Confirmation and extension of a mini-valence Wigner-like energy in Sm, Gd, and Dy

R. B. Cakirli,[1,2,3] K. Blaum,[2] and R. F. Casten[4]

[1]*Department of Physics, Istanbul University, 34134 Turkey*
[2]*Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany*
[3]*ExtreMe Matter Institute EMMI, GSI Helmholtzzentrum für Schwerionenforschung GmbH, Planckstraße 1, 64291 Darmstadt, Germany*
[4]*Wright Lab, Yale University, New Haven, Connecticut 06520, USA*

(Received 17 October 2024; accepted 13 December 2024; published 3 January 2025; corrected 24 January 2025)

An earlier study revealed an unexpected enhancement in valence proton-neutron interaction strengths in certain heavy deformed even-even nuclei with equal numbers of valence protons and valence neutrons. The paper reports on an extension of those results to the entire span of deformed nuclei from Sm to W using mass data taken in the interim.

DOI: 10.1103/PhysRevC.111.014305

I. INTRODUCTION

In 2010, we published a paper [1], based on nuclear mass data then available [2], that suggested a special enhancement in valence proton-neutron interaction strengths in certain heavy nuclei. Maxima in *p*-*n* interaction strengths were found in nuclei with equal or nearly equal numbers of valence protons and neutrons in the deformed Er, Yb, Hf, and W nuclei. We called this a *mini-valence Wigner effect* in analogy with a well-known enhancement found in light nuclei with $N = Z$ [3–5]. The effect was hinted at in Sm, Gd, and Dy, but the mass data available at the time did not extend far enough to the neutron rich side to make a firm claim for these nuclides at $Z = 62, 64, 66$.

In the interim, many new nuclear masses have been measured. The point of this short paper is simply to update the earlier work and to show that the effect is now established from Sm all the way through the deformed region to W.

The close relationship between nuclear masses and changes in nuclear structure has been demonstrated in many studies, see, e.g., [6–11]. In particular, the differences between binding energies and separation energies (for example, the separation energies of two neutrons versus neutron number) can easily point to changes in nuclear structure. While the effects at magic numbers are very clear, there are also examples where the onset of nuclear deformation can be clearly seen from the separation energies (e.g., the shape transitional nuclei in the $N = 90$ region).

The relationship between binding energies and valence proton-neutron interactions was first described in 1989 [6]. The valence proton-neutron interaction, denoted δV_{pn}, is given by the double difference of binding energies:

$$\delta V_{pn}^{ee}(Z,N) = \tfrac{1}{4}[(B_{Z,N} - B_{Z,N-2}) - (B_{Z-2,N} - B_{Z-2,N-2})],$$

where B is the binding energy. The formula gives the average interaction between the last two protons and the last two neutrons. Later, using the 2003 Mass Data Evaluation (AME2003) [2], many more δV_{pn} values were obtained [7]. For different regions, δV_{pn} was interpreted in terms of the overlap between the shell model orbits filled by protons and neutrons of nuclei close to closed shells. For deformed nuclei, Nilsson orbitals are considered. More information can be found in Refs. [1,7,12–15].

In N=Z light nuclei, where both protons and neutrons fill exactly the same orbits, the short range proton-neutron interaction is expected to be large due to the maximal overlap between the shell-model orbit wave functions. Experimentally, for $N = Z$ nuclei, δV_{pn} reaches a maximum value (see Fig. 2 of Ref. [1]). Using Wigner's SU(4) symmetry [3,4] it was possible to interpret these peaks theoretically [5].

In the case of heavy nuclei, as the mass increases, the SU(4) symmetry is expected to be broken by the increase of the accumulated Coulomb force and the spin-orbit effect in the nuclear mean field potential. Therefore it was expected that the Wigner effect would disappear in heavy nuclei. The purpose of our earlier paper [1] was to show that an analogous (much weaker) peaking in δV_{pn} occurs in special cases in heavy nuclei.

