EPJ Web of Conferences **337**, 01229 (2025) https://doi.org/10.1051/epjconf/202533701229
CHEP 2024

Application of Linear and Non-Linear Constraints in a Brute-Force-Based Alignment Approach for CBM

Nora Bluhme[1,2,*], *Sergey* Gorbunov[3], and *Volker* Lindenstruth[1,2,3]

[1]Frankfurt Institute for Advanced Studies, Frankfurt am Main, Germany
[2]Johann Wolfgang Goethe-Universität Frankfurt, Frankfurt am Main, Germany
[3]GSI Helmholtz Centre, Darmstadt, Germany

Abstract. The future Compressed Baryonic Matter experiment (CBM), which is currently being planned and will be realized at the Facility for Antiproton and Ion Research (FAIR), is dedicated to the investigation of heavy-ion collisions at high interaction rates. For this purpose, track-based software alignment is necessary to determine the precise detector component positions with sufficient accuracy. This information is crucial as it enables adequate utilization of the high intrinsic accuracy of the sensors.
The alignment parameters to be determined are typically translations and rotations of individual sensors in relation to their intended nominal positions. They are usually determined by minimizing a χ^2 function of a set of high-quality reconstructed tracks.
To complement the available alignment tools, an additional approach is being developed that is based on brute-force χ^2 minimization. This approach opens up the possibility of integrating different types of constraints into the minimization, such as inequality and non-linear constraints.
This contribution presents the concept of the brute-force alignment procedure. The implementation of constraints and the question of how the results of optical detector measurements, which usually precede software alignment, can be taken into account in this procedure is also addressed.

1 Introduction

The Compressed Baryonic Matter (CBM) experiment, currently under construction at the Facility for Antiproton and Ion Research (FAIR) in Darmstadt, Germany, is designed to explore the QCD phase diagram in regions of highest net baryon densities [1]. The Silicon Tracking System (STS) is the main tracking device within the CBM experiment. Consisting of eight layers (stations) of double-sided silicon microstrip sensors positioned between 30 and 100 cm downstream from the target within the magnetic dipole field, the STS is engineered to handle up to 1000 charged particles per interaction [2].

Operating at interaction rates of 10^5 to 10^7 collisions per second, the fixed-target experiment requires precise detector alignment to fully exploit the high intrinsic resolution of its sensors. Track-based software alignment aims to determine small corrections to the nominal positions and orientations of detector components. These alignment parameters $\vec{p}$ typically

*e-mail: bluhme@compeng.uni-frankfurt.de

consist of translations and rotations that describe the actual positions of individual sensors or sets of sensors relative to the ideal geometry. The parameters are typically determined by minimizing the track residual χ^2 for a set of high-quality reconstructed tracks [3]. This χ^2 is defined as the sum of squared residuals between actual track hit positions and the hit positions predicted by the track model, each weighted by the corresponding hit uncertainty. The function depends on both the alignment parameters $\vec{p}$ and the track parameters $\vec{t}$:

$$\min_{\vec{p}} \chi^2 = \min_{\vec{p}} \sum_{\text{tracks}} \sum_{\text{hits}} \frac{(\text{hit} - \text{hitmodel}(\vec{p}, \vec{t}))^2}{\sigma^2}$$

A particular challenge in detector alignment is the mitigation of weak modes - parameter changes that have minimal or no impact on the χ^2 function. These weak modes are introduced by unconstrained degrees of freedom and can lead to systematic distortions in the detector geometry. Common examples include overall translations or rotations of the entire detector system, as well as more complex deformations that introduce biases to the results of track reconstruction (e.g., scaling or shearing of detector volumes).

One desirable functionality for an alignment algorithm is the ability to integrate precise external measurement data into the alignment process in order to reduce these effects. Optical surveys provide such data, where for example commercial photogrammetry software delivers highly accurate global coordinates of marked reference points throughout the detector assembly. By taking these independently measured spatial constraints into account in the alignment, physically implausible solutions can be eliminated from the χ^2 minimization.

