16th International Particle Accelerator Conference, Taipei, Taiwan JACoW Publishing
ISBN: 978-3-95450-248-6 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2025-TUPB021

EFFECTS OF BEAM PLANE CORRELATION ON INJECTION EFFICIENCY

A. Lauterbach*, G. Franchetti[1,2], Institute for Applied Physics, Goethe Univ., Frankfurt, Germany
[1]GSI Helmholtzzentrum für Schwerionenforschung GmbH, Darmstadt, Germany
[2] Helmholtz Research Academy Hesse for FAIR (HFHF), Frankfurt, Germany

Abstract

The effectiveness and efficiency of a beam injection scheme is crucial to achieve high beam intensities while minimizing possible beam losses. The classical method for injecting from a linac to a synchrotron is the multiturn injection (MTI). In this scheme the quality of the injected beam as well as of the injection scheme depends on factors as beam emittance, type of local bump ramp, chromaticity, dispersion and beam intensity. This approach relies on the decorrelation between the planes of the injected beams. However, investigations on the beam coming from the linac have suggested the possibility that a beam correlation may exist [1]. We present here an investigation of the effect of a correlated beam on the efficiency of the MTI for several degrees of correlation.

MULTITURN INJECTION

The injection process referred to in this study is the well-implemented process of multiturn injection (MTI). During the MTI process, several beamlets are injected via electrostatic septum turn by turn, utilizing a closed orbit bump to fill up the available phase space. During this so-called "phase space painting", the injected particles get accumulated to the maximum available intensity whilst avoiding any particle losses. This topic has been the subject of extensive research in the past [2–4], and recent studies continue to explore its optimization possibilities under varying parameters and extending to a 2-plane-approach [5, 6].

The injection scheme referred to in this work is a standard MTI with a local bump of the closed orbit, utilizing four bumper magnets with a linear steerer strength decrease between the first and the last injection turn. General settings can be extracted from Table 1. An example for phase space painting a can be seen in Fig. 1.

The injection coordinates of the beam are set according to the matching condition $X'(s_{inj}) = X(s_{inj})[-\alpha_x(s_{inj})/\beta_x(s_{inj})]$, where s_{inj} is the longitudinal position of the injection in the ring and $\alpha(s_{inj})$, $\beta(s_{inj})$ are the Twiss parameters at the injection location.

Injection Efficiency

For the purpose of this work, we define the injection efficiency as the number of particles which survive until the end of the simulation NPAR_{end} with respect to the number of particles that were aimed to be injected in total, which is the number of particles injected per beamlet NPARb multiplied by the number of injection turns j:

$$\mathrm{Eff}_{Inj} = \frac{\mathrm{NPAR}_{end}}{\mathrm{NPAR}b \cdot j} \quad (1)$$

In this work, we focus on the effect an exisiting interplane correlation of the incoming beam entering the synchrotron from the linac may have on the injection and the phase space evolution of the injected beam.

* lauterbach@iap.uni-frankfurt.de

Table 1: Simulation Settings Used in This Work for SIS18-lattice with U^{238}_{73+} Ions

Parameter	Value	Unit	Description
NPAR_b	10^5	[1]	Particles per beamlet
ϵ_x	5	[mm-mrad]	Emittance, x-plane
ϵ_y	5	[mm-mrad]	Emittance, y-plane
Q_x	3.29	[1]	Tune, x-plane
Q_y	4.29	[1]	Tune, y-plane
E_{nuc}	11.4	[MeV/u]	Energy per nucleon
I	50	[mA]	Beam current
$\alpha_x(s_{Septum})$	-0.90248	[rad]	Twiss α, x-plane
$\beta_x(s_{Septum})$	8.9851	[m]	Twiss β, x-plane