II. METHODS AND DISCUSSION

Figure 1(top) shows the earlier δV_{pn} values of even-even Sm-W nuclei as a function of both neutron number and the number of valence neutrons [1]. As can be seen in Fig. 1(top), each δV_{pn} for a given nucleus reaches a large value at a certain number of neutrons. For example, $_{68}$Er has a maximum in δV_{pn} at $N = 100$. Clearly, ^{168}Er does not have $N = Z$. However, Er has 18 valence protons, $Z_{val} = 18$, and ^{168}Er has 18 valence neutrons, $N_{val} = 18$, above the closed

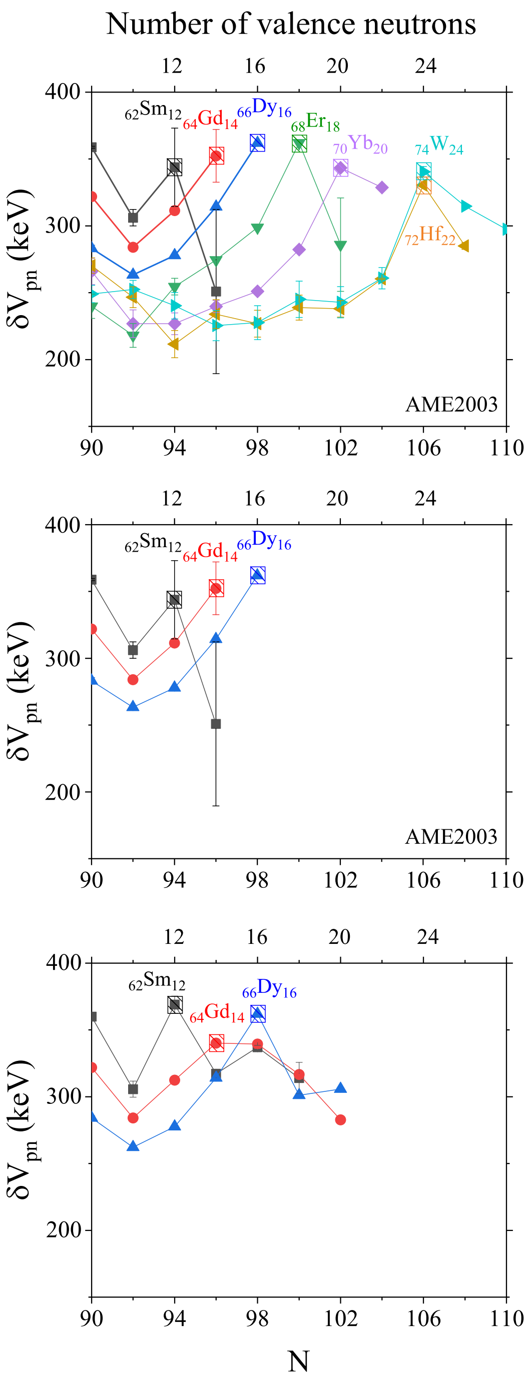

FIG. 1. δV_{pn} values with even-Z and even-N nuclei for the rare-earth region as a function of neutron number and the number of valence neutrons (at the top x axis). The number of valence protons is written for each nucleus. (Top) The figure is based on Refs. [1,2]. (Middle) Same data as (top) but δV_{pn} values focusing on Sm, Gd, and Dy where the earlier mass data were inconclusive. (Bottom) Similar to (middle) but δV_{pn} values using the masses recently measured [16] and masses from AME2020 [17] including specifically Refs. [18,19].