In existing analytical alignment methods like Millepede-II [4], degrees of freedom can be eliminated using linear equality constraints (e.g., through Lagrange multipliers) [5]. However, this approach presents limitations when dealing with survey measurements. The spatial information from these surveys inherently describes non-linear relationships between individual alignment parameters. Furthermore, since all physical measurements contain some degree of uncertainty, they cannot be strictly formulated as equality constraints. Instead, they require a more flexible constraint framework that can accommodate both their non-linear nature and inherent measurement errors.

To address these challenges, we have developed a complementary generic brute-force minimization approach that allows for a more flexible implementation of both linear and non-linear constraints. The initial tests of this approach, presented in this contribution, focus on the alignment of the CBM STS detector.

2 Concept of the Iterative Brute-Force Alignment Algorithm

2.1 Cost Function Minimization

The brute-force alignment approach follows an iterative trial-and-error strategy during which the parameter space is scanned in small steps and tested for the χ^2 cost function. As prerequisites, the χ^2 must be defined as a function of the alignment parameters $\vec{p}$. Additionally, validity intervals in the parameter space must be specified based on the expected mechanical precision of the setup. Minimum and maximum step sizes $\Delta\vec{p}_{\text{min}}$ and $\Delta\vec{p}_{\text{max}}$ are also required for all parameters, which influence the resolution of the results and the convergence speed of the procedure.

The core minimization procedure works by iterating over the parameters p_i and applying minimal changes Δp_i within the respective validity ranges (see Figure 1). At each step, the χ^2 function is evaluated to determine if the parameter changes improve the alignment. Changes in $\vec{p}$ are retained only if they result in a reduction of the χ^2 value, otherwise, they are discarded. This leads to convergence towards the minimum along the χ^2 gradient. Since evaluating the χ^2 function is computationally intensive, involving track refitting and residual calculations, this process is parallelized through multi-threading.

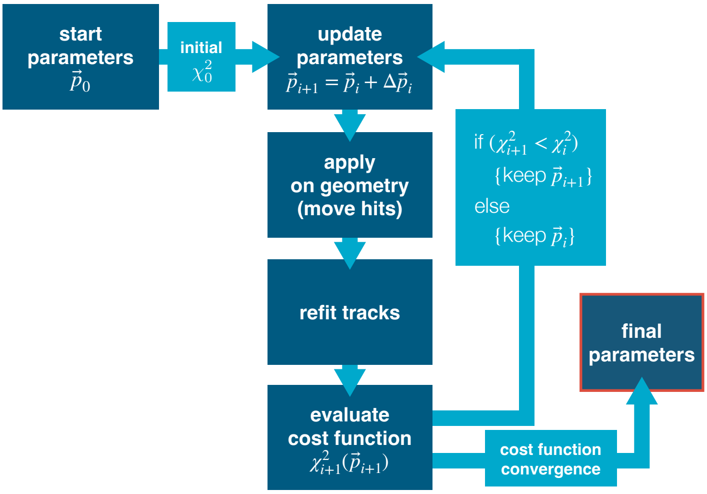

Figure 1. Concept of the iterative brute-force minimization procedure.

2.2 Linear Constraints

As mentioned previously, constraints play a critical role in the alignment process, limiting the multidimensional minimum of the χ^2 and ensuring physically meaningful results. Notably, the parameter boundaries required by the brute-force algorithm serve as an inherent constraint: each alignment parameter is confined to a validity range based on the known setup accuracy of the detector. This restriction prevents exploration of implausible regions of the parameter space and significantly limits weak modes such as global detector shifts or rotations.

To prevent global shifts of the entire detector setup even more, we implemented the most straight-forward linear equality constraint where the sum of all translations in a given dimension is forced to equal zero: If the constraint is applied, all corresponding parameters are jointly adjusted to maintain minimum distance to the respective axis while preserving their relative positions.

