

Bubble and spike reacceleration in nonlinear, single mode, two-dimensional Rayleigh-Taylor instability

A. R. Piriz^{1,*}, S. A. Piriz^{2,†}, N. A. Tahir³, and F. Cobos²

¹*Instituto de Investigaciones Energéticas (INEI), CYTEMA, Universidad de Castilla-La Mancha, 13071 Ciudad Real, Spain*

²*Instituto de Investigaciones Energéticas (INEI), E.I.I.A., and CYTEMA, Universidad de Castilla-La Mancha, 45071 Toledo, Spain*

³*GSI Helmholtzzentrum für Schwerionenforschung Darmstadt, Planckstrasse 1, 64291 Darmstadt, Germany*



(Received 8 March 2026; revised 11 August 2026; accepted 2 September 2026; published 23 September 2026)

A theoretical model is presented for the reacceleration process of the bubble and the spike that follows after the velocity saturation phase in the nonlinear, single-mode, two-dimensional Rayleigh-Taylor instability in ideal media. A previous irrotational model is extended to include the vorticity effects consistently with the existing evidence that vorticity drains momentum from the translational motion. Contradictions and paradoxes arising from the current paradigm based on the acceleration by the vortices centrifugal force are solved. The model shows that the time when the reacceleration occurs depends on the density ratio between both fluids, and that it is determined by achieving the corresponding final widths of the bubble and the spike, which thereafter remains constant.

DOI: [10.1103/px86-rwx6](https://doi.org/10.1103/px86-rwx6)

I. INTRODUCTION

Rayleigh-Taylor instability (RTI) is one of the most common hydrodynamic instabilities occurring in a wide variety of natural situations, as well as in laboratory conditions. It develops at the interface between two fluids of densities ρ_1 and $\rho_2 > \rho_1$, respectively, when gravity in the reference frame of the interface points from the heavier to the lighter fluid [1–3].

It is present in astrophysical [4,5] and geophysical [6,7] phenomena, and it plays a central role in inertial confinement fusion as one of the main factors determining the minimum energy necessary for the ignition of a fuel capsule and the subsequent burn propagation [8–13]. It is present from the first instants of the implosion process at the ablation front, until the final phase of slowing down at the interface between the hot spot and the surrounding cold fuel.

It is also key in the performance of high energy-density physics experiments [14–23], and it has been used as a unique method for studying the mechanical properties of elastic-plastic media under extreme conditions [24–28].

*Contact author: roberto.piriz@uclm.es

†Contact author: sofiaayelen.piriz@uclm.es

Although the intense theoretical, experimental, and numerical simulation research pursued mainly during the last eight decades, there are still many important aspects of the instability that are not well understood. This is probably because the hydrodynamic instabilities are not very accessible to the intuition, and the sophisticated mathematical, numerical, and experimental tools, although providing even exact descriptions of the phenomenon in many cases, are not always able to shed light on the underlying physical process that governs the instability. This is, in part, because the mere description may not allow for distinguishing the concomitance of certain processes from their causal relationship.

A paradigmatic example of this situation is the reacceleration of the bubble and the spike in the nonlinear, single-mode RTI taking place after a phase in which the velocity of the perturbation achieves a saturation value [29–31]. Observed for the first time in numerical simulations by Betti and Sanz [29], and Ramaprabhu *et al.* [30] in 2006, it was experimentally confirmed by Wilkinson and Jacobs [31] in 2007 and, since then, it has been reported in many research works [32–34]. However, the process has not yet been satisfactorily explained in physical terms.

In fact, a common argument claims that the bubble is accelerated by the centrifugal force due to the vorticity accumulated into the bubble and is based on the observed correlation between the presence of vorticity and the reacceleration process [29,30,32–37]. However, as we have already noticed in Ref. [38], by assuming that such correlation implies that vorticity is the direct cause of the bubble and spike reacceleration, we may run in a modern version of the Aristoteles “arrow paradox.” The latter arose 23 centuries ago when Aristoteles tried to explain the flight of an arrow once it separated from the propeller bow, by postulating that in its motion the arrow created a vacuum at the rear of the shaft that, when filled by the air coming from the front of the arrow (a vortice), pushed it ahead and kept it flying. Then, the paradox came up as follows: Why does the arrow not continue flying forever?

In the present context, by assuming that the centrifugal force created by the vortices can accelerate the bubble and the spike, we are driven into a similar (false) paradox. In fact, vorticity is produced by the shear due to the relative motion between the two fluids. If the centrifugal force causes the acceleration of such a motion, then it would further increase the shear, thus creating more vorticity and leading to a continuous increase of the bubble and spike acceleration. This is unphysical and, of course, not observed. Instead, under a constant gravity, experiments and simulations show that after the onset of reacceleration, bubble and spike accelerations are constants [39,40].

However, it does not mean that vorticity is not playing a role in the process, because it is a matter of fact that irrotational theories cannot explain the bubble and spike reacceleration following the saturation phase [38,41], and that the correlation between vorticity and reacceleration has been observed in numerical simulations and experiments [29–36].

