

Isoscalar octet axial form factor of the nucleon from lattice QCD

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(Received 31 March 2025; accepted 16 June 2025; published 2 July 2025)

The isoscalar axial-vector form factor of the nucleon plays a key role in understanding the electroweak interaction of nucleons. For the interpretation of the spin structure of the nucleon the nonsinglet isoscalar axial charge is indispensable. Moreover, $G_A^{u+d-2s}(Q^2)$ together with the isovector and singlet isoscalar form factors are needed for the flavor decomposition of the axial-vector form factor. Detailed knowledge of the flavor decomposition facilitates extractions of Standard Model (SM) parameters from low energy experiments such as the electroweak charge of the proton in the P2 experiment. Here we present a lattice determination of $G_A^{u+d-2s}(Q^2)$ on $N_f = 2 + 1$ $\mathcal{O}(a)$ improved Wilson fermions, with a full error budget concerning extrapolations and interpolations to the continuum and infinite volume and physical quark masses.

DOI: 10.1103/dc9s-35tb

I. INTRODUCTION

The axial form factor of the nucleon is crucial for the understanding of the electroweak interaction of nucleons, mediated by W or Z boson exchange. In the isospin symmetric case the W boson exchange is sensitive to the $u - d$ flavor combination of the axial current, while the Z boson exchange also receives contributions from the strange quark. The latter enters prominently in asymmetry measurements of parity violating electron-proton scattering, which allows for the determination of the weak charge of the proton [1]. The isovector axial-vector form factor of the nucleon is instrumental for the analysis of upcoming neutrino experiments [2,3], with a recent increased effort from lattice QCD [4–11] to calculate the form factor with the precision necessary to facilitate the interpretation of experimental results.

A determination of the octet axial-vector charge, i.e., the flavor combination $u + d - 2s$, is crucial in order to constrain the flavor structure of the proton spin decomposition at the quark level, e.g., based on data from polarized deep inelastic scattering [12]. Assuming SU(3) flavor symmetry, the axial-vector charge for the octet combination can be obtained from semileptonic beta decay of hyperons [13]. However, SU(3) may be badly broken [14], and in baryon chiral perturbation theory at next-to-leading-order [15] the

charge receives a contribution from an unknown SU(3)-breaking low-energy constant, which may not be extracted from semileptonic hyperon decays. Thus, a nonperturbative determination of the octet axial-vector charge is necessary to reduce the model dependence when interpreting polarized deep inelastic scattering data. Moreover, the octet combination is another step toward the full flavor decomposition of the axial-vector form factors, needed e.g., for the precise extraction of the weak charge of the proton in the P2 experiment [16]. The full flavor decomposition relies on the computation of both octet and singlet contributions. However, these renormalize differently: we therefore focus only on the former, leaving the computation of the appropriate renormalization factor and form factor for the latter for future studies. For a recent review on the status of the spin decomposition as obtained from lattice QCD see Ref. [17].

Here we present the determination of the axial-vector octet combination form factor from lattice QCD, where (in Minkowskian notation) we use the following parametrization for the flavor diagonal currents:

$$\begin{aligned} \langle N_{p^+} | A_\mu^a | N_{p^+} \rangle &= \bar{u}(p') \left[G_A^a(Q^2) \gamma_\mu + \frac{q_\mu}{2m} G_P^a(Q^2) \right] \gamma_5 u(p), \\ A_\mu^a &= \bar{\psi} T^a \gamma_\mu \gamma_5 \psi, \quad a \in \{0, 3, 8\}, \\ T^0 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad T^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ T^8 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \quad \psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \end{aligned} \quad (1)$$

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where p (p') is the momentum of the incoming (outgoing) proton, $q = p' - p$ the momentum transfer, $p^2 = p'^2 = m^2$ and $q^2 = -Q^2$. Since we will only be concerned with the octet flavor combination we will drop the superscript in the following. The calculation follows closely the procedures presented in Ref. [8] for the connected part of the calculation. For the disconnected contributions we use the same techniques applied in Ref. [18].

II. SETUP

The calculation is based on $N_f = 2 + 1$ lattice simulations with $\mathcal{O}(a)$ improved Wilson fermions [19] and tree-level improved Lüscher-Weisz gauge action [20], performed within the coordinated lattice simulation (CLS) effort [21]. We perform the necessary reweighting for the twisted-mass and rational approximation using [22,23]. The set of ensembles used in this work is identical to [8], and a summary is given in Table I.

For the extraction of the axial-vector form factor $G_A(Q^2)$ of Eq. (1), the pertinent Wick contractions lead to quark-line connected and quark-line disconnected contributions (cf. Fig. 1). The calculation of the connected part follows the strategy from [8]: the number of sources, the available source-sink separations, and the computational setup are identical. We first calculate the two- and three-point functions, which read as

$$C_2^\Gamma(t, \mathbf{p}) = \Gamma_{\alpha\beta} \sum_{\mathbf{x}} e^{-i\mathbf{p}\cdot\mathbf{x}} \langle \Psi_\beta(\mathbf{x}, t) \bar{\Psi}_\alpha(0) \rangle, \quad (2)$$

$$C_3^{\Gamma'}(t, t_s, \mathbf{q}) = \Gamma'_{\alpha\beta} \sum_{\mathbf{x}, \mathbf{y}} e^{i\mathbf{q}\cdot\mathbf{y}} \langle \Psi_\beta(\mathbf{x}, t_s) A_\mu^8(\mathbf{y}, t) \bar{\Psi}_\alpha(0) \rangle, \quad (3)$$

where the interpolating operator of the nucleon reads as

TABLE I. Summary of ensembles used, where the values for $\beta = 3.40, 3.46, 3.55, 3.70$ correspond to a lattice spacing of roughly $a \sim 0.086, 0.076, 0.064, 0.050$ fm, respectively [24].

ID	β	T/a	L/a	M_π (MeV)	M_N MeV
H102	3.40	96	32	354	1103
H105	3.40	96	32	280	1045
C101	3.40	96	48	225	980
N101	3.40	128	48	281	1030
S400	3.46	128	32	350	1130
N451	3.46	128	48	286	1011
D450	3.46	128	64	216	978
N203	3.55	128	48	346	1112
N200	3.55	128	48	281	1063
D200	3.55	128	64	203	966
E250	3.55	192	96	129	928
N302	3.70	128	48	348	1146
J303	3.70	192	64	260	1048
E300	3.70	192	96	174	962

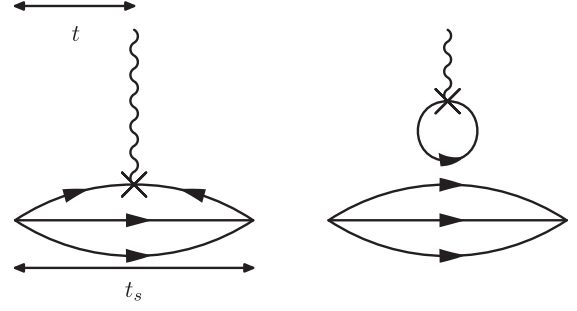


FIG. 1. Wick contractions of the axial current A_μ^a of Eq. (1), denoted by a cross, with the nucleon interpolating operator of Eq. (4). Left: the quark-line connected contribution, where t denotes the distance between source time slice and operator insertion time slice, and t_s denotes the distance between source and sink time slice, respectively. Right: the quark-line disconnected diagram.