2.3 Integration of Survey Measurement as a Non-Linear Constraint

Given well defined visually accessible points x_i in individual detector components with precisely known positions in the respective local coordinate system $\vec{x}_{i_{\rm local}}$, their corresponding global coordinates $\vec{m}_{i_{\rm global}}$ within the entire detector geometry can be determined by optical surveys with high accuracy. This information can be used to extract more realistic start parameters for individual sensors or stations. To retrieve the start parameters from such data, a point cloud recognition algorithm was implemented, as proposed in [6]. Instead of fixing these parameters strictly to the resulting values, a more flexible point measurement constraint was formulated: The global coordinates of $\vec{x}_{i_{\text{local}}}$ depend on the alignment parameters of the respective detector station:

$$\vec{x}_{i_{\text{global}}}(\vec{x}_{i_{\text{local}}}, \vec{p}_{\text{station}})$$

This means, the coordinates are subject to change during the alignment process as the parameters of the station are changed for the scan of the parameter space ($\vec{p}_{i_{\text{station}}} + \Delta\vec{p}_{i_{\text{station}}}$). In order to restrict the optimization to parameter values in accordance with this constraint, in each iteration of the alignment algorithm $\Delta\vec{p}_{i_{\text{station}}}$ is only allowed when the following condition is satisfied:

$$|\vec{x}_{i_{\text{global}}} - \vec{m}_{i_{\text{global}}}| \leq 3.5\sigma$$

where σ is the measurement uncertainty of the optical survey. This flexible handling of the measurement data potentially allows the alignment algorithm to compensate for measurement errors if multiple of these measurement constraints are applied simultaneously.

3 Validation Studies and Initial Results

3.1 Proof of Concept

The brute-force alignment algorithm has been implemented as an external library for the CBMROOT analysis framework. As a proof of concept initial validation tests were performed using a simulation scenario of the STS. The test dataset consisted of ~4500 selected tracks from 100 UrQMD-generated Au-Au central collision events at 10 GeV. Track selection criteria were implemented to ensure reconstruction quality and reduce the impact of effects like multiple scattering: each track was required to have hits in all STS stations and a minimum momentum threshold of 1 GeV was applied.

To evaluate the algorithm's performance, we introduced randomized artificial misalignments in the range of ± 0.5 cm and ± 1.0 cm in the form of translational shifts to six of the eight STS stations. Rotations were not considered in this first validation test scenario. To constrain the system's degrees of freedom, three reference points were established: the positions of both the first and last stations were fixed, along with the x shift of the fifth station. This leaves 17 alignment parameters to be determined (five x shifts plus six y and z shifts). The misalignment was virtually applied on the hit positions of the reconstructed tracks. During each iteration of the alignment procedure, the correction parameters $\vec{p}$ were applied directly to the hit coordinates. The χ^2 evaluation was performed by refitting the tracks with the corrected hit positions and summing the track χ^2 over the entire track sample.

Figures 2 and 3 illustrate the effectiveness of the alignment procedure by comparing three residual distributions for the two misalignment scenarios: before alignment, after alignment, and a reference distribution from an ideally aligned STS detector. For both scenarios, x and y residual distributions are shown for one individual station (Station 2) as well as accumulated distributions from all stations. In both scenarios, the alignment procedure successfully restored the residual distributions to match the reference, bringing both the mean values and standard deviations in line with those of the perfectly aligned detector.

3.2 Point Constraint Test

The point constraint was tested in a simplified toy detector scenario, where the STS tracking stations are represented as 2D planes, and tracks as straight lines, taking no noise, multiple scattering or magnetic field into account. Hits are defined as the intersection points of tracks and stations. Every station is considered as one rigid body with 6 degrees of freedom: Three shifts (x, y, z) and three rotations (α, β, γ), which constitute the alignment parameters to be

