

## Sensitivity of nuclear clocks to new physics

Andrea Caputo<sup>1,\*</sup>, Doron Gazit<sup>2,†</sup>, Hans-Werner Hammer<sup>3,4,‡</sup>, Joachim Kopp<sup>1,5,§</sup>,  
Gil Paz<sup>6,||</sup>, Gilad Perez<sup>7,¶</sup> and Konstantin Springmann<sup>7,8,\*\*</sup>

<sup>1</sup>Theoretical Physics Department, CERN, 1211 Geneva 23, Switzerland

<sup>2</sup>Racah Institute of Physics, The Hebrew University, Jerusalem 9190401, Israel

<sup>3</sup>Department of Physics, Technische Universität Darmstadt, 64289 Darmstadt, Germany

<sup>4</sup>Helmholtz Forschungsakademie Hessen für FAIR (HFHF) and Extreme Matter Institute EMMI,  
GSI Helmholtzzentrum für Schwerionenforschung GmbH, 64291 Darmstadt, Germany

<sup>5</sup>PRISMA Cluster of Excellence & Mainz Institute for Theoretical Physics, Johannes Gutenberg University, 55099 Mainz, Germany

<sup>6</sup>Department of Physics and Astronomy, Wayne State University, Detroit, Michigan 48201, USA

<sup>7</sup>Department of Particle Physics and Astrophysics, Weizmann Institute of Science, Rehovot 761001, Israel

<sup>8</sup>Physik-Department, Technische Universität München, 85748 Garching, Germany



(Received 20 August 2024; accepted 8 August 2025; published 11 September 2025)

The recent demonstration of laser excitation of the  $\approx 8$  eV isomeric state of  $^{229}\text{Th}$  is a significant step towards a nuclear clock. The low excitation energy likely results from a cancellation between electromagnetic and strong contributions, which new physics can disrupt. In this Letter, we quantify the enhancement of a nuclear clock's sensitivity to new physics using a geometric model and a novel  $d$ -wave halo model of the nucleus that reproduces measured differences between  $^{229}\text{Th}$  states. We find likely enhancements of order  $10^4$ , while a worst case with enhancement  $\ll 1$  is unlikely.

DOI: [10.1103/129n-gt5j](https://doi.org/10.1103/129n-gt5j)

**Introduction.** The common understanding in theoretical physics today is that roughly 80% of the Universe's matter is dark matter (DM). Despite ample astrophysical and cosmological evidence,<sup>1</sup> its nature remains a unknown. Ultralight DM (ULDM) theories, including axion [1–5], dilaton [6,7], relaxion [8,9], and Higgs-portal models [10], offer simple explanations. The Nelson-Barr framework also proposes a viable ULDM candidate [11]. These models suggest that ULDM couples mainly to the standard model (SM) QCD sector, causing nuclear parameter oscillations [12–14], and subdominantly to the QED sector, affecting  $\alpha_{\text{em}}$ .

Scalar ULDM couples linearly to hadron masses [15], whereas pseudoscalars (axions) couple quadratically [16–18]. There are also natural ULDM models with quadratic DM interaction with SM fields [19], leading to nuclear parameter oscillations.

Searching for variations in SM parameters involves comparing two clocks with different parameter dependencies [6,14,20]. Laboratory limits come from clock-comparison experiments using atomic/molecular spectroscopy, cavities, and mechanical oscillators. Hyperfine transitions and mechanical oscillators sensitive to nuclear parameters [21–26] lack the accuracy of optical clocks. Optical clocks involve nuclear properties via hyperfine structure and reduced mass, contributing order of magnitude  $\sim 10^{-6}$  and  $\sim 10^{-5}$ , respectively, with charge radius oscillation contributing  $\sim 10^{-3}$  [27]. Molecular rovibrational transitions promise  $O(1)$  sensitivity to nuclear parameter modulation [28,29], though current constraints exist above 10 Hz [30]. See, e.g., Ref. [31] for a review of the use of atoms and molecules in searches for new physics.

A nuclear clock, monitoring an ultranarrow nuclear transition, could significantly advance ULDM searches [32–35]. A suitable isotope is  $^{229}\text{Th}$ , with an isomeric state  $^{229\text{m}}\text{Th}$  at 8 eV above the ground state, excitable with a laser [36,37]. Recent laser excitation observations mark progress towards a nuclear clock [38–40]. The  $^{229}\text{Th}$  data's line-shape analysis already shows high sensitivity to physics beyond the SM [41].

A nuclear clock's sensitivity to new physics comes from an accidental cancellation between nuclear and electromagnetic (EM) energy shifts. If disrupted by new physics this can lead to high sensitivity to beyond-SM physics,  $\sim 10^5$  times [42–48]. For recent reviews see [15,37,49].

\*Contact author: [andrea.caputo@cern.ch](mailto:andrea.caputo@cern.ch)

†Contact author: [doron.gazit@mail.huji.ac.il](mailto:doron.gazit@mail.huji.ac.il)

‡Contact author: [Hans-Werner.Hammer@physik.tu-darmstadt.de](mailto:Hans-Werner.Hammer@physik.tu-darmstadt.de)

§Contact author: [jkopp@cern.ch](mailto:jkopp@cern.ch)

||Contact author: [gilpaz@wayne.edu](mailto:gilpaz@wayne.edu)

¶Contact author: [gilad.perez@weizmann.ac.il](mailto:gilad.perez@weizmann.ac.il)

\*\*Contact author: [konstantin.springmann@weizmann.ac.il](mailto:konstantin.springmann@weizmann.ac.il)

<sup>1</sup>Notice that in our work we assume dark matter, thus new massive particles, to be the most likely explanation for a wide set of observables, from the CMB anisotropies to galactic rotation curves, passing for lensing probes. However, alternatives to dark matter, such as modified theories of gravity, are also being considered.

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP<sup>3</sup>.