On the other hand, recent results about the role of vorticity in the linear RTI in nonideal and/or finite thickness media could shed some light on its function in the present nonlinear problem [42–45]. In fact, it has been shown that, despite the considerable mathematical complexity introduced into the problem by the presence of vorticity, it has a rather insignificant physical effect on the dynamics of the instability and that, in any case, such a slight effect consists in reducing the growth rate, not in increasing it. This is because the generation of vorticity drains momentum (or kinetic energy) from the translational motion and transfers it to the rotational motion, so that the amplitude of the perturbation grows more slowly. The effect is small, but it seems to be contrary to the argument that the vorticity accelerates the growth of the instability.

In addition, such results seem to be consistent with the finding by Mikaelian that in the turbulent regime the rotational kinetic energy is only 2% of the translational kinetic energy [46]. However, a small amount of momentum diverted toward the rotational motion in no way implies that it cannot affect the instability evolution at late times. However, its effects should be compatible with a mechanism of momentum drainage and not with a process of input of momentum (or kinetic energy) that would directly accelerate the amplitude growth. Clearly, the only source of momentum, translational plus rotational, is the buoyancy force.

A correct comprehension of the nonlinear stage of single-mode RTI is an essential previous step for understanding the late evolution of the instability when the situation involves a broad unstable spectrum of modes, leading to the formation of a mixing layer [39,40,47–52]. Recently, we have presented an irrotational model for the nonlinear, two-dimensional RTI in ideal incompressible and immiscible media, which shows that the dynamics of the instability evolution is controlled by the masses of both fluids that are involved in the motion [38]. Such masses are, respectively, determined by two characteristic lengths $l_1 = c_1 \xi$ and $l_2 = c_2 k \xi^2$ that account, respectively, for the mass affected by the vertical motion and by the lateral motion, of the bubble and the spike ($k = 2\pi/\lambda$ is the wave number, λ is the wavelength, and ξ is the amplitude of the perturbation). These two lengths become relevant in the nonlinear regime, when the amplitude ξ is no longer negligible in comparison with k^{-1} and, thus, the treatment extends the linear model of Ref. [53] in which the affected mass was constant and determined by the unique characteristic length k^{-1} .

This irrotational theory yields asymptotic expressions for the saturation velocities of the bubble and the spike, respectively, when $k\xi_b \gg 1$ and $(1 - A_T)k\xi_s \gg 1$ (ξ_b and ξ_s are the bubble and spike amplitudes, and $A_T = (\rho_2 - \rho_1)/(\rho_2 + \rho_1)$ is the Atwood number), and shows that it is caused by the increase of the respective masses that are put in motion as a consequence of the instability evolution, including the continue lateral expansion (contraction) of the bubble (spike). This expansion (contraction) proceeds in such a manner that the bubble (spike) approaches asymptotically to its maximum (minimum) width corresponding to the respective Atwood number, so that in the irrotational approximation the lateral motion progressively slows down but never stops completely.

In this work, we model the effect of the vorticity by assuming, consistently with the above discussion, that the small amount of translational momentum that is drained by the vorticity actually causes the stoppage of the lateral motion at a finite time t_R , whereupon the bubble and the spike can only move vertically affecting an amount of mass determined by characteristic lengths ξ_b and ξ_s , respectively. These lengths are smaller than the ones that would dominate the process if lateral motion remains active ($k\xi_{b,s}^2$) and that determine the masses participating in the motion during the previous phase for $t < t_R$. Consequently, for $t > t_R$ less mass is affected by the motion, thus driving the acceleration of both bubble and spike.

Since during the early nonlinear phase ($t < t_R$) the dynamics of the instability is well described by the irrotational model of Ref. [38], in Sec. II we start with a brief overview of the main features of the model. In Sec. III, we present the new results by first introducing the equation for the time evolution of the bubble and the spike widths, and then extending the model of Sec. II for describing the behavior during the reacceleration phase ($t > t_R$). Further discussion and remarks are given in Sec. IV.

II. THE NONLINEAR SINGLE-MODE RTI: OVERVIEW OF THE IRROTATIONAL MODEL

We briefly review here the main results of the model in Ref. [38]. We start by considering the situation shown in Fig. 1(a) in which at $t = 0$ a fluid of density ρ_2 lies above a lighter one of density $\rho_1 < \rho_2$ in a uniform gravity field $g = g(t)$. When the interface separating them is submitted to a perturbation $\xi(x)$, a hydrostatic pressure difference $\Delta p = p'_2 - p'_1 = (\rho_2 - \rho_1)g\xi$ is created that drives the motion of the interface according to Newton's second law. During the initial linear phase, the equations of motion of peaks and valleys are coupled, and the mass m^L (per unitary area) participating in the motion due to the instability is constant and determined by the unique relevant characteristic length k^{-1} , so that $m^L = (\rho_1 + \rho_2)/k$ and the equation of motions reads [53]

$$\frac{d(m^L \dot{\xi})}{dt} = (\rho_2 - \rho_1)g\xi, \quad (1)$$

where ξ is the amplitude, either of the peaks or the valleys. This linear phase lasts until the time t_{NL} when the amplitude becomes $k\xi \sim 1$ [54]. At $t = t_{NL}$ starts the early nonlinear