$$\Psi_\alpha(x) = \epsilon_{abc} (\bar{u}_a^T(x) C \gamma_5 \tilde{d}_b(x)) \tilde{u}_{c,\alpha}(x), \quad (4)$$

built from Gaussian smeared quark fields [25]

$$\tilde{q} = (1 + \kappa_G \Delta)^{N_G} q \quad (5)$$

using spatially APE-smeared gauge links in the covariant Laplacian [26]. We then form the ratio

$$R(t, t_s, \mathbf{q}) = \frac{C_3^\Gamma(t, t_s, \mathbf{q})}{C_2^\Gamma(t_s, \mathbf{0})} \sqrt{\frac{C_2^\Gamma(t_s - t, \mathbf{q}) C_2^\Gamma(t, \mathbf{0}) C_2^\Gamma(t_s, \mathbf{0})}{C_2^\Gamma(t_s - t, \mathbf{0}) C_2^\Gamma(t, \mathbf{q}) C_2^\Gamma(t_s, \mathbf{q})}}, \quad (6)$$

which in the limit of large source-sink separations allows for the extraction of the axial form factor $G_A(Q^2)$. The calculation of the disconnected part is described in [18]: we first calculate the one-point functions

$$\mathcal{L}(z_0, \mathbf{q}) = - \sum_{\mathbf{z}} e^{i\mathbf{q}\cdot\mathbf{z}} \text{Tr}[S_q^{-1}(z, z) \gamma_\mu \gamma_5], \quad (7)$$

where S_q^{-1} is the propagator for the quark q , and then build the disconnected three-point functions

$$C_3^{\text{disc}, \Gamma'}(t, t_s, \mathbf{q}) = \langle \mathcal{L}(t, \mathbf{q}) C_2^{\Gamma'}(t_s, \mathbf{0}) \rangle, \quad (8)$$

where C_2 denotes the Wick contraction of the interpolating operators of Eq. (4),

$$C_2^{\Gamma'}(t_s, \mathbf{p}') = \sum_{\mathbf{x}} e^{-i\mathbf{p}'\cdot\mathbf{x}} \text{Tr}[\Gamma' \cdot \overline{\Psi(\mathbf{x}, t_s)} \bar{\Psi}(0)], \quad (9)$$

and the traces are understood in Dirac space. For the connected part we always choose the polarization matrix $\Gamma = \frac{1}{2}(1 + \gamma_0)(1 + i\gamma_5\gamma_3)$, whereas for the disconnected

we average over all three different polarizations, i.e.,

$$\Gamma'_i = \frac{1}{2}(1 + \gamma_0)(1 + i\gamma_5\gamma_i), \quad i = 1, 2, 3. \quad (10)$$

The disconnected diagrams are notoriously difficult to calculate precisely, and consequently we increased the statistics for the two-point functions entering Eq. (8) with respect to the number of sources of the two-point correlators entering the ratio Eq. (6) for the connected pieces. When combining the connected and disconnected part, we match the analyzed source-sink separations t_s of the disconnected to the setup of the connected diagrams. The effective form factors are then calculated separately for the connected and disconnected parts. Nevertheless, we perform a jackknife analysis to preserve correlations among the configurations of each ensemble. For the connected piece we use the strategy of [8], where the spatial momentum combinations and the component of the axial current are chosen such that the ratio of Eq. (6) is only sensitive to $G_A(Q^2)$ and automatically $\mathcal{O}(a)$ improved. For the disconnected pieces we choose to solve the system of equations as outlined in [27], restricting the current to its spatial components only. We show an example of the raw data for vanishing momentum on the ensemble E300 in Fig. 2. This illustrates the quality of the signal as well as the magnitude of both connected and disconnected contributions. We emphasize that the signal relies on the difference $2(l-s)$, where l refers to the light quark, and it benefits from the correlations between the two flavors, as shown in Fig. 3.

The renormalization pattern of the improved operators is given in Ref. [28], where the improved operators read as

$$A_\mu^{a,I}(x) = (A_\mu^a(x) + ac_A \tilde{\partial}_\mu P^a(x)), \quad (11)$$

with

$$P^a(x) = \bar{q}(x)\gamma_5 T^a q(x), \quad \tilde{\partial}_\mu = \frac{1}{2}(\partial_\mu + \partial_\mu^*), \quad (12)$$

and the renormalization for the octet current is

$$A_\mu^{R,8,I} = Z_A \left[\left(1 + 3\bar{b}_A a m_q^{\text{av}} + \frac{b_A}{3} a(m_l + 2m_s) \right) A_\mu^{8,I} + 2 \left(\frac{1}{3} b_A + f_A \right) a(m_l - m_s) A_\mu^{0,I} \right], \quad (13)$$

with $m_q^{\text{av}} = (2m_l + m_s)/3$ and where we use the improvement coefficients for c_A , b_A from [29,30], and the renormalization constant of the axial-vector current Z_A from [31], respectively. We expect the coefficient $\bar{b}_A$ to be small, as it is a genuine sea-quark effect, and since a nonperturbative

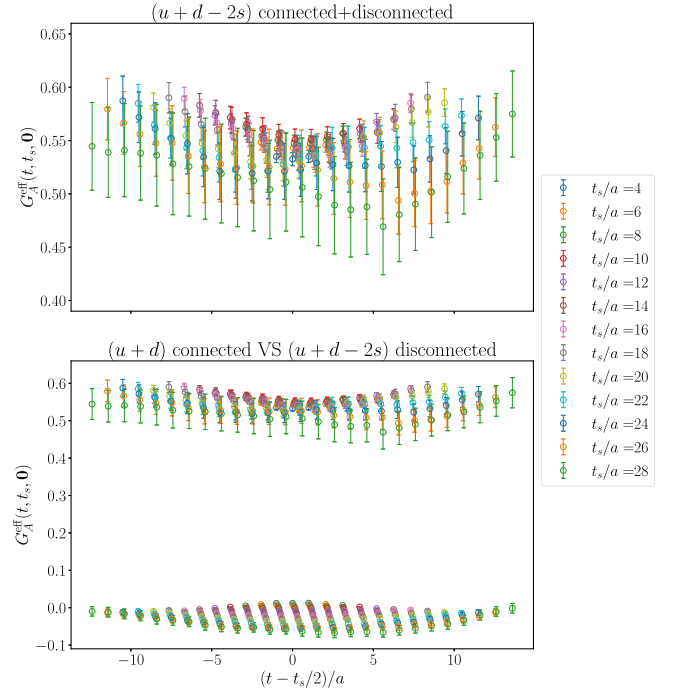


FIG. 2. Effective form factor G_A^{eff} extracted from the ratio Eq. (6) at vanishing Q^2 on the ensemble E300 for all the available source-sink separations t_s . The upper plot shows the full contribution to the octet combination, whereas the lower plot shows the breakdown into connected pieces (upper points) and disconnected pieces (lower points). The plots already include the relevant renormalization factors.

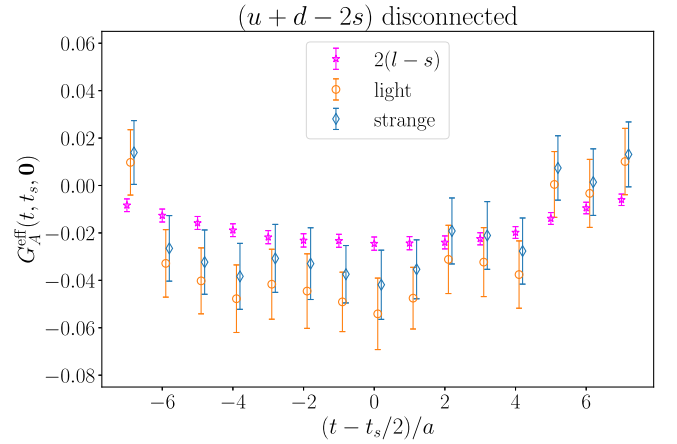


FIG. 3. Effective form factor G_A^{eff} extracted from the ratio Eq. (6) at vanishing Q^2 on the ensemble E300 at $t_s = 0.79$ fm for the light (l) and strange (s) disconnected contributions separately. The plot already includes the relevant renormalization factors for the octet contributions.

determination is not available we will neglect it. For more details on the calculation of the connected and disconnected pieces we refer to [8,18,27,32].

III. ANALYSIS STRATEGY

The analysis strategy follows closely the one of [8,33]. Namely, we make use of the summation method [34,35] to address the excited-state contamination and perform a linear fit to the summed expression

$$S(t_s, \mathbf{q}) = a \sum_{t=a}^{t_s-a} G_A^{\text{eff}}(t, t_s, \mathbf{q})$$

$$t_s \gg 1 \Rightarrow b_0(Q^2) + t_s G_A(Q^2) + \mathcal{O}(t_s e^{-\Delta t_s}), \quad (14)$$

where G_A^{eff} is the effective form factor extracted from the ratios [Eq. (6)], either applying suitable projections or solving a system of equations explicitly. We fit simultaneously for the ranges in Q^2 and t_s using two parametrizations for the form factor, i.e., z expansion

$$G_A^z(Q^2) = \sum_{k=0}^n a_k z^k(Q^2), \quad (15)$$

and a dipole modified with a z expansion

$$G_A^{dz}(Q^2) = \frac{1}{(1 + Q^2/M^2)^2} \left(\sum_{k=0}^{n-1} a_k z^k(Q^2) \right), \quad (16)$$

where M is the axial mass and

$$z(Q^2) = \frac{\sqrt{t_{\text{cut}} + Q^2} - \sqrt{t_{\text{cut}}}}{\sqrt{t_{\text{cut}} + Q^2} + \sqrt{t_{\text{cut}}}}. \quad (17)$$

More precisely, we fix $n = 2$ for both *Ansätze*, substitute them in Eq. (14) and fit for all source-sink separations t_s above all possible choices of minimum $t_{s,\text{min}}$ and up to $Q^2 = 0.7 \text{ GeV}^2$. We set $t_{\text{cut}} = (4M_\pi)^2$ using the physical pion mass $M_\pi = 134.977 \text{ MeV}$ across all the ensembles in order to simplify the chiral extrapolation. This choice also defines the physical pion mass in our work. We introduce a broad Gaussian prior for the axial mass $M^{\text{prior}}[\text{GeV}] \sim \mathcal{N}(1.285, 0.2)$ in order to center it on the $f_1(1285)$ resonance [36]. The presence of the prior does not affect the final shape of the effective form factors, but it helps stabilize the fitting procedure by including physical information about the process.