Accurately quantifying this sensitivity requires understanding the degree of fine tuning in the transition energy. All current models of atomic nuclei have intrinsic imperfections, and a worst case with no cancellation, where EM and nuclear energy differences are individually at the eV scale, cannot be ruled out, leaving the nuclear clock with an enhancement factor of  $O(1)$ .

Our goal is to quantify how likely the worst case is. We do so by calculating the Coulomb energy shift between the ground state and the isomer of  $^{229}\text{Th}$  first in a classical, geometric model of the nucleus, parametrized by its charge radius, quadrupole and higher moments, as well as the skin thickness. We then derive similar results using a quantum halo model, in which  $^{229}\text{Th}$  is described as a  $^{228}\text{Th}$  core orbited by a halo neutron, and the two states differ in the spin alignment of the halo neutron. Finally, we conclude and propose a number of auxiliary measurements and further steps that could shed more light on the likelihood of the worst case and the sensitivity of a nuclear clock to physics beyond the SM.

*The  $^{229}\text{Th}$  nucleus.*  $^{229}\text{Th}$  is a heavy nucleus, making its modeling from first principles challenging. The isomeric state's energy difference with the ground state,  $\Delta E = 8.355\,733\,554\,021(8)$  eV [40], is much lower than typical nuclear physics scales. This low scale is thought to result from a cancellation between the EM ( $\Delta E_{\text{em}}$ ) and nuclear ( $\Delta E_{\text{nuc}}$ ) contributions to the transition energy:

$$\Delta E = \Delta E_{\text{em}} + \Delta E_{\text{nuc}}, \quad (1)$$

with  $|\Delta E_{\text{em}}| \sim |\Delta E_{\text{nuc}}| \gg \Delta E$ . Physics beyond the SM could disrupt this fine-tuned cancellation.

The sensitivity enhancement of a nuclear clock compared to an optical lattice clock is  $|\Delta E_{\text{em}}|/\Delta E \simeq |\Delta E_{\text{nuc}}|/\Delta E$ . To compute this enhancement, it is necessary to determine  $\Delta E_{\text{em}}$  independently. Since  $\Delta E_{\text{em}}$  is not observable and cannot be computed from first principles, modeling is required.

If new physics affects the strong coupling constant,  $\delta\alpha_s$ , and/or the EM coupling constant,  $\delta\alpha_{\text{em}}$ , we can express the transition energy variation as

$$\delta(\Delta E) = \frac{\partial \Delta E_{\text{em}}}{\partial \alpha_{\text{em}}} \delta\alpha_{\text{em}} + \frac{\partial (\Delta E_{\text{em}} + \Delta E_{\text{nuc}})}{\partial \alpha_s} \delta\alpha_s, \quad (2)$$

where  $\partial(\Delta E_{\text{em}} + \Delta E_{\text{nuc}})/\partial \alpha_{\text{em}} = \partial \Delta E_{\text{em}}/\partial \alpha_{\text{em}}$ . DM-induced variations of  $\alpha_{\text{em},s}$  are typically  $\delta\alpha_{\text{em},s} = \alpha_{\text{em},s}[1 + \mathcal{F}(t)g_{\text{em},s}]$ , with  $g_{\text{em},s}$  denoting the DM coupling strength and  $\mathcal{F}(t)$  depending on DM oscillations.

We focus on computing the EM enhancement factor,

$$K_{\text{em}} \equiv \frac{1}{\Delta E} \frac{\partial \Delta E_{\text{em}}}{\partial \ln \alpha_{\text{em}}} \simeq \frac{\Delta E_{\text{em}}}{\Delta E}, \quad (3)$$

where we used the fact that, to leading order,  $\Delta E_{\text{em}}$  depends linearly on  $\alpha_{\text{em}}$  in the second equality.  $K_{\text{em}}$  is the quantity of interest for DM candidates coupled to the EM sector; however, such candidates are typically well constrained by other probes. But some of the most appealing DM candidates couple to the strong sector, and hence it is of great interest to compute the corresponding enhancement factor  $K_s = K_s^{\text{em}} + K_s^{\text{nuc}}$  related to variations of the effective value of  $\alpha_s$ . The effect of such variations is very difficult to model. Nevertheless, if  $\Delta E_{\text{em}}$  has a polynomial dependence on  $\alpha_s$  (e.g., from variation of

nuclear radius with  $\alpha_s$ ; see [27]), we expect

$$K_s^{\text{em}} \equiv \frac{1}{\Delta E} \frac{\partial \Delta E_{\text{em}}}{\partial \ln \alpha_s} \sim \beta \frac{\Delta E_{\text{em}}}{\Delta E} \sim \beta K_{\text{em}}, \quad (4)$$

with  $\beta$  expected to be order 1. This implies  $K_{\text{em}}$  can be a good proxy for  $K_s$ . Barring cancellations between  $K_s^{\text{em}}$  and  $K_s^{\text{nuc}} \equiv \frac{1}{\Delta E} \frac{\partial \Delta E_{\text{nuc}}}{\partial \ln \alpha_s}$ , we expect our computation of  $K_{\text{em}}$  to be a good proxy for  $K_s$ , and our conclusions to apply also to DM models which couple to the strong sector. This statement can be invalidated only if the functional dependences of  $\Delta E_{\text{em}}$  and  $\Delta E_{\text{nuc}}$  on  $\alpha_s$  are exactly the same.

We now determine  $\Delta E_{\text{em}}$  and its variation with  $\alpha_{\text{em}}$ , defining the enhancement factor as  $K \equiv |K_{\text{em}}|$ . Recent studies modeled the  $^{229}\text{Th}$  nucleus using a geometric model [43,50], describing the nucleus as a three-dimensional charged body. This model, though simple, has shortcomings, including neglecting quantum effects and exchange terms [51]. An alternative halo model, describing  $^{229}\text{Th}$  as a  $^{228}\text{Th}$  core with a halo neutron, is supported by neutron separation energy data and charge radii comparisons [52–54]. We investigate the predictions of both models quantitatively. We neglect any contribution from the electrons' density change inside the nucleus. Following Ref. [55], the electrons' contribution to the energy shift is negligible,  $\Delta E = F\delta\langle r^2 \rangle \simeq 10^{-7}$  eV (where  $F$  is the field shift,  $\sim O(1)$  GHz/fm<sup>2</sup> [55]).