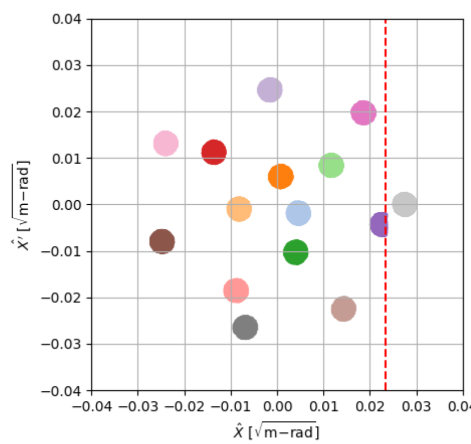

Figure 1: Example for the accumulation of beamlets during MTI (phase space painting) in normalized phase space coordinates for a linear orbit bump decrease MTI of a KV distributed $U238^{73+}$ beam of 10^5 particles at the 15th accumulation turn. The colors indicate the different injection turns of the beamlets. The red dashed line marks the septum position in normalized coordinates.

ISBN: 978-3-95450-248-6 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2025-TUPB021

BEAM CORRELATION

The correlation between the x- and y-plane of a beam is described by the second order beam moments matrix or Covariance matrix C [1]:

$$C = \begin{pmatrix} \langle xx\rangle & \langle xx'\rangle & \langle xy\rangle & \langle xy'\rangle \\ \langle x'x\rangle & \langle x'x'\rangle & \langle x'y\rangle & \langle x'y'\rangle \\ \langle yx\rangle & \langle yx'\rangle & \langle yy\rangle & \langle yy'\rangle \\ \langle y'x\rangle & \langle y'x'\rangle & \langle y'x\rangle & \langle y'y'\rangle \end{pmatrix}. \tag{2}$$

This matrix provides information on how certain coordinates of the beam in the phase space are correlated with each other. Correlation or coupling sources can be all elements of the machine lattice that act on the cross-plane elements of C. In an ideal, decoupled beamline, these cross-plane terms are zero. However, experimental observations show measurable coupling effects, particularly originating from the linear accelerator section [1]. This indicates that practical beam dynamics often deviate from the uncoupled ideal.

Next, we model the inter-plane correlations introduced by an unidentified coupling element, which is suspected to adversely affect the efficiency of the Multi-Turn Injection (MTI) process. To simulate such coupling, we introduce a parameterized 4×4 transformation matrix M of the form:

$$M = \begin{pmatrix} 1 & 0 & \epsilon & 0 \\ 0 & 1 & 0 & \epsilon \\ -\epsilon & 0 & 1 & 0 \\ 0 & -\epsilon & 0 & 1 \end{pmatrix}. \tag{3}$$

The matrix M represents an idealized model of inter-plane coupling which fulfills the symplectic condition [7], ensuring that the transformation is physically consistent. At the same time, the parameterized form of M is beneficial for this exploratory study: Keeping all input factors constant for different simulation runs, direct comparison of the simulation outcome with respect to the parameter ϵ is possible.

In practical beamlines, such coupling effects may arise from skew quadrupole fields or imperfections in beamline alignment and optics.

Since Multi-Turn Injection (MTI) is performed via a closed orbit bump, it is crucial to apply the coupling transformation only to the incoming beam, not to the reference closed orbit. To achieve this, the closed orbit coordinates $\vec{X}_{CO}$ must be explicitly accounted for. While the beam coordinates $\vec{X}$ are defined with respect to the nominal beamline center at $(0,0,0,0)^T$, the coupling transformation must be applied relative to the offset from the closed orbit:

$$\vec{X}\text{corr} = \vec{X}_{CO} + M(\vec{X} - \vec{X}_{CO}). \tag{4}$$

This ensures that the introduced correlation affects only the injected beam, preserving the integrity of the closed orbit configuration.