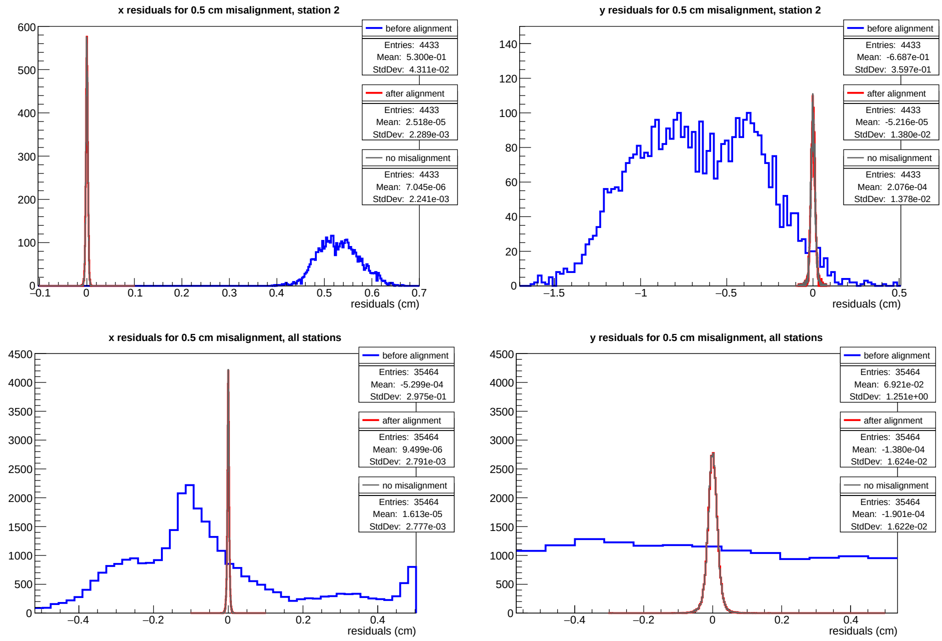

Figure 2. x and y residuals for 0.5 cm misalignment scenarios of the CBM STS. The residual distributions before alignment (blue) and after alignment (red) are compared to the reference distribution from an ideally aligned detector (gray). Upper row: results for Station 2, lower row: results for all stations.

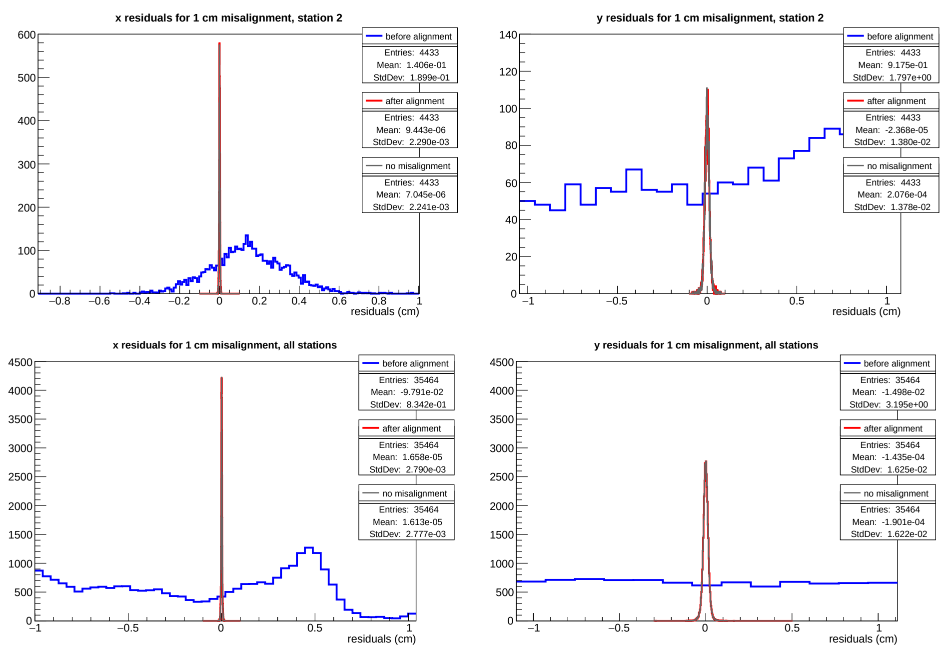

Figure 3. x and y residuals for 1.0 cm misalignment scenarios of the CBM STS. The residual distributions before alignment (blue) and after alignment (red) are compared to the reference distribution from an ideally aligned detector (gray). Upper row: results for Station 2, lower row: results for all stations.

Table 1. RMSD of Toy Detector Alignment Test with Point Constraint. Scenario 1: Two fixed stations, Scenario 2: One station with 3 point constraints. The RMSD of the individual alignment parameter types were calculated for both scenarios across 10 test runs with randomized misalignments (up to 0.5 cm for x, y, z, and 0.008 rad for α, β, γ). The values are given in cm for the shifts and rad for the rotations.