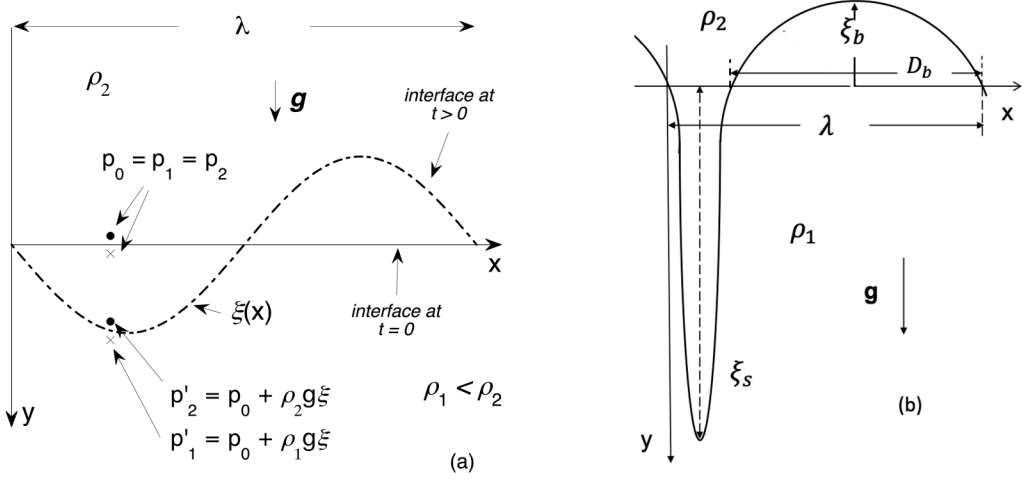


FIG. 1. Schematic of the perturbed interface between two fluids. (a) Early linear phase ($t < t_{NL}$). (b) Early nonlinear phase ($t_{NL} < t < t_R$).

phase [Fig. 1(b)] during which the dynamics of the resulting bubbles and spikes becomes controlled also by the lateral motion in addition to the vertical one. This feature leads to the decoupling of the peak and bubble motions and they must be described by two independent equations:

$$\frac{d(m_s^{NL} \dot{\xi}_s)}{dt} = (\rho_2 - \rho_1) g \xi_s, \quad (2)$$

$$\frac{d(m_b^{NL} \dot{\xi}_b)}{dt} = (\rho_2 - \rho_1) g \xi_b, \quad (3)$$

where ξ_b and ξ_s are the amplitudes of the bubble and the spike, respectively, and m_b^{NL} and m_s^{NL} are their respective masses involved in the motion during this phase [38]:

$$m_b^{NL} = \rho_2 C_2 k \xi_b^2 + \rho_1 C_1 \xi_b, \quad (4)$$

$$m_s^{NL} = \rho_1 C_2 k \xi_s^2 + \rho_2 C_1 \xi_s. \quad (5)$$

The linear and nonlinear regimes in this irrotational phase of the evolution can be described together by interpolating the corresponding masses:

$$\frac{d}{dt} \left\{ \left[\frac{\rho_2}{k} [1 + C_2 (k \xi_b)^2] + \frac{\rho_1}{k} \sqrt{1 + (C_1 k \xi_b)^2} \right] \dot{\xi}_b \right\} = (\rho_2 - \rho_1) g(t) \xi_b, \quad (6)$$

$$\frac{d}{dt} \left\{ \left[\frac{\rho_1}{k} [1 + C_2 (k \xi_s)^2] + \frac{\rho_2}{k} \sqrt{1 + (C_1 k \xi_s)^2} \right] \dot{\xi}_s \right\} = (\rho_2 - \rho_1) g(t) \xi_s, \quad (7)$$

where $C_1 = 1/3$ and $C_2 = 3/2$ for two-dimensional perturbations ($C_2 = 1/2$ for three-dimensional perturbations).