The direct parametrization of the form factor with the above expressions has a stabilizing effect in the fitting strategy. On the other hand, it requires the estimation of a large covariance matrix, which may become noisy or ill conditioned, possibly leading to unstable results due to the most poorly estimated lowest eigenvalues having the largest influence on χ^2 . We therefore use the singular value decomposition (SVD) to reduce the influence of these poorly known and noisy contributions and to reduce the condition number of the covariance matrix, requiring it to

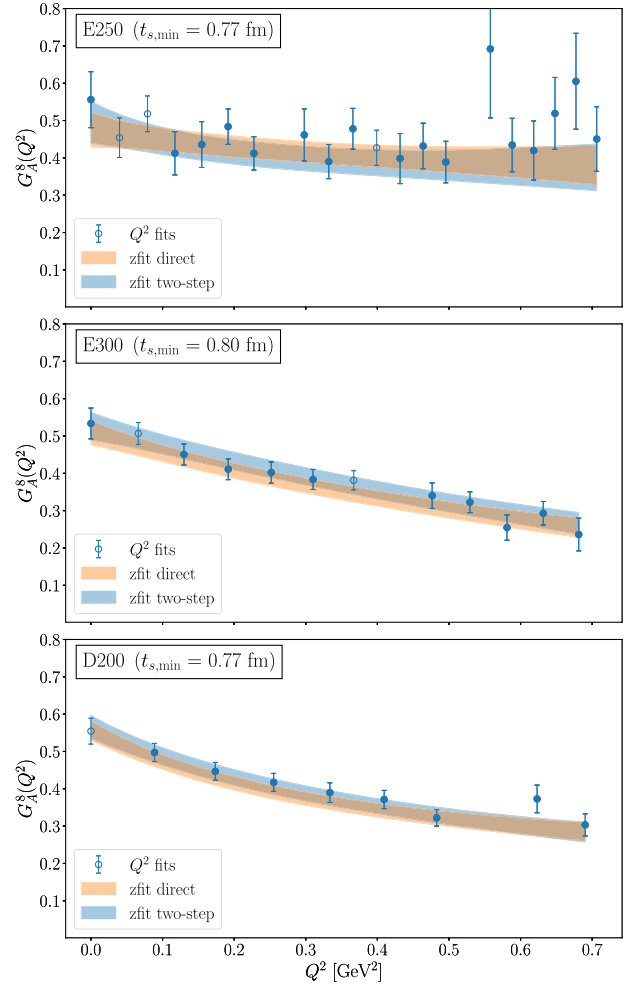


FIG. 4. Comparison of the direct z fit and the two-step procedure for the ensembles E250 at $t_{s,\text{min}} = 0.77 \text{ fm}$ (top), E300 at $t_{s,\text{min}} = 0.80 \text{ fm}$ (center), D200 at $t_{s,\text{min}} = 0.77 \text{ fm}$ (bottom). The empty points indicate a p value smaller than 5% or larger than 95% for the first step of the two-step procedure, i.e., the linear fit to the summation expression in Eq. (14).

be the same for all choices of $t_{s,\text{min}}$ entering the linear fit to the summation expression in Eq. (14) within a given ensemble. The results from this approach are compatible with the two-step procedure of first fitting the t_s range at a fixed Q^2 and then performing a z expansion on those points. As an example, we show a comparison for three of our most chiral ensembles in Fig. 4 for a given choice of $t_{s,\text{min}}$. Furthermore, the results obtained with an SVD cut are compatible with the values obtained by introducing a small damping in the off diagonal elements, as shown in Fig. 5 (cf. also with [8]). The results of the fits performed with an SVD cut, which are the ones used for our final analysis, are tabulated in the Appendix in Table II (z expansion) and in Table III (modified z expansion) ensemble-by-ensemble. We refer to the Appendix for more details on the fitting strategy. For completeness, we also report the values of $G_A^8(Q^2)$ obtained from the linear fit to

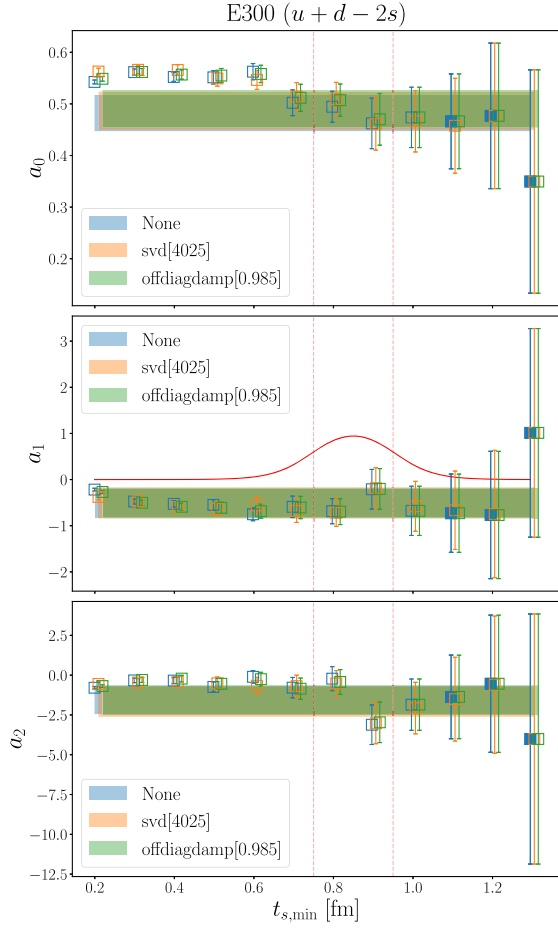


FIG. 5. Results of the direct fit to the coefficients of the z expansion as a function of the minimum source-sink separation $t_{s,\min}$ for different regularizations of the covariance matrix on the E300 ensemble. The numbers in brackets next to “svd” and “offdiagdamp” represent the chosen condition number to perform the SVD cut and the off diagonal damping, respectively. For the unregulated case, labeled as “None,” filled symbols indicate that the p value is between 0.05 and 0.95. The horizontal bands illustrate the final result of the window average on all points. The red curve represents an enlarged version of the window Eq. (18), and the vertical lines correspond to t_w^{low} and t_w^{up} .

the summation expression in Eq. (14) at fixed Q^2 for the most relevant choices of $t_{s,\min}$ in Tables IV–XVII.

The choice of $t_{s,\min}$ influences both the contamination by excited states and the statistical error. In order to strike the right balance between the two, we perform a weighted average over all the results assigning the weights through a window function

$$\frac{1}{N_w} \left[\tanh\left(\frac{t_{s,\min} - t_w^{\text{low}}}{\Delta t_w}\right) - \tanh\left(\frac{t_{s,\min} - t_w^{\text{up}}}{\Delta t_w}\right) \right], \quad (18)$$

where N_w is a normalization factor and $t_w^{\text{low}} = 0.75$ fm, $t_w^{\text{up}} = 0.95$ fm, $\Delta t_w = 0.1$ fm on each ensemble to reduce

any human bias. In Fig. 5 we show an example of the procedure on the ensemble E300, which illustrates that the chosen window correctly assigns more weight to the points that start exhibiting a plateau, i.e., the region where the excited states are efficiently suppressed, before entering the large source-sink separation region, where the error increases rapidly. In addition, we show that different regularizations of the covariance matrix (as opposed to the straightforward jackknife estimation, labeled as “None”) produce a negligible effect, proving that the fits are stable and reliable regardless of the chosen strategy to estimate the covariance matrix.