*The classical geometric model.* Despite its aforementioned shortcomings, the geometric model of the  $^{229}\text{Th}$  nucleus has been adopted in several studies of nuclear clocks' potential to look for new physics [43,50]. We therefore study this model first, advancing it beyond previous calculations. Calculating  $\Delta E_{\text{em}}$  at the required level of  $\sim 8$  eV is considered to be intractable in nuclear physics, and we do not claim to achieve this either. Instead, we aim to estimate a preferred range of  $\Delta E_{\text{em}}$ , and answer the following two questions:

- (1) What is the expected enhancement factor  $K$ ?
- (2) How likely is the worst case, where  $\Delta E_{\text{em}}$  is accidentally close to zero, jeopardizing new physics searches?

Our starting point is the nuclear charge density, which we describe with a Woods-Saxon distribution [56]

$$\rho(r, \theta) = \rho_0 \left[ 1 + \exp\left(\frac{r - R(\theta)}{z}\right) \right]^{-1}, \quad (5)$$

where  $z$  is the skin thickness of the nucleus,  $R$  is its radius, and the constant  $\rho_0$  is determined by the normalisation condition  $Ze = \int d^3\mathbf{r} \rho(r, \theta)$  with  $e$  the electric charge unit and  $Z = 90$  for thorium. We model nonsphericity of the nucleus through  $R(\theta)$ :

$$\frac{R(\theta)}{R_0} = 1 + \beta_2 Y_{20}(\theta) + \beta_3 Y_{30}(\theta) + \beta_4 Y_{40}(\theta) + \dots, \quad (6)$$

where  $R_0$  is the scale radius,  $Y_{lm}(\theta, \phi)$  denote spherical harmonics, and  $\beta_2$ ,  $\beta_3$ , and  $\beta_4$  are coefficients of the quadrupole, octupole, and hexadecapole, respectively. Higher multipole moments are neglected in this work. Azimuthal symmetry sets the magnetic quantum number  $m$  to zero, and  $R(\theta)$  depends only on the polar angle  $\theta$ . Note that  $\beta_3$  and  $\beta_4$  are not constrained by experiments yet, contrary to  $\beta_2$ , but they all impact  $\Delta E_{\text{em}}$ .

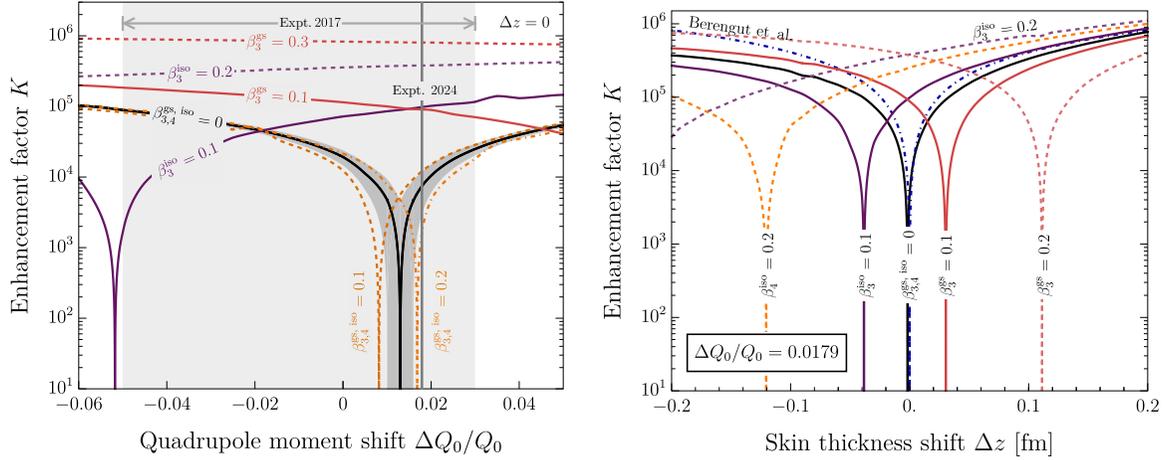


FIG. 1. Left panel: The enhancement factor  $K$  benefiting new physics searches with  $^{229}\text{Th}$  as a function of the quadrupole moment difference between the ground and isomer states. Colored lines correspond to different values of the higher EM multipole moments  $\beta_{3,4}^{\text{gs, iso}}$  of the ground state (gs) and the isomer (iso); cf. Eq. (6). Labels give the multipole moments, which are chosen different from zero. The vertical gray line indicates the latest, very precisely measured value of  $\Delta Q_0/Q_0$  [40], while the wide shaded gray band corresponds to the  $1\sigma$  uncertainty from previous measurements of the quadrupole moments [57]. The dark-gray shaded area denotes the  $2\sigma$  uncertainty from  $\langle r^2 \rangle$  for the case of  $\beta_{3,4}^{\text{gs, iso}} = 0$ . We see that, for specific values of  $\Delta Q_0/Q_0$  and the values of  $\beta_{3,4}^{\text{gs, iso}}$ , the  $K$  factor may be drastically smaller than  $10^5$ . Right panel:  $K$  as a function of  $\Delta z$ , the difference in the skin thickness between the ground and isomer states of  $^{229}\text{Th}$ . Also in this case we see that  $K$  can go to zero for specific values of  $\Delta z$  and  $\beta_{3,4}^{\text{gs, iso}}$ , though, for most values of these unknown parameters,  $K$  is of order  $10^4$ – $10^5$ . The dot-dashed blue line shows the approximate expression for  $K$  from Eq. (7) of Ref. [50], which does not take into account the impact of  $\beta_{3,4}$  and is a good approximation only for small  $\Delta z$  [58].