It is convenient to write the previous equations in dimensionless form by introducing the following dimensionless magnitudes:

$$z_{s,b} = k \xi_{s,b}, \quad T = t \sqrt{kg}. \quad (8)$$

Then, the evolution equations for the bubble and the spike amplitudes, respectively, read

$$\frac{d}{dT} \left\{ \left[(1 + A_T)(1 + C_2 z_b^2) + (1 - A_T) \sqrt{1 + (C_1 z_b)^2} \right] \dot{z}_b \right\} = 2A_T z_b, \quad (9)$$

$$\frac{d}{dT} \left\{ [(1 - A_T)(1 + C_2 z_s^2) + (1 + A_T)\sqrt{1 + (C_1 z_s)^2}] \dot{z}_s \right\} = 2A_T \dot{z}_s. \quad (10)$$

We can easily obtain the asymptotic solutions. For the bubble, when $z_b \gg 1$, we get

$$z_{b\infty} = \sqrt{\frac{A_T}{C_2(1 + A_T)}} T. \quad (11)$$

Instead, for the spike when $z_s \gg 1$ we must take into account the value of A_T . If it is not too close to $A_T = 1$, then it is $(1 - A_T)z_s \gg 1$, and it turns out that [38,41]

$$z_{s\infty} = \sqrt{\frac{A_T}{C_2(1 - A_T)}} T. \quad (12)$$

In the opposite case, when A_T is sufficiently close to 1, it will be $(1 - A_T)z_s \ll 1$ even for large values of z_s . Then, we get

$$z_{s\infty}(A_T \sim 1) = \frac{A_T}{3C_1(1 + A_T)} T^2. \quad (13)$$

Equations (11) and (12) show the velocity saturation of the bubble and the spike, respectively, and that it occurs at progressively later times as A_T approaches to 1. In the limit, when A_T is very close to 1, the spike velocity grows linearly in time as it results from Eq. (13), which correctly yields the spike free fall for $A_T = 1$.

III. REACCELERATION PHASE

A. Bubble width

We can describe the evolution of the width D_b of the bubble and the width $D_s = \lambda - D_b$ of the spike from the model presented above. In fact, because of mass conservation, the areas of the bubble and the spike must be the same [55]:

$$\beta_b \xi_b D_b = \beta_s \xi_s (\lambda - D_b), \quad (14)$$

where β_b and β_s are shape factors that account for the difference between the real areas and the ones corresponding to perfect rectangles. For simplicity and without too much error, we can take $\beta_b \approx \beta_s$ and then it turns out that

$$\frac{D_b}{\lambda} = \frac{1}{1 + \frac{z_b}{z_s}}. \quad (15)$$

In Fig. 2, we show the time evolution of D_b/λ resulting from the introduction of the solutions of Eqs. (9) and (10) for z_b and z_s into Eq. (15) [with $z_0 = z_b(t=0) = z_s(t=0) = 0.01$ and $\dot{z}_0 = \dot{z}_b(t=0) = \dot{z}_s(t=0) = 0$]. We can see that the bubble width grows continuously and approaches asymptotically to the value

$$\frac{D_{b\infty}}{\lambda} = \frac{1}{1 + \sqrt{\frac{1-A_T}{1+A_T}}}, \quad (16)$$

such as it is determined by the asymptotic amplitudes given by Eqs. (11) and (12). That is, in the irrotational model, the bubble (spike) expands (contracts) continuously achieving its asymptotic value corresponding to a given Atwood number A_T at an infinite time, moving progressively more slowly but never stopping. Consequently, the bubble and the spike approach to their asymptotic velocities given by Eqs. (11) and (12), provided that $A_T \neq 1$.

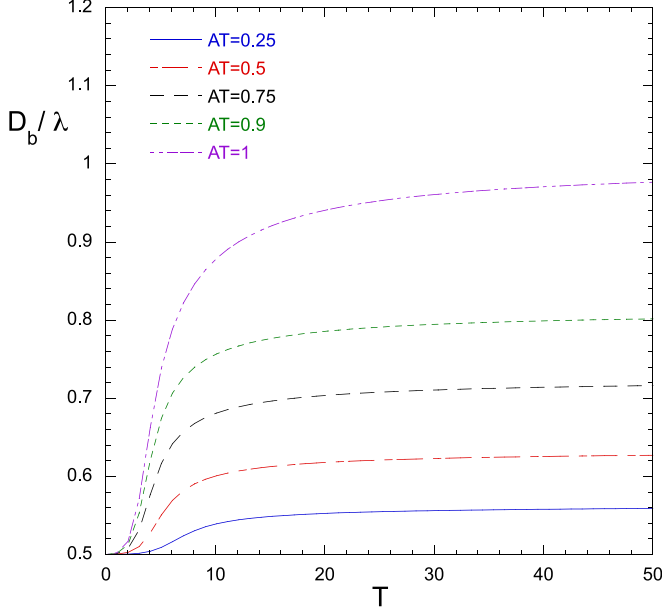


FIG. 2. Normalized width D_b/λ of the bubble as a function of time for different values of the Atwood number, given by the irrotational model.

B. Effect of the vorticity

The behavior of the bubble described above is clearly unphysical because the maximum bubble width must be achieved in a finite time. In fact, it is direct consequence of neglecting the shear generated vorticity between the bubble and the spike, which drains part of the momentum that in the irrotational situation is transferred only to translational motion, and diverts it to the rotational motion of the vortices.

