\text{DT}} = 2832.7 \pm 56.8$. Consequently, the BF of $D^+ \rightarrow \mu^+ \nu_\mu$ is found to be $\mathcal{B}_{D^+ \rightarrow \mu^+ \nu_\mu} = (4.034 \pm 0.080_{\text{stat}}) \times 10^{-4}$.

TABLE II. Systematic uncertainties in the BF measurement.

Source	Uncertainty (%)
M_{BC} fit	0.30
μ^+ tracking	0.06
μ^+ PID	0.10
$E_{\text{max}}^{\text{extra } \gamma}$ & $N_{\text{charge}}^{\text{extra}}$	0.08
M_{miss}^2 fit	0.90
Effect of $D^+ \rightarrow \gamma \mu^+ \nu_\mu$	0.17
MC statistics	0.16
Total	0.99

Table II summarizes the sources of systematic uncertainties in the measurement of the BF of $D^+ \rightarrow \mu^+ \nu_\mu$. The systematic uncertainty in the M_{BC} fit is estimated with alternative signal and background shapes. The alternative signal shapes are obtained by varying the parameters of the smeared normal distributions by $\pm 1\sigma$. The alternative background shape is obtained by varying the endpoint of the ARGUS function by $\pm 0.2 \text{ MeV}/c^2$. The relative difference of the ST yields between the data and the inclusive MC sample $\mathcal{R}(N_{\text{data}}/N_{\text{MC}})$ is assigned as the systematic uncertainty. Adding these systematic effects in quadrature gives a systematic uncertainty of 0.30% due to the M_{BC} fit.

The μ^+ tracking and PID efficiencies are studied with the control sample of $e^+ e^- \rightarrow \gamma \mu^+ \mu^-$ events. After correcting the signal efficiency by $f_{\mu \text{PID}}^{\text{cor}}$, we assign 0.06% and 0.10% as the uncertainties in the μ^+ tracking and PID, respectively.

The efficiency for the $E_{\text{max}}^{\text{extra } \gamma}$ requirement is studied with a control sample of DT hadronic events, i.e., events where both D^+ and D^- decay to one of the eight ST hadronic final states. The systematic uncertainty is taken to be 0.08% considering the efficiency differences between the data and MC simulation.

The systematic uncertainty associated with the signal shape in the M_{miss}^2 fit is estimated by using an alternative signal shape represented by a double normal distribution. The relative change of the signal yield, 0.89%, is assigned as the systematic uncertainty from this source. The systematic uncertainty due to the peaking background is estimated by varying the world average BFs of the two background components within $\pm 1\sigma$ [2]. The larger relative change of the fitted signal yield, 0.06% and 0.12%, is assigned as the systematic uncertainty associated with the estimated yields of $D^+ \rightarrow \tau^+ \nu_\tau$ and $D^+ \rightarrow \pi^+ \pi^0$, respectively. Additionally, the kernel bandwidth in the kernel density estimation of the combinatorial background PDF is varied through 0.5, 1.0, 1.5, and 2.0. The largest difference in the fitted signal yield, 0.10%, is taken as a systematic uncertainty. The total systematic uncertainty arising from the M_{miss}^2 fit is determined to be 0.90% by adding the four individual uncertainties in quadrature.

To consider the systematic uncertainty of the background of $D^+ \rightarrow \gamma\mu^+\nu_\mu$, we vary the fixed yield of $D^+ \rightarrow \gamma\mu^+\nu_\mu$ in the M_{miss}^2 fit within $\pm 1\sigma$. The larger change of the measured BF, 0.17%, is taken as the systematic uncertainty. The systematic uncertainty arising from the limited MC sample size, including both ST and DT MC samples, is 0.16%.

Assuming all components are independent, the relative total systematic uncertainty in the BF measurement is determined to be 0.99% by adding the contributions in quadrature. Accounting for this, the BF of $D^+ \rightarrow \mu^+\nu_\mu$ is measured to be $(4.034 \pm 0.080_{\text{stat}} \pm 0.040_{\text{syst}}) \times 10^{-4}$.

To determine $f_{D^+}|V_{cd}|$, radiative corrections for $\mathcal{B}(D^+ \rightarrow \mu^+\nu_\mu)$ are necessary as stated by Ref. [12]. The $D^+ \rightarrow \gamma\ell^+\nu_\ell$ decay rate, partially due to structure-dependent bremsstrahlung was predicted to be in a large range of $(0.46\text{--}8.2) \times 10^{-5}$ [46–52], but was recently determined to be $(0.54 \pm 0.38) \times 10^{-5}$ by BESIII [44]. The BESIII result gives its fraction of $(0.23 \pm 0.17)\%$, which has been fixed in the signal extraction. The known short-distance electroweak correction increases $\mathcal{B}(D^+ \rightarrow \mu^+\nu_\mu)$ by 1.8% [12,53], and the long-distance electroweak correction lowers $\mathcal{B}(D^+ \rightarrow \mu^+\nu_\mu)$ by 2.5% [54], with an uncertainty of 0.6% due to unknown electromagnetic corrections that depend upon the mesons' structure [12]. Combining our BF with the world average values of G_F , m_{μ^+} , m_{D^+} and the D^+ lifetime $\tau_{D^+} = (1.033 \pm 0.005) \times 10^{-12}$ [2] in $\Gamma_{D^+ \rightarrow \mu^+\nu_\mu}^{(0)}$ and incorporating the above corrections, we obtain

$$f_{D^+}|V_{cd}| = (48.02 \pm 0.48_{\text{stat}} \pm 0.24_{\text{syst}} \pm 0.12_{\text{input}} \pm 0.15_{\text{EM}}) \text{ MeV}.$$