Given  $\rho(r, \theta)$ , we compute its two moments, which are accessible experimentally: the mean squared charge radius

$$\langle r^2 \rangle \equiv \frac{1}{eZ} \int d^3\mathbf{r} r^2 \rho(r, \theta) \quad (7)$$

and the intrinsic quadrupole moment<sup>2</sup>

$$Q_0 \equiv \frac{1}{eZ} \int d^3\mathbf{r} r^2 \rho(r, \theta) [3 \cos^2(\theta) - 1]. \quad (8)$$

The measurements of  $\langle r^2 \rangle$  and  $Q_0$  fix two parameters of our model. In particular, the scale radius  $R_0$  depends mainly on  $\langle r^2 \rangle = [5.7557(143) \text{ fm}]^2$  [57,59], while the parameter  $\beta_2$  is closely related to the intrinsic quadrupole moment,  $Q_0 = 9.8(1) \text{ fm}^2$  [57]. From this value,  $\beta_2 \sim 0.2$  with weak dependence on  $\beta_{3,4}$ . For small  $\beta_{2,3,4}$  one finds  $\langle r^2 \rangle \sim R_0^2$  plus small  $O(z^2/R_0^2)$  corrections (typically  $z \sim 0.5 \text{ fm}$ ). Similarly, for small  $\beta_2$ ,  $Q_0$  can be expanded in  $\beta_2$ . In the following, we do not assume any expansions and numerically solve for various parameters of our model.

Note that Ref. [43] relates  $\langle r^2 \rangle$  to  $\beta_2$  (or equivalently  $Q_0$ ) by taking a constant volume approach; that is, by assuming equal  $R_0$  for the ground state and the isomer. Furthermore, [43] studies the impact of small  $\beta_3$  values on the energy difference  $\Delta E_{\text{em}}$ , but not the impact of  $\beta_4$ . Our study does not assume constant volume. We numerically scan over the parameters

<sup>2</sup>We use the notation  $Q_0$  here to distinguish the intrinsic quadrupole moment from the spectroscopic quadrupole moment  $Q_s = Q_0[(j(2j-1))/(j+1)(2j+3)]$ , which is often the quantity that is reported in the experimental literature.

$\beta_{3,4} \in [0, 0.3]$ , i.e., of the order of  $\beta_2$ , and  $\Delta z$ , the difference in the skin thickness between the ground and isomer states of  $^{229}\text{Th}$ , to carefully assess their combined impact on the sensitivity.

To calculate the EM energy for both the ground state and the isomer of  $^{229}\text{Th}$ , we approximate them with the direct Coulomb energy contribution

$$E_{\text{cm}} \simeq E_C \equiv \frac{1}{2} \int d^3\mathbf{r} d^3\mathbf{r}' \frac{\rho(r, \theta) \rho(r', \theta')}{|\mathbf{r} - \mathbf{r}'|}. \quad (9)$$

The integral is calculated numerically. We show the results in Fig. 1.

In the left panel of Fig. 1 we plot the enhancement factor  $K$  as a function of the relative quadrupole moment difference between the isomer state and the ground state of  $^{229}\text{Th}$ ,  $\Delta Q_0/Q_0$ , for various combinations of  $\beta_{3,4}$ . We find that the worst case, in which  $K \simeq 0$ , implying no enhanced sensitivity to new physics, is fairly unlikely. It would only be realized for finely tuned values of  $\beta_{3,4}^{\text{gs, iso}}$  of the ground state (gs) and the isomer (iso) such that the funnel region is aligned with the measured value of  $\Delta Q_0/Q_0$  from Ref. [40] (vertical dark gray line in left panel of Fig. 1), but typical values of  $K \sim 10^4$ – $10^5$  are far more likely. The same conclusions can be drawn from Fig. S1 of the Supplemental Material [60], showing  $K$  factor as a function of  $\beta_{3,4}^{\text{gs, iso}}$ . Similar fine tuning is needed for a given value of  $\langle r^2 \rangle$ . We illustrate this by the dark-gray shaded area denoting the  $2\sigma$  uncertainty from  $\langle r^2 \rangle$  [59]. For a precisely measured value of  $\langle r^2 \rangle$ ,  $\beta_{3,4}^{\text{gs, iso}}$  must be fine tuned to zero to obtain small values of  $K$ .

In the right panel of Fig. 1 we show the impact of  $\Delta z$  for both the ground state and the isomer. We see again that  $K$

might be very small or zero. Note that the treatment of  $\Delta z$  discussed in [50] is valid only for small  $\Delta z$  and does not simultaneously take into account the effects of  $\beta_{3,4}$ .

<sup>229</sup>Th as a halo nucleus. To describe <sup>229</sup>Th as a halo nucleus, we follow the halo effective field theory (EFT) framework for shallow  $d$ -wave states developed in [61,62]. We consider the ground state ( $j^\pi = 5/2^+$ ) and the isomer ( $j^\pi = 3/2^+$ ) as a doublet resulting from the  $l = 2$  coupling of the halo neutron to the <sup>228</sup>Th core system. We stress that halo EFT should only be considered a model for the neutron-<sup>228</sup>Th system, not a proper EFT, as the separation between the low and high energy scales is marginal. If the expansion parameter is estimated as the ratio of the neutron separation energies of <sup>229</sup>Th ( $S_n \approx 5.2$  MeV) and <sup>228</sup>Th ( $S_n \approx 7.1$  MeV), it would be  $\sqrt{5.2/7.1} \approx 0.86$ . In addition, the predictivity of halo EFT is limited for  $d$ -wave nuclei as the centrifugal barrier creates a strong dependence on short-distance (small  $r$ ) effects and therefore on the overlap of the halo nucleon's wave function with the core. This is reflected in a leading-order dependence of most observables on counter-terms whose exact magnitude depends on the core-halo interaction model.