	$RMSD_x$	$RMSD_y$	$RMSD_z$	$RMSD_\alpha$	$RMSD_\beta$	$RMSD_\gamma$
Scenario 1	1.069e-04	9.671e-05	9.671e-05	1.292e-05	1.417e-05	4.061e-06
Scenario 2	4.139e-03	7.457e-03	2.213e-02	5.969e-04	6.707e-04	3.716e-04

determined. Two alignment scenarios were tested for comparison. In Scenario 1 two stations were fixed, which is sufficient to constrain all excess degrees of freedom. In Scenario 2, the last station was subjected to virtual measurements instead of being fixed: Three points were arbitrarily defined in the local coordinate system of the station. Using the known Monte Carlo misalignment of the simulation, the global coordinates of these points were determined and subjected to a random virtual measurement error in the range of ± 100 µm. The start parameters for the last station were calculated using the point cloud registration (Section 2.3). The parameters of the station were then constrained by the measurements as described in Section 2.3. For both scenarios the detector accuracy was set to 5 µm. 10 tests with randomized misalignments of up to 0.5 cm for the shifts and 0.008 rad for the rotations were conducted. For each test 200 tracks were generated and the alignment procedure was executed. The root mean square deviation (RMSD) of the individual alignment parameter types (x, y, z, α, β, γ) were calculated for both scenarios across all test runs and are summarized in Table 1. The RMSD values for the point constraint scenario are notably higher than those for the fixed station scenario, which is expected due to the additional uncertainty introduced by the virtual measurements. However, the results are still within an acceptable range with respect to the applied misalignment, indicating that the point constraint is a viable method for constraining the alignment parameters in the brute-force alignment approach.

4 Conclusion

The brute-force minimization approach for detector alignment presented in this work demonstrates promising capabilities for addressing complex alignment challenges in high-interaction rate experiments such as CBM. The iterative trial-and-error procedure, while conceptually straightforward, has proven effective in recovering the introduced misalignment. The residual distributions before and after alignment confirm that the procedure successfully restores detector geometry to closely match the undisturbed reference system. This was consistently demonstrated across different misalignment scenarios, indicating the robustness of the method.

Despite these successes, the method is still subject to a number of limitations that still need to be addressed. The current implementation is computationally intensive, particularly for complex detector systems with many degrees of freedom. While parallelization of the χ^2 evaluation helps mitigate this issue, further optimization is needed for practical application to the full CBM detector system. Additionally, the simplified test scenario, which focused on translational shifts and ignored effects like multiple scattering, does not yet fully represent the complexity of a real experimental setup. The performance of the algorithm under more realistic conditions remains to be thoroughly evaluated.

However, the successful integration of non-linear point measurement constraints in the simplified toy detector scenario demonstrates promising potential for broader applications.

This capability suggests that in the future the approach can incorporate optical survey measurement data, including their associated uncertainties, into more realistic simulations or real-data processing of the complete CBM system. This potentially represents a significant extension of the existing analytical alignment toolkit, particularly for handling complex spatial constraints that are challenging to implement in conventional approaches.

This work is supported by BMBF (05P21RFFC1).

References

[1] T. Ablyazimov et al., Challenges in QCD matter physics –The scientific programme of the Compressed Baryonic Matter experiment at FAIR. Eur. Phys. J. A **53**, 60 (2017). doi:10.1140/epja/i2017-12248-y

[2] J. Heuser, W. Müller, V. Pugatch, P. Senger, C. J. Schmidt, C. Sturm, U. Frankenfeld, *Technical Design Report for the CBM Silicon Tracking System (STS)* (GSI, Darmstadt, 2013)

[3] Volker Blobel, Software alignment for tracking detectors. Nucl. Instrum. Methods Phys. Res. A **566**, 5-13 (2006). doi:10.1016/j.nima.2006.05.157

[4] Millepede-II manual. https://www.desy.de/~kleinwrt/MP2/doc/html/index.html

[5] Volker Blobel, Alignment algorithms. Proceedings of the first LHC Detector Alignment-Workshop, 5-12 (2007). doi:10.5170/CERN-2007-004.5

[6] K. S. Arun, T. S. Huang, S. D. Blostein, Least-squares fitting of two 3-d point sets. IEEE Trans. Pattern Anal. Mach. Intell. **9(5)**, 698-700 (1987). doi:10.1109/tpami.1987.4767965