A. Physical-point extrapolation

We obtain the coefficients for both fit *Ansätze* for G_A through the procedure described above and then extrapolate the coefficients of these z expansions to the chiral and continuum limit using a simple *Ansatz* linear in M_π^2 and a^2 for all the coefficients a_i (and axial mass M), i.e.,

$$a_i = d_{i,0} + d_{i,\pi} M_\pi^2 + d_{i,a} a^2. \quad (19)$$

We include the term [37]

$$\frac{M_\pi^2}{\sqrt{M_\pi L}} e^{-M_\pi L} \quad (20)$$

in the axial charge a_0 to account for possible finite-volume effects.¹ We perform multiple fits with cuts in the pion mass, $M_\pi^{\text{cut}}[\text{MeV}] = \{300, 285, 265\}$, and by removing the coarsest lattice spacing, preserving the correlations among the three coefficients on each ensemble.

This *Ansatz* appears to be sufficient to describe the data well thanks to their flat behavior in the variables M_π^2 , a^2 and the lattice spatial extent L . We show an example of the chiral-continuum extrapolation in Fig. 6 for the z -expansion and modified z -expansion *Ansätze*.

B. Model average

We perform a weighted average [39] inspired by the Akaike Information Criterion (AIC) [40] over the resulting a_i in the version proposed in [41]. In particular, we assign to each fit k the weight

$$w_k^{\text{AIC}} \propto e^{-\frac{1}{2}(\chi_k^2 + 2n_{\text{par},k} - n_{\text{data},k})}, \quad (21)$$

with $n_{\text{par},k}$ being the number of parameters and $n_{\text{data},k}$ the number of data points entering the fit. With these we build the weighted joint distribution

¹We do not include any higher order terms [38] as we are not sensitive to them with our current statistics.

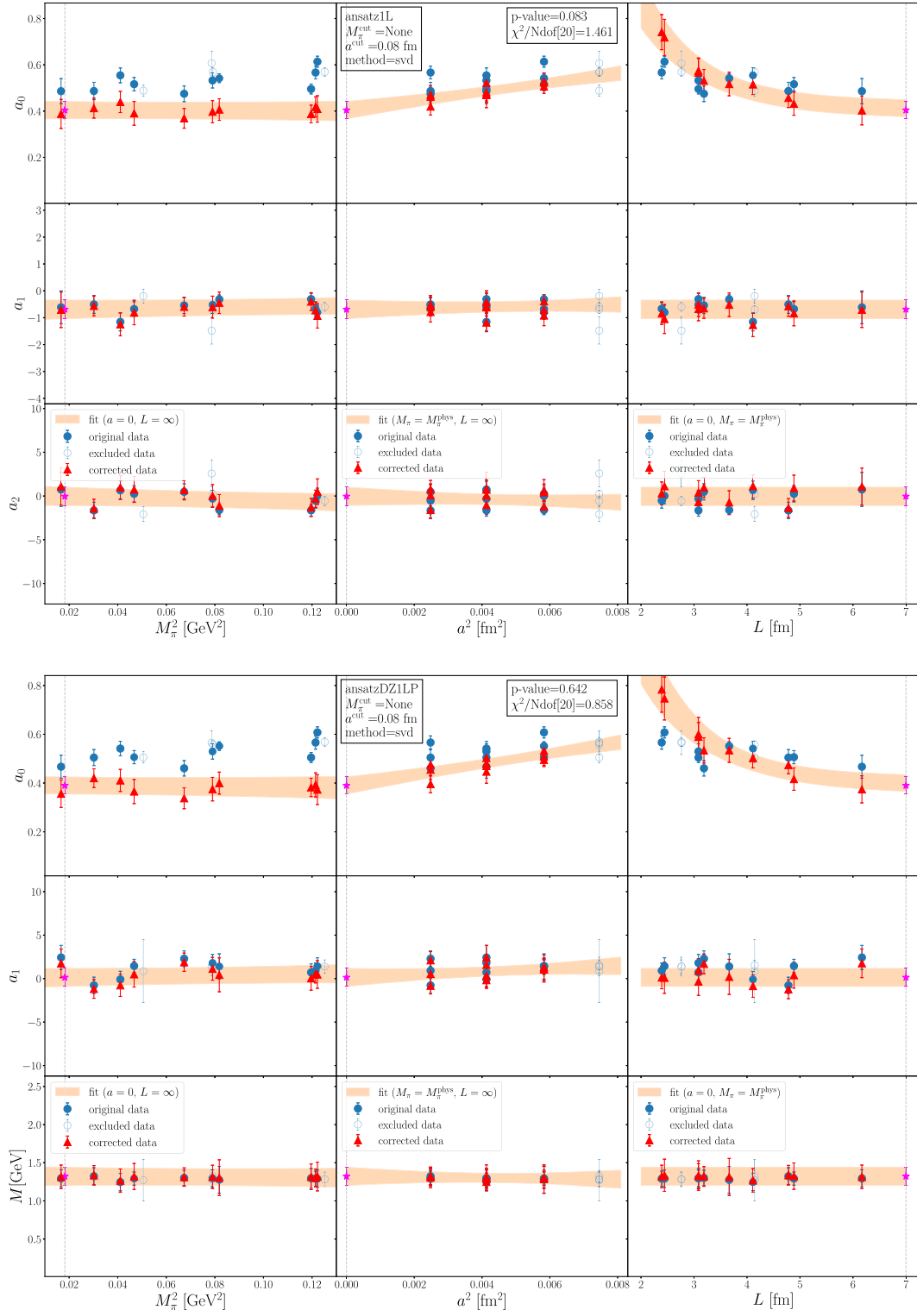


FIG. 6. Chiral continuum extrapolation for the z expansion (upper plot) and the modified z expansion (lower plot) *Ansätze* with finite-volume term, with no cut on M_π and without the coarsest lattice spacing. The results are plotted as a function of M_π^2 (left), a (center) and L (right). The blue circles indicate the original data, and the filled ones are the ones used after applying the cuts; the red triangles represent the corrected data with respect to the fit *Ansatz*, as indicated in the legend for the orange bands.

$$D(\mathbf{x}) = \sum_k w_k^{\text{AIC}} \mathcal{M}(\mathbf{x}; \langle \mathbf{x}^{(k)} \rangle, \mathbf{cov}^{(k)}) \quad (22)$$

for each of the coefficients, where $\langle \mathbf{x}^{(k)} \rangle$ indicates the mean value of the coefficients $\mathbf{x}$ in the k -th fit and $\mathbf{cov}^{(k)}$ is the corresponding covariance matrix of the multivariate normal distribution $\mathcal{M}$. In this case, depending on the fit *Ansatz* for $G_A(Q^2)$, we have either $\mathbf{x} = (a_0, a_1, a_2)$ or $\mathbf{x} = (M, a_0, a_1)$. The final results for the coefficients are obtained from the standard definition of mean and covariance: labeling two generic coefficients as y and z , we have

$$\langle y \rangle = \int dy dz y D(y, z) = \sum_k w_k^{\text{AIC}} \langle y^{(k)} \rangle \quad (23)$$

for the mean value $\langle y \rangle$, and

$$\text{Cov}[y, z] = \langle yz \rangle - \langle y \rangle \langle z \rangle \quad (24)$$

for the covariance of each pair of variables y and z . We find that this procedure is essentially equivalent to the one of [41], with the advantage that it includes both statistical and systematic error automatically, while at the same time preserving the correlations among the coefficients of the $G_A(Q^2)$ parametrization, as discussed in the Appendix.