Coupling Coefficient

The evolution of the covariance of the beam coordinates can be calculated at each instant of the process by applying Eq. (2). As a measure of the resulting correlation factor of the beam at the end of the injection process, a coefficient T is introduced (adapted from [8]):

$$T = \frac{t}{1+t}, \tag{5}$$

where

$$t = \frac{\sqrt{\langle xx\rangle\langle x'x'\rangle - \langle xx'\rangle^2}\sqrt{\langle yy\rangle\langle y'y'\rangle - \langle yy'\rangle^2}}{\sqrt{\det C}} - 1. \tag{6}$$

Defining T like in Eqs. (5) and (6) ensures a norming to a maximum of 1 for better comparison of the outcome. For a strongly coupled beam $T \rightarrow 1$, whereas in absence of coupling $T = 0$.

PROOF OF CONCEPT

The hypothesis is that the higher the induced coupling per beamlet is, the more losses will occur, resulting in a lower injection efficiency after the injection process is completed. With the settings presented in Table 1, we run the simulation for 20 injection turns and several values of ϵ.

Simulation Procedure

For the purpose of this work, a one-plane MTI simulation code was implemented in FORTRAN95, utilizing the MICROMAP library for beam initialization and tracking. As a representative synchrotron lattice, the SIS18 lattice is used. The code calculates the beam injected for a certain number of j injection turns and is tracked for a total number of n turns in the ring. At each j, a beamlet of NPARb particles is injected, which means it is initialized with a certain distribution around an injection point in phase space at a longitudinal position s_{inj}. Right after injection, the particle data is manipulated by matrix multiplication with M as shown in Eq. (4).

The beam performs one revolution in the ring before the next injection of NPARb particles takes place. The total particle number NPAR is tracked at every turn, as well as the phase space coordinates of each particle, together with a flag for the turn number of which the corresponding particle was injected. Figure 1 shows an example of the phase space painting during injection.

The simulation is performed for several stregth of correlation, varying the value of ϵ in ten equidistant steps between 0 and 0.45. The results are listed in Table 2 and shown graphically in Fig. 2.

The simulation results show a decreasing injection Efficiency when increasing the correlation grade ϵ. For this is the only variable changed within the simulation settings it is feasible to say the injection decrease is caused by the applied interplane correlation. At the same time, the magnitude of efficiency decrease is not very high for a simulated MTI with ideal conditions like the absence of chromaticity and dispersion effects.

ISBN: 978-3-95450-248-6 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2025-TUPB021

Table 2: Simulation Results for a simulation sample MTI of $j = 20$ Injection turns of $U238^{73+}$ ions, example for SIS18-lattice.

Induced Correlation Strength ϵ[-]	**Injection Efficiency** $\mathbf{Eff}_{Inj}$ [-]	**Coupling Coefficient** T[-]
0	0.8490	5.8781×10^{-5}
0.05	0.8490	0.2781
0.1	0.8485	0.6027
0.15	0.8483	0.7680
0.2	0.8474	0.8495
0.25	0.8469	0.8931
0.3	0.8458	0.9187
0.35	0.8447	0.9346
0.4	0.8431	0.9452
0.45	0.8419	0.9524

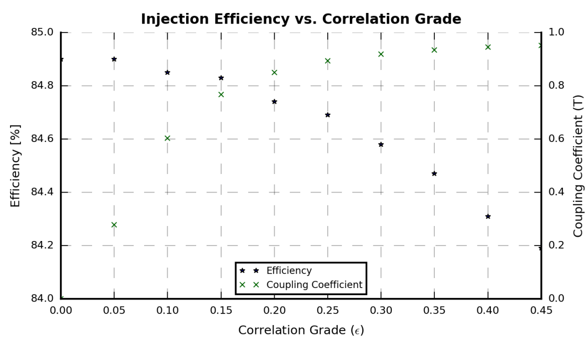

Figure 2: Evolution of injection efficiency and correlation factor of the resulting injected beam after injection.