However,  $d$ -wave halo EFT suggests that the neutron-halo doublet states,  $j = 3/2$  and  $j = 5/2$ , are degenerate at leading order, and that therefore the *differences* between their charge radii, quadrupole moments, etc., vanish at leading order [61,62]. Thus, we develop here a model, inspired by halo EFT, that focuses on these differences, motivated by the nature of the isomeric transition. We begin by computing the two quantities to which we have experimental access:  $\langle r^2 \rangle$  and  $Q_0$ . Within our halo model, the charge density of the ground and isomer states are identical, except for a difference from the spin-orbit interaction between the halo neutron's spin and its  $l = 2$  orbital angular momentum. This spin-orbit contribution to the charge density is given by [63]

$$\rho_{\text{SO}} = \frac{\mu_n}{2m_n^2} i\sigma_{\alpha\beta} [(\nabla\Phi_\alpha^\dagger) \times (\nabla\Phi_\beta)], \quad (10)$$

where  $\mu_n = -1.91$  is the magnetic moment of the neutron in units of the nuclear magneton,  $\Phi_\alpha$  is the (nonrelativistic two-component) neutron spinor with  $\alpha, \beta = 1, 2$  spinor indices. With  $\rho_{\text{SO}}$  at hand we can compute the corresponding mean squared charge radius and the quadrupole moment, in analogy to our calculations for the geometric model, using Eqs. (7) and (8). The calculations appear in the Supplemental Material [60].

The difference of the mean squared radii,

$$\Delta\langle r^2 \rangle_{\text{SO}} \equiv \langle r^2 \rangle_{\text{SO}}^{\text{iso}} - \langle r^2 \rangle_{\text{SO}}^{\text{gs}} = 0.0047 \text{ fm}^2, \quad (11)$$

is smaller than the measured  $\Delta\langle r^2 \rangle = 0.012(2) \text{ fm}^2$  [57] by a factor of  $\sim 2$ , which we consider excellent agreement given the inherent uncertainties to the halo model. For the quadrupole moment, we find

$$Q_{\text{SO}}^{\text{iso}} - Q_{\text{SO}}^{\text{gs}} = 0.185 \text{ fm}^2, \quad (12)$$

in excellent agreement with the latest experimental value [40]

$$(Q^{\text{iso}} - Q^{\text{gs}})_{\text{exp}} = 0.173(9) \text{ fm}^2. \quad (13)$$

We consider this a further validation of the halo model for <sup>229</sup>Th. We also notice that the computations of  $\Delta\langle r^2 \rangle_{\text{SO}}$  and

$Q_{\text{SO}}^{\text{iso}} - Q_{\text{SO}}^{\text{gs}}$  are *not* sensitive to ultraviolet (UV) physics, i.e., insensitive to the structure of the halo neutron wave function near the origin. This is not expected to be true for higher multiple moments.

The main assumption in these calculations is that the contribution originates from single nucleons [63], i.e., consistent with a halo model [52]. This leaves room for a possible renormalization of the halo-neutron magnetic moment, which offers an explanation for the  $\approx 50\%$  deviation from experiment.

Having shown that the halo model gives sensible results for  $\langle r^2 \rangle$  and  $Q_0$ , we compute the spin-orbit contribution to the binding energy of the halo neutron [64]

$$E_{\text{SO}} = \frac{e\mu_n}{2m_n^2} \left[ j(j+1) - l(l+1) - \frac{3}{4} \right] \int_0^\infty \left[ \frac{[u(r)]^2}{r} \frac{dV_C}{dr} \right] dr, \quad (14)$$

where  $l = 2$  and  $j = 5/2$  (ground state) or  $3/2$  (isomer) are its angular momentum quantum numbers,  $V_C(r)$  is the Coulomb potential of the <sup>228</sup>Th core, and  $u(r)$  is the excess neutron's radial  $d$ -wave function [65]:

$$u(r) = A(r)e^{-\gamma r} \left( 1 + \frac{3}{\gamma r} + \frac{3}{(\gamma r)^2} \right). \quad (15)$$

In this expression,  $\gamma \equiv \sqrt{2m_n E_B}$ ,  $E_B = 5.2$  MeV is the binding energy, and  $A(r)$  is determined by short-range physics.  $A(r)$  has a weak dependence on  $r$ , and it asymptotically converges at large  $r$  to a constant  $A$ , called the asymptotic normalization constant (ANC). At leading order in halo EFT,  $A$  is taken to be fully independent of  $r$ , and depends on the UV cutoff scale, which we assume in the following. As usual in EFTs, the behavior of the ANC at short distances should be fixed from additional experimental constraints (see the discussion in the outlook section). Since the neutron separation energy for the ground state of <sup>229</sup>Th is expected to be almost the same as for the isomer, a good assumption is that  $u(r)$  is also the same for  $j = 5/2$  and  $j = 3/2$ . After all, these quantum numbers are just a result of the spin-orbit coupling for fixed  $l = 2$ , and we treat the spin-orbit potential as a small perturbation.

In order to estimate  $E_{\text{SO}}$  from Eq. (14), we also need to specify the Coulomb potential,  $V_C(r)$ . As the halo-neutron radial wave function is dominated by the length scale  $\gamma^{-1}$ , i.e., a few Fermi and much larger than the UV length scale, the charge density can be approximated by a Woods-Saxon form [66]; see Eq. (5). The universal property  $\rho \approx \text{constant}$  leads to  $dV_C/dr \propto r$ , which conveniently cancels the UV sensitivity of the integral in Eq. (14); we then assume a constant  $A(r)$ , constrained by the normalization condition on  $u(r)$ , and numerically find

$$\Delta E_{\text{SO}} = E_{\text{SO}}|_{j=5/2} - E_{\text{SO}}|_{j=3/2} \approx 144p \text{ keV}, \quad (16)$$

where the order-1 factor  $p$  is a function of  $A$  and  $l$ , the latter being a short distance cutoff we introduced to regularize the integrals for  $r \rightarrow 0$  (we find that the integral is almost independent of  $l$  for values  $l \leq 4$  fm). A more complete nuclear model would predict a specific form for  $A(r)$ , eliminating the need for a regulator, and thereby determining  $p$ . Intuitively,  $p$

is related to the probability of finding the halo neutron inside the nucleus. The details of the computation are given in the Supplemental Material [60].