IV. RESULTS

Our results for the coefficients of the z expansion of the nucleon axial form factor in the continuum and at the physical pion mass are

$$\begin{aligned} a_0 &= +0.456 \pm 0.038 \text{ (stat)} \pm 0.035 \text{ (sys)}, \\ a_1 &= -0.664 \pm 0.349 \text{ (stat)} \pm 0.141 \text{ (sys)}, \\ a_2 &= -0.405 \pm 1.067 \text{ (stat)} \pm 0.531 \text{ (sys)}, \end{aligned} \quad (25)$$

with correlation matrix

$$C^z = \begin{pmatrix} 1.00000 & -0.52277 & 0.23098 \\ -0.52277 & 1.00000 & -0.88683 \\ 0.23098 & -0.88683 & 1.00000 \end{pmatrix}. \quad (26)$$

For the modified z expansion we have

$$\begin{aligned} a_0 &= 0.457 \pm 0.032 \text{ (stat)} \pm 0.024 \text{ (sys)}, \\ a_1 &= 0.052 \pm 0.909 \text{ (stat)} \pm 0.182 \text{ (sys)}, \\ M[\text{GeV}] &= 1.312 \pm 0.104 \text{ (stat)} \pm 0.014 \text{ (sys)}, \end{aligned} \quad (27)$$

with correlation matrix

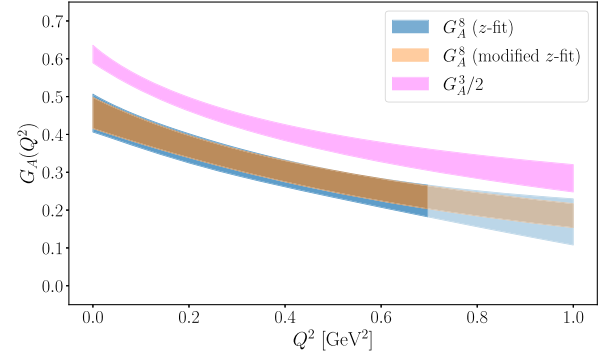


FIG. 7. Final form factor for both *Ansätze*. The data are used up to $Q^2 = 0.7 \text{ GeV}^2$, and the lighter-shaded bands represent extrapolations based on the respective parametrizations.

$$C^{dz} = \begin{pmatrix} 1.00000 & -0.29470 & -0.03725 \\ -0.29470 & 1.00000 & -0.64760 \\ -0.03725 & -0.64760 & 1.00000 \end{pmatrix}. \quad (28)$$

The results for both *Ansätze* are plotted in Fig. 7 and extrapolated up to $Q^2 = 1 \text{ GeV}^2$ (as indicated by the shaded areas). We also include the band for the isovector form factor $G_A^3(Q^2)$ from [8] divided by a factor of 2 to compare the falloff between the two channels.

For the octet axial charge we quote

$$g_A^8 = 0.457(40), \quad (29)$$

which is our most precise result based on the modified z expansion, well consistent with the determination from the z expansion *Ansatz* $g_A^{8,z} = 0.456(51)$. The values are in agreement with the prediction $g_A^8 = 0.46(5)$ obtained using the Cloudy Bag model [12], as well as with the first estimate from the ETMC Collaboration [6] and their most recent result $g_A^8 = 0.490(20)$ [42]. In contrast, the value of $g_A^8 = 0.588(33)$ [13,43] extracted from hyperon decays assuming SU(3) flavor symmetry is disfavored at the 2.5σ level.

V. CONCLUSIONS

This work provides the first physical predictions from lattice QCD of the octet axial form factor $G_A^8(Q^2)$ in a large Q^2 range. We made use of the strategies developed on a companion set of data for the isovector counterpart $u - d$ [8], extending some of the key features. In particular, we parametrized the form factor with either the z expansion or a dipole multiplied by a z expansion, substituting this parametrization directly into the summation expression to perform a simultaneous fit in Q^2 and t_s . We reported the final coefficients obtained through a linear *Ansatz* in M_π^2 and a^2 for the chiral, continuum and infinite-volume extrapolation, followed by a model average inspired by

the AIC. The results include both systematic and statistical errors, and the value obtained for the axial charge g_A^8 is compatible with previous results.

This work provides a further step toward the flavor decomposition of the axial form factor, the completion of which requires a similar analysis for the singlet $u + d + s$ case. The strange-quark contribution is of particular interest, as it provides insight into the composition of the proton spin and serves as theoretical input for neutrino experiments. An important task for the near future is therefore the computation of the appropriate renormalization factors for the singlet form factor on all ensembles.

ACKNOWLEDGMENTS

This work was supported by the Deutsche Forschungsgemeinschaft (DFG) through the Collaborative Research Center 1660 “Hadrons and Nuclei as Discovery Tools,” under Grant No. HI 2048/1-3 (Project No. 399400745) and in the Cluster of Excellence “Precision Physics, Fundamental Interactions and Structure of Matter” (PRISMA+EXC 2118/1) funded by the DFG within the German Excellence strategy (Project ID No. 39083149). Calculations for this project were partly performed on the HPC clusters “Clover” and “HIMster2” at the Helmholtz Institute Mainz, and “Mogon 2” at Johannes Gutenberg-Universität Mainz. The authors gratefully acknowledge the Gauss Centre for Supercomputing e.V. [44] for funding this project by providing computing time on the GCS Supercomputer systems JUQUEEN and JUWELS at Jülich Supercomputing Centre (JSC) via Grants No. HMZ21, No. HMZ23 and No. HMZ36 [the latter through the John von Neumann Institute for Computing (NIC)], as well as on the GCS Supercomputer HAZELHEN at Höchstleistungsrechenzentrum Stuttgart [45] under Project No. GCS-HQCD. Our programs use the QDP++ library [46] and deflated SAP + GCR solver from the OPENQCD package [47], while the contractions have been explicitly checked using [48]. We are grateful to our colleagues in the CLS initiative for sharing the gauge field configurations on which this work is based.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX

1. Method

As described in the main text, our fits rely on a (modified) z expansion for the form factor directly incorporated into the fit to Eq. (14) performed over a simultaneous range of both Q^2 and t_s . This has already been

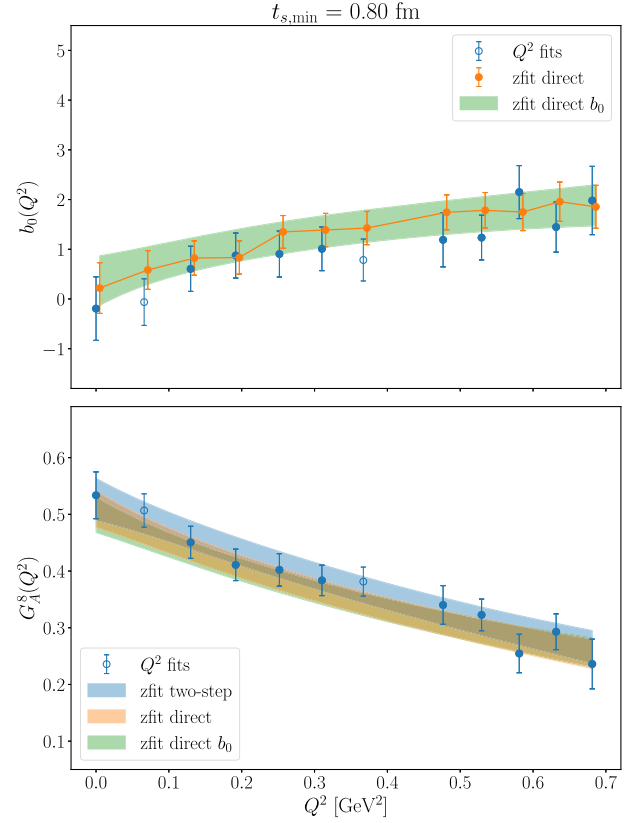


FIG. 8. Comparison of different z -fit procedures for the *Ansatz*, Eq. (14), for the extraction of the form factor $G_A^8(Q^2)$ (lower plot) on E300. The blue points refer to the two-step procedure, whereas the orange points refer to the direct approach. The green bands complement the latter considering the case where $b_0(Q^2)$ (upper plot) is also parametrized by a z expansion of order $n_b = 2$. The empty points indicate a p value larger than 95% for the first step of the two-step procedure, i.e., the linear fit to the summation expression in Eq. (14), highlighting another advantage of using the direct approach.

proven to give stable results compatible with the standard two-step approach of first fitting the t_s range at a fixed Q^2 and then performing the z expansion on those points [8]. Here we further consider the possibility of incorporating a z expansion for the parameter b_0 in Eq. (14), namely $b_0(Q^2) = \sum_{k=0}^{n_b} d_k z^k(Q^2)$ at order $n_b = 2$, to further explore the stability of the fits. An example is shown in Fig. 8, where the two-step procedure (blue points) is compared with our direct approach, with or without a z -expansion parametrization for the term b_0 . All the approaches are shown to give compatible results, and we do not observe any significant deviation among them on any ensembles.