The only source of total momentum is indeed the driving force $(\rho_2 - \rho_1)g\xi$, and the part going to the vorticity must be taken from the translational motion of the bubble and spike. Notice that this process is just the opposite to the one frequently claimed. In fact, momentum is transferred from translational to rotational motion, and not the other way around.

In the same manner, the asymptotic approach to the terminal velocity of the bubble, and of the spike for $A_T < 1$, yielded by the irrotational model turns out from the absence of vorticity. Therefore, we can model the effect of the vorticity by considering that the small amount of momentum taken by the rotational motion is drained from the translational motion, thus producing the stopping of the lateral bubble (spike) expansion (contraction) at a finite time T_R . In other words, the extra translational momentum that in the irrotational case is available for $T > T_R$ is absorbed by the rotational motion. As a consequence, the maximum vertical velocities of the bubble and the spike will be achieved at a finite time T_R with values somewhat lower than their asymptotic ones given by Eqs. (11) and (12):

$$\dot{z}_b(T = T_R) = f_b \dot{z}_{b\infty}, \quad (17)$$

$$\dot{z}_s(T = T_R) = f_s \dot{z}_{s\infty}, \quad (18)$$

where $f_{b,s}$ are fractions of the irrotational asymptotic velocities, smaller but very close to 1, indicating that only a small fraction of the total momentum is assumed to be drained by the vorticity.

In addition, since the irrotational asymptotic velocity of the bubble is reached earlier than that of the spike, it is also $f_s < f_b \sim 1$. Consequently, the width D_b of the bubble remains constant for

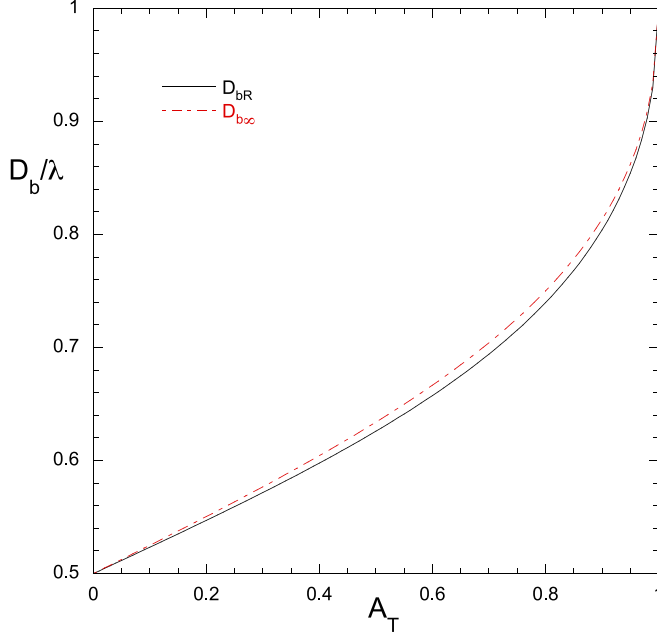


FIG. 3. Normalized asymptotic bubble widths $D_{b\infty}/\lambda$ and D_{bR}/λ at $T \geq T_R$, as a function of the Atwood number A_T .

$T \geq T_R$ with the value D_{bR} :

$$\frac{D_{bR}}{\lambda} = \frac{1}{1 + \frac{z_{bR}}{z_{sR}}} = \frac{1}{1 + \alpha \sqrt{\frac{1-A_T}{1+A_T}}}, \quad (19)$$

where we have used that $z_{bR} = z_b(T_R) = \alpha z_{sR} \sqrt{(1-A_T)/(1+A_T)}$, with $\alpha = f_b/f_s$ being somewhat larger than 1, and $z_{sR} = z_s(T_R)$.

For simplicity, we can take $f_b \approx 1$. Then, since for $A_T = 0$ it must be $D_{bR} = \lambda/2$ and $f_s \approx 1$, we define

$$\alpha \approx 1/f_s \equiv 1 + \epsilon A_T, \quad (20)$$

where ϵ is a parameter that determines the precise instant when the lateral motion stops and, consistently with the assumption that only a small fraction of the total momentum is transferred to the vortices for $T > T_R$, it must be $\epsilon \ll 1$ [46]. Thus, the net effect of the vorticity is contained in the ϵ parameter, which takes back to the irrotational situation when $\epsilon = 0$.

Figure 3 shows D_{bR} for $T \geq T_R$ and $D_{b\infty}$ as a function of A_T for $\epsilon = 0.07$. The exact value of ϵ is irrelevant for the present purpose, provided that it is a small number, but we have chosen it in order to have $\xi_{bR} \sim \lambda$ for $A_T \leq 0.9$, as observed in numerical simulations [30]. The small difference between the two values of D_b is consistent with a small amount of momentum drained by the vorticity from the translational motion of the spike and transferred to the rotational motion of the vortices generated by the shear and the resulting Kelvin-Helmholtz instability.