Summarizing, we find the contribution of the spin-orbit interaction to  $\Delta E_{\text{em}}$  to be of order 100 keV, which corresponds to an enhancement factor  $K \approx 10^4$  in searches for new physics using nuclear clocks.

*Summary and outlook.* In this work, we studied how the sensitivity of a  $^{229}\text{Th}$  nuclear clock to new physics depends on nuclear modeling. We showed that within a classical geometric model of the  $^{229}\text{Th}$  nucleus, a worst case where the sensitivity goes to zero is possible but requires fine tuning of the model parameters at the percent or per mille level. Conversely, we presented a more realistic quantum halo model, where an unpaired neutron of  $^{229}\text{Th}$  is loosely bound to a  $^{228}\text{Th}$  spin-0 core in a  $d$ -wave state with orbital angular momentum  $l = 2$ . We validated this model against experimentally measured differences in charge radii and quadrupole moments of the  $^{229}\text{Th}$  ground state and isomer  $^{229\text{m}}\text{Th}$ , correctly predicting these differences to be close to zero. Within the halo model, the worst case is never realized, and the enhancement factor  $K_{\text{em}}$  in searches for new physics is always  $\gtrsim 10^4$ .

Our work suggests several future research directions. The main obstacle to accurately determining the enhancement factor is the nuclear model, with its many unknown parameters. Future neutron capture experiments could refine and verify the halo model. The amplitude of neutron capture in  $d$ -wave halos

is proportional to the asymptotic normalization constant  $A$ , so such experiments can constrain the probability of finding the halo neutron inside the core, affecting the spin-orbit contribution to the EM interaction. We assumed that  $A$  is identical for the ground state and isomer, an assumption that can be tested by measuring the neutron capture cross section,  $\sigma_{\text{cap}}$ , for both states. Halo EFT predicts  $\sigma_{\text{cap}}^{\text{iso}} = \frac{2}{3}\sigma_{\text{cap}}^{\text{gs}}$  [61].

It is important to confirm the latest results for  $\Delta Q_0/Q_0$  [40] and improve the determination of  $\Delta(r^2)$ . Additionally, devising tests sensitive to higher-order multipoles  $\beta_{3,4}$  and skin thickness  $z$  is highly desirable. These quantities are crucial for determining the enhancement factor within the classical geometric model, which, despite its shortcomings, is still of interest and represents an alternative to the  $d$ -wave halo model.

Finally, we argued that  $K_{\text{em}} \sim K_s$ , so our results apply to new physics coupled to either the EM or strong sectors. This holds unless the EM and nuclear energy differences between the ground and isomer states of thorium-229 have the exact same dependence on  $\alpha_s$ . EFT approaches to the expansion of nuclear forces in the QCD scale predict different dependencies on this scale [67,68]. Verifying this within concrete frameworks is key. We leave this and other developments for future work.

*Data availability.* The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

- 
- [1] J. Preskill, M. B. Wise, and F. Wilczek, Cosmology of the invisible axion, *Phys. Lett. B* **120**, 127 (1983).
- [2] L. F. Abbott and P. Sikivie, A cosmological bound on the invisible axion, *Phys. Lett. B* **120**, 133 (1983).
- [3] M. Dine and W. Fischler, The not so harmless axion, *Phys. Lett. B* **120**, 137 (1983).
- [4] A. Hook, TASI lectures on the strong CP problem and axions, *PoS TASI 2018*, 004 (2019).
- [5] L. Di Luzio, M. Giannotti, E. Nardi, and L. Visinelli, The landscape of QCD axion models, *Phys. Rept.* **870**, 1 (2020).
- [6] A. Arvanitaki, J. Huang, and K. Van Tilburg, Searching for dilaton dark matter with atomic clocks, *Phys. Rev. D* **91**, 015015 (2015).
- [7] J. Hubisz, S. Ironi, G. Perez, and R. Rosenfeld, A note on the quality of dilatonic ultralight dark matter, *Phys. Lett. B* **851**, 138583 (2024).
- [8] A. Banerjee, H. Kim, and G. Perez, Coherent relaxion dark matter, *Phys. Rev. D* **100**, 115026 (2019).
- [9] A. Chattrchyan and G. Servant, Relaxion dark matter from stochastic misalignment, *J. Cosmol. Astropart. Phys.* **2023**, 036 (2023).
- [10] F. Piazza and M. Pospelov, Sub-eV scalar dark matter through the super-renormalizable Higgs portal, *Phys. Rev. D* **82**, 043533 (2010).
- [11] M. Dine, G. Perez, W. Ratzinger, and I. Savoray, Nelson-Barr ultralight dark matter, *Phys. Rev. D* **111**, 015049 (2025).
- [12] V. V. Flambaum and A. F. Tedesco, Dependence of nuclear magnetic moments on quark masses and limits on temporal variation of fundamental constants from atomic clock experiments, *Phys. Rev. C* **73**, 055501 (2006).
- [13] T. Damour and J. F. Donoghue, Phenomenology of the equivalence principle with light scalars, *Class. Quantum Grav.* **27**, 202001 (2010).
- [14] T. Damour and J. F. Donoghue, Equivalence principle violations and couplings of a light dilaton, *Phys. Rev. D* **82**, 084033 (2010).
- [15] D. Antypas *et al.*, New horizons: Scalar and vector ultralight dark matter, [arxiv:2203.14915](https://arxiv.org/abs/2203.14915).
- [16] H. Kim and G. Perez, Oscillations of atomic energy levels induced by QCD axion dark matter, *Phys. Rev. D* **109**, 015005 (2024).
- [17] A. Hook and J. Huang, Probing axions with neutron star inspirals and other stellar processes, *J. High Energy Phys.* **06** (2018) 036.
- [18] R. Balkin, J. Serra, K. Springmann, S. Stelzl, and A. Weiler, White dwarfs as a probe of exceptionally light QCD axions, *Phys. Rev. D* **109**, 095032 (2024).
- [19] A. Banerjee, G. Perez, M. Safronova, I. Savoray, and A. Shalit, The phenomenology of quadratically coupled ultra light dark matter, *J. High Energy Phys.* **10** (2023) 042.
- [20] Y. V. Stadnik and V. V. Flambaum, Can dark matter induce cosmological evolution of the fundamental constants of nature? *Phys. Rev. Lett.* **115**, 201301 (2015).
- [21] A. Hees, J. Guéna, M. Abgrall, S. Bize, and P. Wolf, Searching for an oscillating massive scalar field as a dark matter candidate using atomic hyperfine frequency comparisons, *Phys. Rev. Lett.* **117**, 061301 (2016).