We also explored more options to regulate a large covariance matrix, introducing either an off diagonal damping or an SVD cut. The latter have been determined by removing the smallest singular values λ iteratively (and setting all values $\lambda < \lambda_{\min}$ to $\lambda = \lambda_{\min}$) until the fit

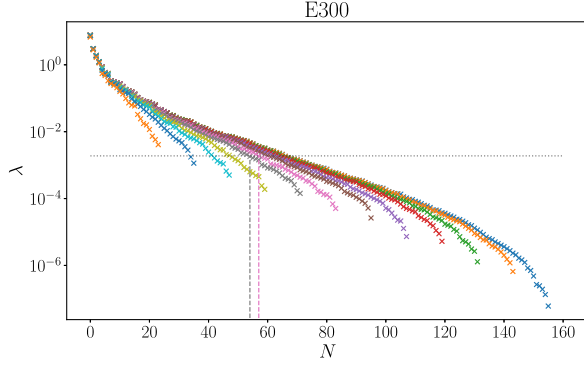


FIG. 9. Example of the scan performed to determine the SVD cut on E300. The x axis labels the index of the singular values for each of the choices of $t_{s,\min}/a \in \{4, 6, \dots, 26\}$, indicated with different colors, while the y axis indicates the singular values of the matrix. Thus, the last dot of each color indicates the size of the corresponding covariance matrix. The horizontal line corresponds to our choice for the cut, applied to all values of $t_{s,\min}$ in the analysis. The vertical lines indicate the point where the cut is applied on the two $t_{s,\min}$ in the center of the window average, i.e., $t_w^{\text{low}} \leq t_{s,\min} \leq t_w^{\text{up}}$.

becomes stable against poorly estimated fluctuations from the lowest eigenmodes, as shown in Fig. 9. In order to estimate when this aim has been realized, we consider the reduced χ^2 of the fit as an indicator of the stability of the fit, although we note that this loses its statistical meaning in the presence of SVD cuts. Operationally, we define a p value derived from the reduced χ^2 to be acceptable if it lies between 0.05 and 0.95, irrespective of whether the covariance matrix has been regulated or not. We find that the choice of the cut has little impact on the final outcome. The results are in agreement both with the naive estimate of the covariance matrix from the jackknife analysis (which in some cases leads to an unacceptable χ^2) and the introduction of a small off diagonal damping (see Fig. 5). This agreement indicates that the results obtained are not significantly affected by the poorly estimated lowest eigenmodes of the covariance matrix, and that we are justified in employing SVD cuts to suppress the impact of these modes without significantly disturbing the final result.

2. Model average

In this work we perform a simple AIC-inspired weighted average over all the variations in the chiral-continuum extrapolation, which gives a compatible result with the strategy of [41]. The latter involves the construction of a cumulative distribution function (CDF)

$$P(x) = \int_{-\infty}^x dx' \sum_k w_k^{\text{AIC}} \mathcal{N}(x'; \langle x^{(k)} \rangle, \sigma_{x^{(k)}}) \quad (\text{A1})$$

for every coefficient $x^{(k)}$ of each k -th fit. The final mean $\langle x \rangle$ is determined from the 50th percentile, i.e., $\langle x \rangle \equiv x|_{50}$ such that $P(x|_{50}) = 0.5$, and the error is obtained similarly from the 16th and 84th percentiles, i.e., $\sigma_x = (x|_{84} - x|_{16})/2$. The latter already contains both systematic and statistical error, which can be disentangled repeating the procedure by uniformly inflating the variances in Eq. (A1) $\sigma_{x^{(k)}} \rightarrow \lambda \sigma_{x^{(k)}}$, $\lambda \neq 1$, to assess the magnitude of the systematic effects, which are independent from λ . To obtain the correlations among coefficients, labeled generically as x and y , the procedure must be repeated by building a new CDF for the product $z = xy$.

However, the simpler mean and error estimation of a weighted sum of the multivariate distributions of the coefficients obtained from each fit gives the same results preserving by default the correlations and allowing a simple disentangling of systematic and statistical errors. We start from the definition

$$\text{Cov}[x, y] = \langle xy \rangle - \langle x \rangle \langle y \rangle \quad (\text{A2})$$

with

$$\langle x \rangle = \int dx dy x D(x, y) = \sum_k w_k^{\text{AIC}} \langle x^{(k)} \rangle, \quad (\text{A3})$$

$$\langle xy \rangle = \int dx dy xy D(x, y) = \sum_k w_k^{\text{AIC}} \langle x^{(k)} y^{(k)} \rangle, \quad (\text{A4})$$

where the distribution $D(x, y)$ has been defined in Eq. (22). It can be shown explicitly that

$$\text{Cov}[x, y] = \text{Cov}^{\text{sys}}[x, y] + \text{Cov}^{\text{stat}}[x, y], \quad (\text{A5})$$

where

$$\begin{aligned} \text{Cov}^{\text{stat}}[x, y] &= \sum_k w_k^{\text{AIC}} \text{Cov}[x^{(k)}, y^{(k)}] \\ &= \sum_k w_k^{\text{AIC}} [\langle x^{(k)} y^{(k)} \rangle - \langle x^{(k)} \rangle \langle y^{(k)} \rangle], \end{aligned} \quad (\text{A6})$$

and

$$\begin{aligned} \text{Cov}^{\text{sys}}[x, y] &= \sum_k w_k^{\text{AIC}} [\langle x^{(k)} \rangle - \langle x \rangle] [\langle y^{(k)} \rangle - \langle y \rangle] \\ &= \sum_k w_k^{\text{AIC}} \langle x^{(k)} \rangle \langle y^{(k)} \rangle - \langle x \rangle \langle y \rangle. \end{aligned} \quad (\text{A7})$$

We show in Fig. 10 the comparison between the two approaches to justify our choice. The upper plot corresponds to the AIC average performed on the fit results (red points) through the use of the CDF (blue curve), where the vertical blue bands indicate the final result in terms of an

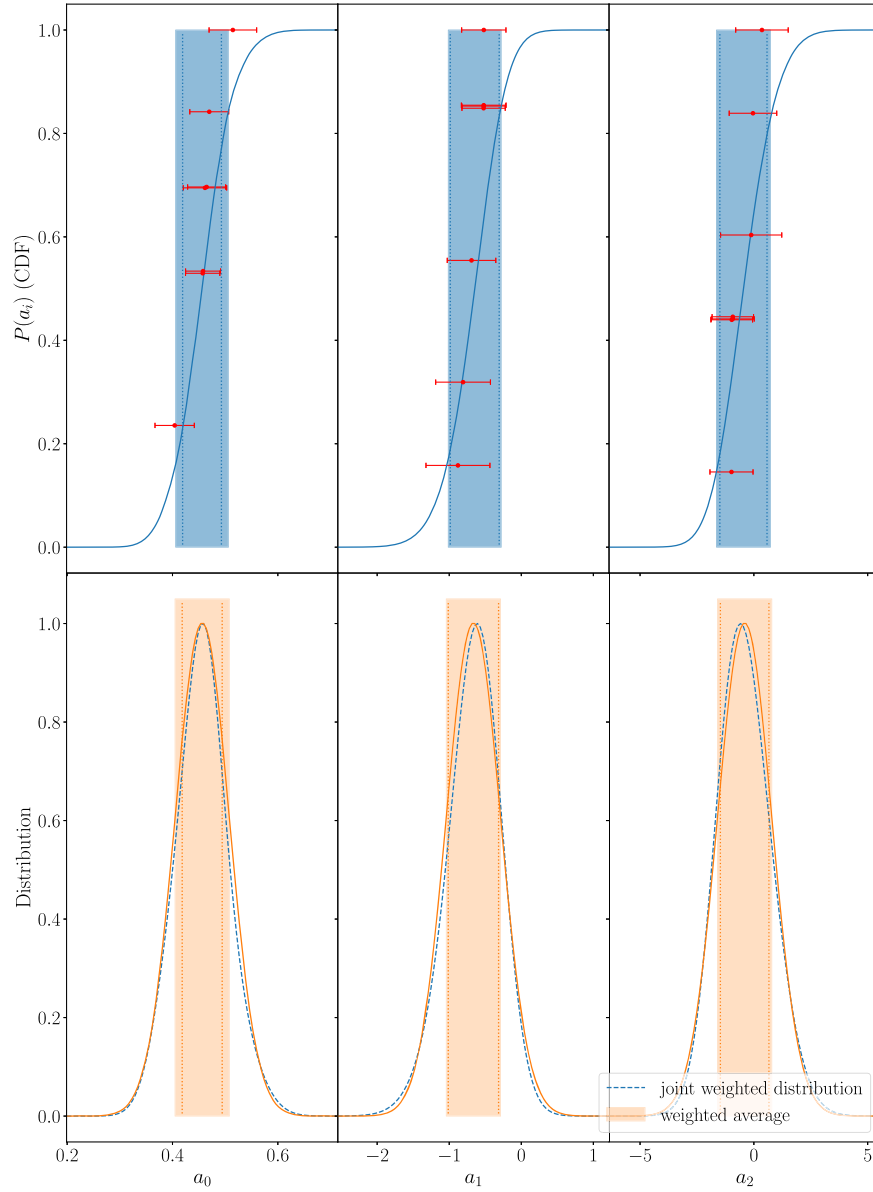


FIG. 10. Final result obtained after model average with AIC (blue bands, upper plots) or weighted average (orange bands, lower plots) of the three coefficients a_0 , a_1 , a_2 of the z expansion. The vertical dotted lines indicate the statistical error for both averages.