Then, since $D_b = D_{bR}$ for $T \geq T_R$, the equations of motion for bubble and spike given by Eqs. (6) and (7) are not independent any longer; they become coupled, and then we have

$$\frac{z_b(T)}{z_s(T)} = \frac{z_{bR}}{z_{sR}}. \quad (21)$$

Notice that hereafter $z_{b,s}$ denote the values for $T \geq T_R$.

As we have already mentioned, in this regime, $\xi_{b,s}$ become the only characteristic lengths determining the masses participating in the motion of the bubble and the spike, respectively, due to the evolution of the instability. That is, the characteristic lengths $l_{2b,s} = C_2 k \xi_{v,s}^2$ become, respectively,

$$l_{2b} = C_2 z_{bR} \xi_b, \quad l_{2s} = C_2 z_{sR} \xi_s. \quad (22)$$

Then, for $T \geq T_R$, the new mass incorporated to the motion turns out to be smaller than the one in the irrotational case. That is, since $z_{b,sR}$ are smaller than the irrotational values $z_{b,s}$ for $T > T_R$, less mass has to be pushed, leading to the acceleration of the bubble and the spike.

In Ref. [38], it was assumed on empirical basis that $z_{bR} = z_{sR} = 2\pi$, and the motion of the bubble and the spike remained independent for $T > T_R$, which is inconsistent with Eq. (15). This led to results in disagreement with numerical simulations and experiments for the highest values of the Atwood number A_T .

Here, instead, the constants z_{bR} and z_{sR} are determined from the momentum conservation equations, together with T_R for $T \geq T_R$, and T_R occurring in the nonlinear regime. The masses participating in the motion to be used in Eqs. (9) and (10) are calculated using Eq. (22). The resulting equations are formally similar to the ones used in Ref. [38], but now the values of z_{bR} and z_{sR} are consistently determined in order to keep constant the bubble and spike widths for $T > T_R$:

$$\frac{d}{dT}(C_{b3} z_b \dot{z}_b) = A_T z_b, \quad (23)$$

$$\frac{d}{dT}(C_{s3} z_s \dot{z}_s) = A_T z_s, \quad (24)$$

where

$$C_{b3} = \frac{1 - A_T}{2} C_1 + z_{bR} \frac{1 + A_T}{2} C_2, \quad (25)$$

$$C_{s3} = \frac{1 + A_T}{2} C_1 + z_{sR} \frac{1 - A_T}{2} C_2. \quad (26)$$

The asymptotic solutions of the previous equations read

$$z_b = \frac{A_T}{6C_{b3}} T^2, \quad z_s = \frac{A_T}{6C_{s3}} T^2, \quad (27)$$

and since D_b is constant, from Eq. (15) and using Eqs. (11) and (12), we get

$$\frac{z_b}{z_s} = \frac{z_{bR}}{z_{sR}} = \frac{C_{s3}}{C_{b3}} = \alpha \sqrt{\frac{1 - A_T}{1 + A_T}}. \quad (28)$$

Introducing Eqs. (25) and (26) into Eq. (30) and solving for z_{sR} , after some short algebra, we obtain

$$z_{sR} = \frac{C_1}{C_2} \frac{1 + A_T - \alpha(1 - A_T) \sqrt{\frac{1 - A_T}{1 + A_T}}}{(\alpha^2 - 1)(1 - A_T)}, \quad (29)$$

and using the relationship given again by Eq. (28),

$$z_{bR} = \frac{C_1}{C_2} \frac{\alpha}{(\alpha^2 - 1)} \frac{1 + A_T - \alpha(1 - A_T) \sqrt{\frac{1 - A_T}{1 + A_T}}}{\sqrt{1 - A_T^2}}. \quad (30)$$

Figure 4(a) shows z_{bR} and z_{sR} as a function of the Atwood number A_T . It is seen that both threshold amplitudes grow with A_T and achieve very high values for A_T approaching to 1, thus making it more difficult to observe reacceleration for such large values of A_T because of the finite time available. This behavior of z_{bR} and z_{sR} is in agreement with the experimental and numerical simulation results