- [22] C. J. Kennedy, E. Oelker, J. M. Robinson, T. Bothwell, D. Kedar, W. R. Milner, G. E. Marti, A. Derevianko, and J. Ye, Precision metrology meets cosmology: Improved constraints on ultralight dark matter from atom-cavity frequency comparisons, *Phys. Rev. Lett.* **125**, 201302 (2020).
- [23] W. M. Campbell, B. T. McAllister, M. Goryachev, E. N. Ivanov, and M. E. Tobar, Searching for scalar dark matter via coupling to fundamental constants with photonic, atomic and mechanical oscillators, *Phys. Rev. Lett.* **126**, 071301 (2021).
- [24] T. Kobayashi *et al.*, Search for ultralight dark matter from long-term frequency comparisons of optical and microwave atomic clocks, *Phys. Rev. Lett.* **129**, 241301 (2022).
- [25] X. Zhang, A. Banerjee, M. Leyser, G. Perez, S. Schiller, D. Budker, and D. Antypas, Search for ultralight dark matter with spectroscopy of radio-frequency atomic transitions, *Phys. Rev. Lett.* **130**, 251002 (2023).
- [26] N. Sherrill *et al.*, Analysis of atomic-clock data to constrain variations of fundamental constants, *New J. Phys.* **25**, 093012 (2023).
- [27] A. Banerjee, D. Budker, M. Filzinger, N. Huntemann, G. Paz, G. Perez, S. Porsev, and M. Safronova, Oscillating nuclear charge radii as sensors for ultralight dark matter, [arxiv:2301.10784](https://arxiv.org/abs/2301.10784).
- [28] I. Kozyryev, Z. Lasner, and J. M. Doyle, Enhanced sensitivity to ultralight bosonic dark matter in the spectra of the linear radical sroh, *Phys. Rev. A* **103**, 043313 (2021).
- [29] E. Madge, G. Perez, and Z. Meir, Prospects of nuclear-coupled-dark-matter detection via correlation spectroscopy of  $I_2^+$  and  $Ca^+$ , *Phys. Rev. D* **110**, 015008 (2024).
- [30] R. Oswald *et al.*, Search for dark-matter-induced oscillations of fundamental constants using molecular spectroscopy, *Phys. Rev. Lett.* **129**, 031302 (2022).
- [31] M. S. Safronova, D. Budker, D. DeMille, D. F. J. Kimball, A. Derevianko, and C. W. Clark, Search for new physics with atoms and molecules, *Rev. Mod. Phys.* **90**, 025008 (2018).
- [32] E. Peik and C. Tamm, Nuclear laser spectroscopy of the 3.5 eV transition in Th-229, *Europhys. Lett.* **61**, 181 (2003).
- [33] C. J. Campbell, A. G. Radnaev, A. Kuzmich, V. A. Dzuba, V. V. Flambaum, and A. Derevianko, Single-ion nuclear clock for metrology at the 19th decimal place, *Phys. Rev. Lett.* **108**, 120802 (2012).
- [34] G. A. Kazakov, A. N. Litvinov, V. I. Romanenko, L. P. Yatsenko, A. V. Romanenko, M. Schreitl, G. Winkler, and T. Schumm, Performance of a  $^{229}\text{Th}$  orium solid-state nuclear clock, *New J. Phys.* **14**, 083019 (2012).
- [35] L. von der Wense and B. Seiferle, The  $^{229}\text{Th}$  isomer: Prospects for a nuclear optical clock, *Eur. Phys. J. A* **56**, 277 (2020).
- [36] S. Kraemer *et al.*, Observation of the radiative decay of the  $^{229}\text{Th}$  nuclear clock isomer, *Nature (London)* **617**, 706 (2023).
- [37] P. G. Thirolf, S. Kraemer, D. Moritz, and K. Scharl, The thorium isomer  $^{229m}\text{Th}$ : Review of status and perspectives after more than 50 years of research, *Eur. Phys. J. Spec. Top.* **233**, 1113 (2024).
- [38] J. Tiedau *et al.*, Laser excitation of the Th-229 Nucleus, *Phys. Rev. Lett.* **132**, 182501 (2024).
- [39] R. Elwell, C. Schneider, J. Jeet, J. E. S. Terhune, H. W. T. Morgan, A. N. Alexandrova, H. B. Tran Tan, A. Derevianko, and E. R. Hudson, Laser excitation of the  $^{229}\text{Th}$  Nuclear isomeric transition in a solid-state host, *Phys. Rev. Lett.* **133**, 013201 (2024).
- [40] C. Zhang *et al.*, Frequency ratio of the  $^{229m}\text{Th}$  isomeric transition and the  $^{87}\text{Sr}$  atomic clock, *Nature (London)* **633**, 63 (2024).
- [41] E. Fuchs, F. Kirk, E. Madge, C. Paranjape, E. Peik, G. Perez, W. Ratzinger, and J. Tiedau, Searching for dark matter with the  $^{229}\text{Th}$  nuclear lineshape from laser spectroscopy, *Phys. Rev. X* **15**, 021055 (2025).
- [42] V. V. Flambaum, Enhanced effect of temporal variation of the fine structure constant and the strong interaction in  $^{229}\text{Th}$ , *Phys. Rev. Lett.* **97**, 092502 (2006).
- [43] P. Fadeev, J. C. Berengut, and V. V. Flambaum, Sensitivity of  $^{229}\text{Th}$  nuclear clock transition to variation of the fine-structure constant, *Phys. Rev. A* **102**, 052833 (2020).
- [44] W. G. Rellergert, D. DeMille, R. R. Greco, M. P. Hehlen, J. R. Torgerson, and E. R. Hudson, Constraining the evolution of the fundamental constants with a solid-state optical frequency reference based on the  $^{229}\text{Th}$  nucleus, *Phys. Rev. Lett.* **104**, 200802 (2010).
- [45] M. S. Safronova, The search for variation of fundamental constants with clocks, *Ann. Phys.* **531**, 1800364 (2019).
- [46] E. Peik, T. Schumm, M. S. Safronova, A. Pálffy, J. Weitenberg, and P. G. Thirolf, Nuclear clocks for testing fundamental physics, *Quantum Sci. Technol.* **6**, 034002 (2021).
- [47] A. C. Hayes, J. L. Friar, and P. Möller, Splitting sensitivity of the ground and 7.6 eV isomeric states of  $^{229}\text{Th}$ , *Phys. Rev. C* **78**, 024311 (2008).
- [48] H. Banks, E. Fuchs, and M. McCullough, A nuclear interferometer for ultra-light dark matter detection, [arXiv:2407.11112](https://arxiv.org/abs/2407.11112).
- [49] B. Maheshwari and A. K. Jain, Nuclear isomers at the extremes of their properties, *Eur. Phys. J. Spec. Top.* **233**, 1101 (2024).
- [50] J. C. Berengut, V. A. Dzuba, V. V. Flambaum, and S. G. Porsev, A proposed experimental method to determine  $\alpha$  sensitivity of splitting between ground and 7.6 eV isomeric states in  $^{229}\text{Th}$ , *Phys. Rev. Lett.* **102**, 210801 (2009).
- [51] T. Naito, X. Roca-Maza, G. Colò, and H. Liang, Effects of finite nucleon size, vacuum polarization, and electromagnetic spin-orbit interaction on nuclear binding energies and radii in spherical nuclei, *Phys. Rev. C* **101**, 064311 (2020).
- [52] H. W. Hammer, C. Ji, and D. R. Phillips, Effective field theory description of halo nuclei, *J. Phys. G* **44**, 103002 (2017).
- [53] E. Browne and J. K. Tuli, Nuclear data sheets for  $A = 229$ , *Nucl. Data Sheets* **109**, 2657 (2008).
- [54] K. Abusaleem, Nuclear data sheets for  $A = 228$ , *Nucl. Data Sheets* **116**, 163 (2014).
- [55] V. A. Dzuba and V. V. Flambaum, Effects of electrons on nuclear clock transition frequency in  $^{229}\text{Th}$  ions, *Phys. Rev. Lett.* **131**, 263002 (2023).
- [56] R. D. Woods and D. S. Saxon, Diffuse surface optical model for nucleon-nuclei scattering, *Phys. Rev.* **95**, 577 (1954).
- [57] J. Thielking, M. V. Okhapkin, P. Glowacki, D. M. Meier, L. von der Wense, B. Seiferle, C. E. Düllmann, P. G. Thirolf, and E. Peik, Laser spectroscopic characterization of the nuclear-clock isomer  $^{229m}\text{Th}$ , *Nature (London)* **556**, 321 (2018).
- [58] H. De Vries, C. De Jager, and C. De Vries, Nuclear charge-density-distribution parameters from elastic electron scattering, *At. Data Nucl. Data Tables* **36**, 495 (1987).
- [59] M. S. Safronova, S. G. Porsev, M. G. Kozlov, J. Thielking, M. V. Okhapkin, P. Glowacki, D. M. Meier, and E. Peik, Nuclear charge radii of  $^{229}\text{Th}$  from isotope and isomer shifts, *Phys. Rev. Lett.* **121**, 213001 (2018).