interval spanning between the 16th and 84th percentiles of the distribution in Eq. (A1). The lower plot compares the full distribution used to build the CDF (dashed blue curve) to the one obtained with a weighted average of the fit results (solid orange curve). The vertical dotted lines indicate the statistical error for both cases. In other words, the orange bands in the lower plot are built using the mean in Eq. (A3) and the covariance (variance) in Eq. (A2) and assuming the final result follows a Gaussian distribution. We see that the two approaches yield no significant difference, either in the total error or the decomposition into statistical and systematic uncertainties. This relies on the fact that the average of our results behaves like a Gaussian. Note that this may not be the case in general, e.g., where more fit

Ansätze are used for the chiral-continuum extrapolation, and may not be applicable in all situations.

3. Form factor data

We report the numerical results of the form factor extraction on each ensemble as described in Sec. III. In Tables II–III we collect the results of the simultaneous fits in Q^2 and t_s for the z -expansion and modified z -expansion *Ansätze* after window average, respectively. For completeness, in Tables IV–XVII we report the results of $G_A^8(Q^2)$ obtained from the linear fit to the summation expression in Eq. (14) at fixed Q^2 for $t_w^{\text{low}} \leq t_{s,\text{min}} \leq t_w^{\text{up}}$, with $t_w^{\text{low}} = 0.75$ fm and $t_w^{\text{up}} = 0.95$ fm.

TABLE II. Results for the coefficients a_0 , a_1 , a_2 of the z expansion for each ensemble, as well as their correlations ρ . These are obtained from the window average of z -expansion fits to the sum $S(\vec{q}, t_s)$ in Eq. (14) using different $t_{s,\min}$.

	a_0	a_1	a_2	ρ_{a_0,a_1}	ρ_{a_0,a_2}	ρ_{a_1,a_2}
H102	0.569(19)	-0.60(16)	-0.57(52)	-0.55804	0.25473	-0.88293
H105	0.607(53)	-1.48(51)	2.6(1.5)	-0.68792	0.43632	-0.91044
C101	0.488(24)	-0.19(25)	-2.08(83)	-0.60011	0.37279	-0.88766
N101	0.569(18)	-0.70(13)	0.21(43)	-0.60403	0.31946	-0.84955
S400	0.614(24)	-0.80(26)	0.010(878)	-0.56939	0.31177	-0.91693
N451	0.542(19)	-0.31(16)	-1.60(52)	-0.61864	0.32671	-0.86284
D450	0.516(28)	-0.68(27)	0.25(92)	-0.63480	0.35958	-0.87961
N203	0.496(21)	-0.31(20)	-1.64(65)	-0.58782	0.30535	-0.88447
N200	0.532(33)	-0.51(32)	-0.3(1.0)	-0.58641	0.31684	-0.89698
D200	0.555(33)	-1.15(33)	0.6(1.0)	-0.63127	0.38864	-0.89982
E250	0.486(54)	-0.61(62)	0.7(2.0)	-0.73743	0.52539	-0.92201
N302	0.567(28)	-0.67(26)	-0.53(86)	-0.56357	0.27283	-0.90206
J303	0.475(34)	-0.54(31)	0.48(95)	-0.67460	0.40246	-0.88869
E300	0.487(37)	-0.50(34)	-1.65(96)	-0.69221	0.46788	-0.89354

TABLE III. Results for the coefficients a_0 , a_1 , M of the modified z expansion for each ensemble, with M being the axial mass of the dipole, as well as their correlations ρ . These are obtained from the window average of modified z -expansion fits to the sum $S(\vec{q}, t_s)$ in Eq. (14) using different $t_{s,\min}$.

	a_0	a_1	M (GeV)	ρ_{a_0,a_1}	$\rho_{a_0,M}$	$\rho_{a_1,M}$
H102	0.568(19)	1.38(77)	1.283(95)	-0.18210	-0.04088	-0.87744
H105	0.565(49)	1.4(1.1)	1.284(98)	-0.42137	-0.20474	-0.39070
C101	0.505(23)	0.9(3.6)	1.27(27)	-0.34602	0.30670	-0.95599
N101	0.557(17)	1.55(55)	1.323(79)	-0.13591	-0.10115	-0.79267
S400	0.608(23)	1.43(95)	1.29(11)	-0.20571	-0.08311	-0.84400
N451	0.552(18)	1.4(1.5)	1.28(18)	-0.30308	0.18872	-0.96142
D450	0.506(27)	1.46(76)	1.293(89)	-0.35386	-0.06347	-0.59937
N203	0.505(21)	0.7(1.0)	1.30(13)	-0.29013	0.08477	-0.89023
N200	0.530(31)	1.80(99)	1.30(11)	-0.30242	-0.03475	-0.71410
D200	0.541(31)	-0.09(92)	1.25(12)	-0.23985	-0.18638	-0.60583
E250	0.467(47)	2.4(1.4)	1.29(11)	-0.50752	-0.03163	-0.49149
N302	0.566(27)	0.92(85)	1.291(98)	-0.28218	-0.06353	-0.77017
J303	0.461(31)	2.29(90)	1.301(100)	-0.41322	-0.03005	-0.62716
E300	0.505(33)	-0.78(97)	1.32(11)	-0.37199	0.04028	-0.64084

TABLE IV. Form factor $G_A^8(Q^2)$ extracted from a linear fit of Eq. (14) for $t_w^{\text{low}} \leq t_{s,\min} \leq t_w^{\text{up}}$ on H102.

H102			
$G_A^8(Q^2)$ for $t_{s,\min}$ (fm)			
Q^2 (GeV ²)	0.78	0.86	0.95
0.00	0.589(17)	0.558(28)	0.566(31)
0.19	0.483(15)	0.470(18)	0.480(23)
0.37	0.425(13)	0.407(20)	0.415(24)
0.54	0.363(15)	0.371(22)	0.378(27)
0.70	0.333(20)	0.311(38)	0.318(42)

TABLE V. Form factor $G_A^8(Q^2)$ extracted from a linear fit of Eq. (14) for $t_w^{\text{low}} \leq t_{s,\min} \leq t_w^{\text{up}}$ on H105.

H105			
$G_A^8(Q^2)$ for $t_{s,\min}$ (fm)			
Q^2 (GeV ²)	0.78	0.86	0.95
0.00	0.533(45)	0.754(79)	0.781(85)
0.19	0.454(25)	0.497(52)	0.507(57)
0.37	0.376(27)	0.443(64)	0.444(71)
0.54	0.354(31)	0.360(64)	0.354(69)
0.70	0.333(43)	0.47(11)	0.52(12)

TABLE VI. Form factor $G_A^8(Q^2)$ extracted from a linear fit of Eq. (14) for $t_w^{\text{low}} \leq t_{s,\text{min}} \leq t_w^{\text{up}}$ on C101.

C101			
Q^2 (GeV ²)	$G_A^8(Q^2)$ for $t_{s,\text{min}}$ (fm)		
	0.78	0.86	0.95
0.00	0.463(32)	0.447(42)	0.450(45)
0.09	0.454(25)	0.424(39)	0.429(41)
0.17	0.432(26)	0.358(41)	0.350(43)
0.25	0.444(25)	0.375(45)	0.370(53)
0.33	0.378(22)	0.339(42)	0.317(46)
0.40	0.340(25)	0.289(43)	0.285(44)
0.48	0.336(25)	0.277(54)	0.220(62)
0.62	0.280(35)	0.339(82)	0.313(89)
0.68	0.263(29)	0.235(57)	0.209(61)

TABLE VII. Form factor $G_A^8(Q^2)$ extracted from a linear fit of Eq. (14) for $t_w^{\text{low}} \leq t_{s,\text{min}} \leq t_w^{\text{up}}$ on N101.