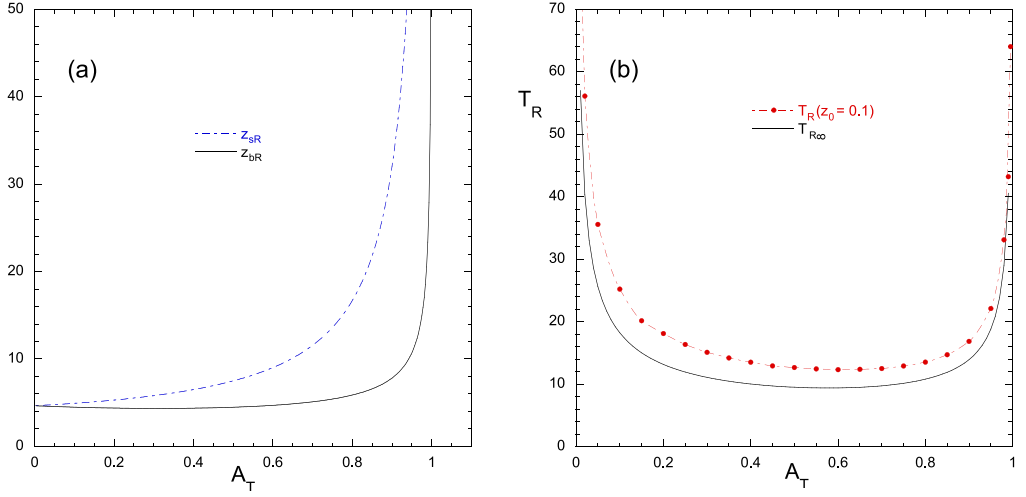


FIG. 4. (a) Dimensionless amplitudes z_{bR} of the bubble and z_{sR} of the spike at the reacceleration time T_R as a function of the Atwood number A_T . (b) Reacceleration time $T_R(z_0 = 0.1)$ for an initial amplitude $z_0 = 0.1$, and $T_{R\infty}$ obtained from the asymptotic solution.

[30,31] and is due to the smaller role of vorticity as A_T approaches to 1. In the same manner, and for the same reason, we can see in Fig. 4(b) that the reacceleration time T_R also grows to infinity for A_T approaching to 1.

In particular, z_{bR} does not change too much for $A_T \leq 0.8-0.9$, going to infinity for $A_T = 1$, consistently with the available evidence [33,34]. This trait was at the basis of the assumption made in Ref. [38] of taking $z_{bR} = 2\pi$ for any value of A_T , which is seen here to be approximately valid only for the intermediate and low values of A_T .

On the other hand, T_R also grows considerably for small values of A_T because it takes more time for the instability to enter in the nonlinear regime. This feature can be seen in Fig. 4(b), where T_R is shown as a function of A_T . Since the reacceleration time depends on the time the instability has evolved in the linear regime, we also show T_R numerically calculated for $z_0 = 0.1$, at the time when z_b (z_s), given by Eqs. (9) and (10), is equal to z_{bR} (z_{sR}), given by Eq. (30) [Eq. (29)].

In Fig. 4(b), we also show $T_{R\infty}$, the value obtained from the asymptotic analytic expression given by Eq. (12):

$$T_{R\infty} \approx \alpha_{z_{sR}} \sqrt{\frac{C_2(1 - A_T)}{A_T}}, \quad (31)$$

with z_{sR} given by Eq. (29), and assuming that A_T is not too close to 1, when Eq. (12) is not the appropriate asymptotic expression for the spike amplitude any longer.

IV. DISCUSSION AND CONCLUDING REMARKS

The bubble and spike reacceleration process taking place after a saturation phase of the perturbation velocities in the early nonlinear regime is known from numerical simulations and experimental evidence [31–37,39,40] and was observed around 20 years ago [29,30]. However, the explanations proposed so far, based on the existing correlation between reacceleration and the presence of vorticity, are not free of contradictions and lead to paradoxes that evidence its unphysical character.

In fact, current claims that bubble is accelerated by the “centrifugal force” of the vortices resemble the ancient arrow paradox and clearly reversed the cause-effect relationship, since it is the momentum originated by the buoyancy force what drives the translational motion, producing

the shear and giving place to the vortices. That is, momentum is transferred from the translational to the rotational motion and not the other way around.

Similarly, the argument that vorticity reduces the “friction” between bubbles and spikes implies to invoke the existence of “friction” in ideal fluids for describing the instability for $T > T_R$. Thus, although the rise of vorticity generated by the Kelvin-Helmholtz instability plays certainly a role, it consists in taking away a small amount of momentum from the vertical motion, sufficient for stopping the lateral motion of the bubble (spike) at a finite time T_R , but otherwise negligible in comparison with the total momentum of the system [46].

The irrotational model valid for $T < T_R$ decouples the equations of motion for the bubble and the spike, allowing for a lateral motion in which the bubble and spike slow down continuously without ever stopping, and approaching asymptotically to its final value corresponding to a given Atwood number. Consequently, the growth of the mass involved in the motion due to the instability leads to the asymptotic bubble and spike (for $A_T < 1$) saturation of their velocities.

When the lateral motion ceases due to the vorticity, bubble and spike can only move vertically, affecting the mass within a characteristic length proportional to $\xi_{b,s}$. As it was discussed in Ref. [38], in the linear phase it is $k\xi \ll 1$ and the only characteristic length is k^{-1} , when it enters in the early nonlinear phase, the motion is governed by the two characteristic lengths ξ and $k\xi^2$, and in the late nonlinear phase, when $k\xi \gg 1$, the only characteristic length is the amplitude ξ . These characteristic lengths determine, respectively, the masses involved in the motion in the different evolution phases.

Thus, in this later phase, the mass participating in the motion becomes less than that in the irrotational regime and, by pushing less mass, the motion accelerates. In addition, since for $T \geq T_R$ the bubble and the spike widths remain constants, their equations of motion become coupled again, as it happens in the linear regime.