Here the third (forth) uncertainty mainly arises from 0.2% (0.3%) uncertainty in τ_{D^+} (radiation corrections). Taking the magnitude of the $c \rightarrow d$ CKM matrix element $|V_{cd}| = 0.22487 \pm 0.00068$ from the global SM fit [2] we obtain

$$f_{D^+} = (213.5 \pm 2.1_{\text{stat}} \pm 1.1_{\text{syst}} \pm 0.8_{\text{input}} \pm 0.7_{\text{EM}}) \text{ MeV}.$$

Alternatively, taking the averaged decay constant $f_{D^+} = (212.0 \pm 0.7) \text{ MeV}$ from recent LQCD calculation [18] as input, we determine

$$|V_{cd}| = 0.2265 \pm 0.0023_{\text{stat}} \pm 0.0011_{\text{syst}} \pm 0.0009_{\text{input}} \pm 0.0007_{\text{EM}}.$$

Here, the uncertainties due to the input values of τ_{D^+} and $|V_{cd}|$ (f_{D^+}) are 0.2% and 0.3% (0.3%), respectively.

Using our measurement, the ratio of $\mathcal{B}_{D^+ \rightarrow \mu^+\nu_\mu}$ over the world average value of $\mathcal{B}_{D^+ \rightarrow \tau^+\nu_\tau} = (1.20 \pm 0.27) \times 10^{-3}$ [2] is determined to be $\mathcal{R}_{\tau/\mu} = 2.97 \pm 0.67$, which agrees with the SM prediction of 2.66 ± 0.01 from Eq. (1) within uncertainties.

Finally, we measure the separate BFs of $D^+ \rightarrow \mu^+\nu_\mu$ and $D^- \rightarrow \mu^-\bar{\nu}_\mu$ to be $(3.99 \pm 0.11_{\text{stat}} \pm 0.04_{\text{syst}}) \times 10^{-4}$ and $(4.07 \pm 0.11_{\text{stat}} \pm 0.04_{\text{syst}}) \times 10^{-4}$, respectively. From these we determine the BF asymmetry to be $A_{\text{CP}} = [(\mathcal{B}_{D^+ \rightarrow \mu^+\nu_\mu} - \mathcal{B}_{D^- \rightarrow \mu^-\bar{\nu}_\mu}) / (\mathcal{B}_{D^+ \rightarrow \mu^+\nu_\mu} + \mathcal{B}_{D^- \rightarrow \mu^-\bar{\nu}_\mu})] = (-1.0 \pm 2.0_{\text{stat}} \pm 0.9_{\text{syst}})\%$, where systematic uncertainties are assigned to account for the uncorrelated contributions between the charge-conjugated modes, arising from the $\mu^\pm$ tracking and PID, the ST yields, the limited MC sample sizes, and the M_{miss}^2 fits. The A_{CP} is compatible with the null hypothesis.

In summary, using the e^+e^- collision data sample corresponding to an integrated luminosity of 20.3 fb^{-1} collected at $E_{\text{c.m.}} = 3.773 \text{ GeV}$ with the BESIII detector, we report the most precise measurements of the BF of $D^+ \rightarrow \mu^+\nu_\mu$, the decay constant f_{D^+} , and the magnitude of the $c \rightarrow d$ CKM matrix element $|V_{cd}|$. The precision is improved by a factor of 2.3 compared to the previous best measurement [9]. In addition, we have searched for LFU and CP violation in $D^+ \rightarrow \ell^+\nu_\ell$ decays, yet no statistical significant violation has been observed.

Acknowledgments—The BESIII Collaboration thanks the staff of BEPCII and the IHEP computing center for their strong support. The authors appreciate helpful discussions with Prof. Maozhi Yang and Prof. Yuming Wang. This work is supported in part by the National Key R&D Program of China under Contracts No. 2023YFA1606000, National Natural Science Foundation of China (NSFC) under Contracts No. 11635010, No. 11735014, No. 11935015, No. 11935016, No. 11935018, No. 12025502, No. 12035009, No. 12035013, No. 12061131003, No. 12192260, No. 12192261, No. 12192262, No. 12192263, No. 12192264, No. 12192265, No. 12221005, No. 12225509, No. 12235017, No. 12361141819; the Chinese Academy of Sciences (CAS) Large-Scale Scientific Facility Program; the CAS Center for Excellence in Particle Physics (CCEPP); Joint Large-Scale Scientific Facility Funds of the NSFC and CAS under Contracts No. U2032104 and No. U1832207; the Excellent Youth Foundation of Henan Scientific Committee under Contract No. 242300421044; 100 Talents Program of CAS; The Institute of Nuclear and Particle Physics (INPAC) and Shanghai Key Laboratory for Particle Physics and Cosmology; German Research Foundation DFG under Contracts No. 455635585, No. FOR5327, No. GRK 2149; Istituto Nazionale di Fisica Nucleare, Italy; Ministry of Development of Turkey under Contract No. DPT2006K-120470; National Research Foundation of Korea under Contract No. NRF-2022R1A2C1092335; National Science and Technology fund of Mongolia; National Science Research and Innovation Fund (NSRF) via the Program Management Unit for Human Resources & Institutional Development,

Research and Innovation of Thailand under Contract No. B16F640076; Polish National Science Centre under Contract No. 2019/35/O/ST2/02907; The Swedish Research Council; U.S. Department of Energy under Contract No. DE-FG02-05ER41374.

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