N101			
Q^2 (GeV ²)	$G_A^8(Q^2)$ for $t_{s,\text{min}}$ (fm)		
	0.78	0.86	0.95
0.00	0.550(21)	0.568(29)	0.617(43)
0.09	0.504(17)	0.517(24)	0.544(38)
0.17	0.481(17)	0.483(29)	0.529(42)
0.25	0.452(14)	0.461(33)	0.535(63)
0.33	0.437(16)	0.429(27)	0.450(38)
0.41	0.424(16)	0.423(27)	0.450(37)
0.48	0.374(16)	0.385(29)	0.408(49)
0.62	0.372(23)	0.408(41)	0.483(77)
0.69	0.372(19)	0.401(32)	0.406(53)

TABLE VIII. Form factor $G_A^8(Q^2)$ extracted from a linear fit of Eq. (14) for $t_w^{\text{low}} \leq t_{s,\text{min}} \leq t_w^{\text{up}}$ on S400.

S400		
Q^2 (GeV ²)	$G_A^8(Q^2)$ for $t_{s,\text{min}}$ (fm)	
	0.76	0.92
0.00	0.596(18)	0.624(36)
0.25	0.488(12)	0.502(27)
0.47	0.417(14)	0.437(33)
0.68	0.383(17)	0.364(43)

TABLE IX. Form factor $G_A^8(Q^2)$ extracted from a linear fit of Eq. (14) for $t_w^{\text{low}} \leq t_{s,\text{min}} \leq t_w^{\text{up}}$ on N451.

N451		
Q^2 (GeV ²)	$G_A^8(Q^2)$ for $t_{s,\text{min}}$ (fm)	
	0.76	0.92
0.00	0.543(19)	0.523(26)
0.11	0.509(15)	0.488(20)
0.22	0.471(17)	0.448(22)
0.32	0.439(15)	0.415(20)
0.42	0.398(19)	0.393(28)
0.51	0.375(21)	0.352(29)
0.60	0.346(18)	0.318(24)

TABLE X. Form factor $G_A^8(Q^2)$ extracted from a linear fit of Eq. (14) for $t_w^{\text{low}} \leq t_{s,\text{min}} \leq t_w^{\text{up}}$ on D450.

D450			
Q^2 (GeV ²)	$G_A^8(Q^2)$ for $t_{s,\text{min}}$ (fm)		
	0.76	0.84	0.92
0.00	0.518(42)	0.539(44)	0.587(55)
0.06	0.486(31)	0.494(33)	0.519(39)
0.12	0.468(27)	0.477(31)	0.506(40)
0.18	0.436(29)	0.429(32)	0.436(42)
0.24	0.452(32)	0.459(38)	0.455(47)
0.30	0.417(26)	0.419(30)	0.440(42)
0.35	0.387(23)	0.382(27)	0.369(38)

TABLE XI. Form factor $G_A^8(Q^2)$ extracted from a linear fit of Eq. (14) for $t_w^{\text{low}} \leq t_{s,\text{min}} \leq t_w^{\text{up}}$ on N203.

N203		
Q^2 (GeV ²)	$G_A^8(Q^2)$ for $t_{s,\text{min}}$ (fm)	
	0.77	0.90
0.00	0.562(14)	0.481(25)
0.16	0.487(11)	0.423(19)
0.30	0.438(10)	0.366(21)
0.44	0.410(11)	0.374(25)
0.58	0.346(14)	0.252(31)
0.71	0.319(14)	0.204(31)

TABLE XII. Form factor $G_A^8(Q^2)$ extracted from a linear fit of Eq. (14) for $t_w^{\text{low}} \leq t_{s,\text{min}} \leq t_w^{\text{up}}$ on N200.

N200		
Q^2 (GeV ²)	$G_A^8(Q^2)$ for $t_{s,\text{min}}$ (fm)	
	0.77	0.90
0.00	0.582(22)	0.458(55)
0.16	0.516(16)	0.487(50)
0.30	0.467(17)	0.381(44)
0.44	0.434(18)	0.376(60)
0.57	0.404(23)	0.425(70)
0.70	0.388(23)	0.353(68)

TABLE XIII. Form factor $G_A^8(Q^2)$ extracted from a linear fit of Eq. (14) for $t_w^{\text{low}} \leq t_{s,\text{min}} \leq t_w^{\text{up}}$ on D200.

D200		
Q^2 (GeV ²)	$G_A^8(Q^2)$ for $t_{s,\text{min}}$ (fm)	
	0.77	0.90
0.00	0.554(35)	0.548(53)
0.09	0.497(24)	0.469(45)
0.17	0.447(24)	0.405(40)
0.26	0.417(24)	0.430(41)
0.33	0.390(26)	0.345(50)
0.41	0.371(24)	0.325(49)
0.48	0.322(22)	0.295(47)
0.62	0.373(37)	0.351(81)
0.69	0.303(30)	0.227(59)

TABLE XIV. Form factor $G_A^8(Q^2)$ extracted from a linear fit of Eq. (14) for $t_w^{\text{low}} \leq t_{s,\text{min}} \leq t_w^{\text{up}}$ on E250.

E250		
Q^2 (GeV ²)	$G_A^8(Q^2)$ for $t_{s,\text{min}}$ (fm)	
	0.77	0.90
0.00	0.556(75)	0.65(10)
0.04	0.454(53)	0.483(71)
0.08	0.518(48)	0.526(59)
0.12	0.412(58)	0.349(85)
0.15	0.436(61)	0.449(74)
0.19	0.484(48)	0.478(58)
0.23	0.412(45)	0.388(55)
0.30	0.462(70)	0.417(85)
0.33	0.390(46)	0.348(61)
0.37	0.478(55)	0.436(77)
0.40	0.427(47)	0.399(66)
0.43	0.398(67)	0.40(10)
0.46	0.432(61)	0.427(83)
0.50	0.389(55)	0.336(79)
0.56	0.69(18)	0.73(24)
0.59	0.434(72)	0.33(10)
0.62	0.420(79)	0.38(10)

(Table continued)

TABLE XIV. (Continued)

E250		
Q^2 (GeV ²)	$G_A^8(Q^2)$ for $t_{s,\text{min}}$ (fm)	
	0.77	0.90
0.65	0.519(96)	0.49(14)
0.68	0.61(13)	0.66(16)
0.71	0.450(87)	0.42(11)

TABLE XV. Form factor $G_A^8(Q^2)$ extracted from a linear fit of Eq. (14) for $t_w^{\text{low}} \leq t_{s,\text{min}} \leq t_w^{\text{up}}$ on N302.

N302		
Q^2 (GeV ²)	$G_A^8(Q^2)$ for $t_{s,\text{min}}$ (fm)	
	0.80	0.90
0.00	0.580(23)	0.544(41)
0.26	0.469(16)	0.438(29)
0.49	0.395(18)	0.324(35)
0.71	0.351(20)	0.331(44)

TABLE XVI. Form factor $G_A^8(Q^2)$ extracted from a linear fit of Eq. (14) for $t_w^{\text{low}} \leq t_{s,\text{min}} \leq t_w^{\text{up}}$ on J303.

J303		
Q^2 (GeV ²)	$G_A^8(Q^2)$ for $t_{s,\text{min}}$ (fm)	
	0.80	0.90
0.00	0.480(29)	0.462(52)
0.15	0.449(20)	0.402(36)
0.28	0.425(20)	0.403(38)
0.41	0.374(20)	0.379(41)
0.54	0.357(28)	0.322(58)
0.66	0.352(26)	0.427(54)

TABLE XVII. Form factor $G_A^8(Q^2)$ extracted from a linear fit of Eq. (14) for $t_w^{\text{low}} \leq t_{s,\text{min}} \leq t_w^{\text{up}}$ on E300.

E300		
Q^2 (GeV ²)	$G_A^8(Q^2)$ for $t_{s,\text{min}}$ (fm)	
	0.80	0.90
0.00	0.534(41)	0.483(61)
0.07	0.507(29)	0.487(48)
0.13	0.451(28)	0.444(47)
0.19	0.411(28)	0.431(44)
0.25	0.402(29)	0.350(46)
0.31	0.384(27)	0.376(47)
0.37	0.381(26)	0.395(44)
0.48	0.340(34)	0.329(63)
0.53	0.323(28)	0.253(52)
0.58	0.255(34)	0.202(63)
0.63	0.293(32)	0.249(60)
0.68	0.236(44)	0.146(79)

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