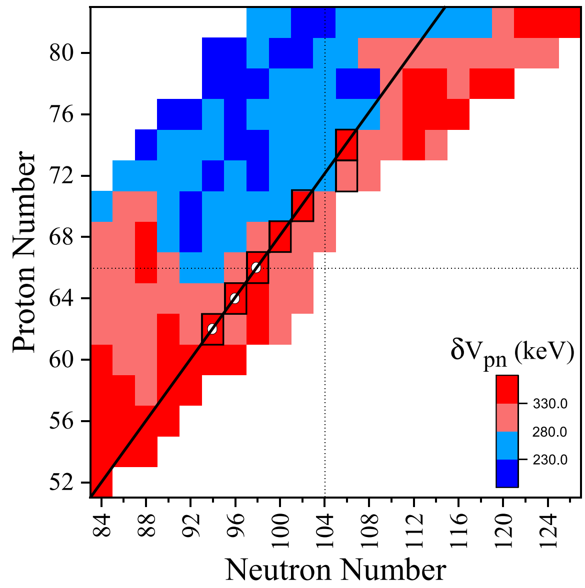

FIG. 2. Color-coded N-Z plot of δV_{pn} values where the peak values are clearly seen. The white dots point out the new maxima in δV_{pn} discussed in this paper. The plot is based on Refs. [7,12,15] but now presented with the recent masses in Refs. [16,17].

shells $Z = 50$ and $N = 82$, respectively. That is, ^{168}Er has a maximum δV_{pn} in the case of $N_{\rm val} = Z_{\rm val}$. Another example from Fig. 1(top) is $_{70}$Yb which has $Z_{\rm val} = 20$. A maximum δV_{pn} appears at $N = 102$, $N_{\rm val} = 20$. Again there is a maximum in δV_{pn} at $N_{\rm val} = Z_{\rm val} = 20$. In occasional cases like Hf the maximum occurs at $N_{\rm val} = Z_{\rm val} + 2$. The reason has to do with the fact that in heavy nuclei the proton and neutron valence shells are not the same size and the unique parity orbit can be filled differently in the two shells (this was discussed in Ref. [1]). By analogy with the Wigner effect at $N = Z$ in light nuclei, we called this $N_{\rm val} = Z_{\rm val}$ effect in heavy nuclei *a mini-valence Wigner effect* [1]. However, while it was clear that the δV_{pn} values increased with neutron number in Sm, Gd, and Dy, due to some unknown masses and some large errors on other masses, it was not clear whether there was an actual maximum for $_{62}$Sm ($Z_{\rm val} = 12$), $_{64}$Gd ($Z_{\rm val} = 14$), and $_{66}$Dy ($Z_{\rm val} = 16$), as seen in Fig. 1(middle), which has the same data as Fig. 1(top) but highlighting the data for Sm, Gd, and Dy nuclei, respectively. New and more accurate mass measurements were needed to test the mini-valence Wigner effect.

The purpose of this study is to present the δV_{pn} values for Sm, Gd, and Dy, which could not be obtained experimentally in Ref. [1], but are now available in Refs. [16,17], and to draw attention to the fact that the above mentioned mini-valence Wigner effect in heavy nuclei persists throughout the deformed region. Figure 1(bottom) is created using recent data and it is analogous to Fig. 1(middle) except that it now incorporates the latest data [16–19]. The previously unknown δV_{pn} values in Gd ($Z_{\rm val} = 14$) at $N = 98$–102 and Dy ($Z_{\rm val} = 16$) at $N = 100$–102 are now available. It is clearly seen that δV_{pn} is maximized at $_{64}$Gd $Z_{\rm val} = N_{\rm val} = 14$ and δV_{pn} of $_{66}$Dy is maximized at $Z_{\rm val} = N_{\rm val} = 16$. The δV_{pn} errors for Sm are reduced (mostly because of reducing Nd mass errors) and a cleaner result for Sm is obtained [compare Fig. 1(top) and (middle)]. There is no significant change in the Er, Yb, Hf, and W δV_{pn} values. These results show that the effect discussed in Ref. [1] extends from Sm to W.