- [60] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/129n-gt5j> for additional material on the classical geometrical model, i.e. density plot of the enhancement factor as a function of the deformability, and details about the charge radius, quadrupole moment and binding energy of the neutron in the Halo model.
- [61] J. Braun, H.-W. Hammer, and L. Platter, Halo structure of  $^{17}\text{C}$ , *Eur. Phys. J. A* **54**, 196 (2018).
- [62] J. Braun, W. Elkamhawy, R. Roth, and H. W. Hammer, Electric structure of shallow  $d$ -wave states in halo EFT, *J. Phys. G* **46**, 115101 (2019).
- [63] A. Ong, J. C. Berengut, and V. V. Flambaum, The effect of spin-orbit nuclear charge density corrections due to the anomalous magnetic moment on halo nuclei, *Phys. Rev. C* **82**, 014320 (2010).
- [64] J. J. Sakurai, *Modern Quantum Mechanics*, 2nd ed. (Addison-Wesley, Reading, MA, 1994).
- [65] V. Zelevinsky and A. Volya, *Physics of Atomic Nuclei* (Wiley-VCH, Weinheim, 2017).
- [66] W. N. Cottingham and D. A. Greenwood, *An Introduction to Nuclear Physics* (Cambridge University Press, Cambridge, 2001).
- [67] U.-G. Meißner, Anthropropic considerations in nuclear physics, *Sci. Bull.* **60**, 43 (2015).
- [68] H. W. Hammer, S. König, and U. van Kolck, Nuclear effective field theory: Status and perspectives, *Rev. Mod. Phys.* **92**, 025004 (2020).