The resulting reacceleration time T_R , as well as the corresponding bubble and spike amplitudes z_{bR} and z_{sR} , grow without limit when the Atwood number approaches to $A_T = 1$. Then, the observation of bubble and spike reacceleration in numerical simulations and experiments requires of growing resolution and available time as larger is A_T . In this regard, it is noticed that, in determining the space resolution, the region ahead of the spike of size on the order of ξ_s must be taken into account, since it contains fluid mass affected by the motion of the spike.

In addition, z_{bR} turns out to be rather insensitive to the Atwood number value in the small and intermediate range, which is also in agreement with the existing evidence [33].

In the same manner, the model predicts a constant acceleration for $T > T_R$, as observed in the numerical simulations [30,33]. The linear growth with time of the bubble and the spike velocities lead, of course, to the consequent increase of shear and vorticity generation. However, always diverting a fraction of the spike and bubble momentum toward the rotational motion, the increasing shear does not produce a continuous grow of the acceleration, as would happen if the acceleration were driven by the centrifugal force created by the vortices.

Therefore, the scenario depicted in this work, besides agreeing with the available evidence (constant acceleration, and $T_R \rightarrow \infty$ for $A_T \rightarrow 1$), is physically self-consistent and free of the conceptual flaws of previous proposals that invoked the existence of “vortices centrifugal force” reversing the causality relationship, or of some kind of “friction” in ideal media, that would drive the reacceleration process.

Finally, we notice that the model has a single parameter ϵ for modeling the effects of the vorticity that determines the maximum velocities of the bubble and the spike before the reacceleration, which are somewhat smaller than the asymptotic values predicted by the irrotational theories, and are achieved at a finite time. This parameter can be related to the vorticity Ω_0 generated for $T \geq T_R$, which is $\Omega_0 \propto (\dot{z}_b + \dot{z}_s)$ [29]. From Eq. (27), we have

$$\Omega_0 \propto \left(\frac{1}{C_{b3}} + \frac{1}{C_{s3}} \right) \sim \epsilon T, \quad (32)$$

where we have used Eqs. (25) and (26) for C_{b3} and C_{s3} , respectively, and Eqs. (29) and (30) for z_{bR} and z_{sR} , respectively. Then, vorticity turns out proportional to ϵ consistently with irrotationality

when $\epsilon = 0$ and grows linearly with time following the shear increase. Besides, Ω_0 vanishes for $A_T \rightarrow 1$ ($z_{b,sR} \rightarrow \infty$).

Unfortunately, is not possible at present to determine ϵ with precision by comparing with numerical simulations because they present a great spread in the values of the magnitudes that could be used with such a purpose, which may be caused by numerical viscosity, resolution, miscibility of the considered fluids, etc. This feature can be appreciated in Fig. 12 of Ref. [56], where the time evolution of the growth rate parameter α_b is shown, as given by the numerical simulations performed with ten different three-dimensional codes.

The late asymptotic value of $\alpha_b \equiv \alpha_{b\infty}$ can be associated with the parameter ϵ through Eq. (27): $\alpha_{b\infty} = 1/(6C_{b3})$. In fact, the evolution of the multimode RTI progresses by an inverse cascade process (bubble merging and bubble competition) for which the dominant wavelength λ continuously grows. Then, provided that sufficient time is available, the minimum mode $n = 1$ corresponding to $\lambda = L$ (L is the lateral size of the system) will be achieved at a certain time $t = t_e$. From then on, the system will evolve as the single mode $n = 1$ reaccelerating at $t = t_R$ and evolving according to Eq. (27), that would correspond with the asymptotic value $\alpha_{b\infty}$ shown in Ref. [56].

Roughly speaking, $\alpha_{b\infty} \propto \epsilon$, and the uncertainty on a factor of the order of 2 or 3 for $\alpha_{b\infty}$ in the numerical simulations is translated to the parameter ϵ . Perhaps in the future, the availability of more precise numerical simulations or the development of better models could delimit its value more rigorously. However, in the meantime, it allows for presenting a more consistent and satisfactory physical picture than the existing one, and opens a door for studying the influence of the reacceleration process in the nonlinear broadband multimode RTI, which so far has not been considered.

ACKNOWLEDGMENTS

This work has been partially supported by MICIU/AEI/10.13039/501100011033 and FEDER, EU (Grant No. PID2024-157933OA-I00), and by UCLM (Grant No. 2025-GRIN-38368) of Spain, by the BMBF of Germany, and by the EUROfusion Enabling Research Project No. CfP-FSD-AWP26-ENR-01 ‘‘Conceptual design for a European High Power Laser Fusion Research Facility’’ (HiPER+RF), funded by the European Union via the Euratom Research and Training Programme (Grant Agreement No. 101052200—EUROfusion).

DATA AVAILABILITY

The data that support the findings of this article are openly available [57].

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