A more synoptic way to see the behavior of δV_{pn} is shown in Fig. 2, which is updated from Ref. [15]. It shows a color coded plot of empirical δV_{pn} values in the $Z = 50$–82 and $N = 82$–126 shells. The peaks in δV_{pn} are clearly seen in the dark red colored boxes. The white dots in some of the boxes are the newly established maximum values. The black line is the locus of $Z_{\mathrm{val}} = N_{\mathrm{val}}$ nuclei running through the peak trajectory. These maxima continue to coincide with the onset of deformation as $R_{4/2}$ values closely approach the 3.33 rotational limit along the entire extended trajectory.

III. CONCLUSIONS

The mini-valence Wigner's effect is confirmed and extended by recent mass measurements. It would be useful to study this effect with systematic theoretical calculations, not only in this region but other deformed heavy nuclei, such as actinides, for isotopes with equal or nearly equal numbers of valence protons and neutrons.

ACKNOWLEDGMENTS

We thank A. Jaries for pointing out the new results in Ref. [16] which initiated this study. R.B.C. expresses gratitude for the support from the Max-Planck-Partner Group and the ExtreMe Matter Institute EMMI at the GSI Helmholtzzentrum für Schwerionenforschung GmbH, Darmstadt, Germany.

[1] R. B. Cakirli, K. Blaum, and R. F. Casten, Phys. Rev. C **82**, 061304(R) (2010).
[2] G. Audi, A. H. Wapstra, and C. Thibault, Nucl. Phys. A **729**, 337 (2003).
[3] E. P. Wigner, Phys. Rev. **51**, 106 (1937).
[4] F. Hund, Z. Phys. **105**, 202 (1937).
[5] P. Van Isacker, D. D. Warner, and D. S. Brenner, Phys. Rev. Lett. **74**, 4607 (1995).
[6] J. D. Garrett, Z.-Y. Zhang, J.-Y. Zhang, R. F. Casten *et al.*, Phys. Lett. B **227**, 1 (1989).
[7] R. B. Cakirli, D. S. Brenner, R. F. Casten, and E. A. Millman, Phys. Rev. Lett. **94**, 092501 (2005); **95**, 119903(E) (2005).
[8] R. B. Cakirli, R. F. Casten, R. Winkler, K. Blaum, and M. Kowalska, Phys. Rev. Lett. **102**, 082501 (2009).
[9] L. Chen *et al.*, Phys. Rev. Lett. **102**, 122503 (2009).
[10] F. Wienholtz, D. Beck, K. Blaum, Ch. Borgmann *et al.*, Nature (London) **498**, 346 (2013).
[11] D. Shubina *et al.*, Phys. Rev. C **88**, 024310 (2013).
[12] D. S. Brenner, R. B. Cakirli, and R. F. Casten, Phys. Rev. C **73**, 034315 (2006).
[13] M. Stoitsov, R. B. Cakirli, R. F. Casten, W. Nazarewicz, and W. Satula, Phys. Rev. Lett. **98**, 132502 (2007).
[14] Y. Oktem, R. B. Cakirli, R. F. Casten, R. J. Casperson, and D. S. Brenner, Phys. Rev. C **74**, 027304 (2006).
[15] D. Bonatsos, S. Karampagia, R. B. Cakirli, R. F. Casten, K. Blaum, and L. Amon Susam, Phys. Rev. C **88**, 054309 (2013).
[16] A. Jaries, S. Nikas, A. Kankainen, T. Eronen *et al.*, Phys. Rev. C **110**, 045809 (2024).
[17] M. Wang, W. J. Huang, F. G. Kondev, G. Audi *et al.*, Chinese Phys. C **45**, 030003 (2021).
[18] R. Orford, N. Vassh, J. A. Clark, G. C. McLaughlin, M. R. Mumpower, G. Savard *et al.*, Phys. Rev. Lett. **120**, 262702 (2018).
[19] M. Vilen, J. M. Kelly, A. Kankainen, M. Brodeur, A. Aprahamian, L. Canete *et al.*, Phys. Rev. Lett. **120**, 262701 (2018).

Correction: The previously published Figure 2 contained an error and has been replaced.