DISCUSSION

The approach of a parameterized one-knob-model like shown in this work is promising, because it enables a direct observation of efficiency change due to the variation of the parameter ϵ. A relation between the development of the injection efficiency in the presence of artificially induced correlation can be shown, resulting also in a larger Coupling Coefficient.

Reduction of possible correlation sources already within the transfer channel is prone to be beneficial for the efficiency of the injection process. Optimization of the general injection parameters may allow a better simulation baseline of higher efficiency at the end of the injection process. At the same time, the simulation can be expanded to the presence of chromaticity, dispersion or even space charge effects for a higher number of particles. This will give a more realistic scenario of how much efficiency decrease may be caused by interplane coupling effects in the injected beam. Furthermore, the extension of the study to the effect on after-injection losses will be of interest.

This work has been performed for a specific way of applying interplane correlation through the Matrix M shown in Eqs.(3) and (4). Other variations of coupling matrices that are not symmetric or have no non-zero elements may show a comparable effect, as long as they fulfill the symplectic condition.

ACKNOWLEDGEMENTS

One of the authors, AL, thanks Oleksiy Dolinskyy for the very useful, informative and constructive discussions on MTI and the practical support at GSI, as well as Michael Maier for the very insightful discussions about beam correlation and the ROSE experiment at GSI. Special appreciation is extended to Areso Sherjan for constructive, challenging, and supportive discussions throughout the course of this work. The authors also thank the members of the accelerator physics group at the Institute of Applied Physics at the Goethe University Frankfurt for valuable discussions.

REFERENCES

[1] M. T. Maier, A. Bechtold, L. Groening, J. M. Maus, and C. Xiao, “ROSE - a Rotating 4D Emittance Scanner”, in *Proc. IBIC’19*, Malmö, Sweden, Sep. 2019, pp. 669–673. doi:10.18429/JACoW-IBIC2019-THAO03

[2] S. Appel and O. Boine-Frankenheim, “Optimization of Multi-turn Injection into a Heavy-Ion Synchrotron using Genetic Algorithms”, in *Proc. IPAC’15*, Richmond, VA, USA, May 2015, pp. 3689–3692. doi:10.18429/JACoW-IPAC2015-THPF007

[3] S. Appel, L. Groening, Y. El Hayek, M. Maier, and C. Xiao, “Injection optimization through generation of flat ion beams,”, *Nucl. Instrum. Methods Phys. Res., Sect. A*, vol. 866, pp. 36–39, Sep. 2017. doi:10.1016/j.nima.2017.05.041

[4] Youssef El Hayek, “Minimierung der systemischen Anfangsverluste im SIS18”, Dissertation, Goethe-University Frankfurt am Main, 2013.

[5] O. Dolinskyy, D. Ondreka, P. Spiller, and Y. El Hayek, “Enhancing beam intensity in SIS18 by a two-plane multi-turn injection approach”, presented at the IPAC’25, Taipei, Taiwan, Jun. 2025, paper MOPS141, this conference.

[6] S. Zhang, G. Shen, W. Chai, J. Liu, L. Yao, H. Ren, G. Wang, L. Hou, J. Yang, “Simulation and parameter optimization of 6-dimensional phase space injection scheme based on HIAF-BRing”, *Nucl. Instrum. Methods Phys. Res. A*, vol. 1064, p. 169348, 2024. doi:10.1016/j.nima.2024.169348

[7] D. A. Edwards and L. C. Teng, “Parametrization of Linear Coupled Motion in Periodic Systems,”*IEEE Trans. Nucl. Sci.*, vol. 20, no. 3, pp. 885–888, 1973. doi:10.1109/tns.1973.4327279

[8] C. Xiao *et al.*, “Measurement of the transverse four-dimensional beam rms-emittance of an intense uranium beam at 11.4 MeV/u,”, *Nucl. Instrum. Methods Phys. Res., Sect. A*, vol. 820, pp. 14–22, Jun. 2016. doi:10.1016/j.nima.2016.02.090