New approach for the induced charge calculation for cylindrical electrodes[*]

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The HITRAP facility is part of the GSI accelerator complex and it is designed to store and cool up to 10^5 bare uranium ions. One of the cooling mechanisms is the resistive cooling where energy is dissipated in an external RLC circuit, which is connected to the so called pick-up electrodes. These electrodes pick up the induced charge and the related current caused by the ion dynamics and give a feedback to the ion dynamics due to an imbalance of the potential on these electrodes. But since there exists no theoretical treatment of resistive cooling which is able to accurately predict cooling time and cooling behavior numerical simulations are necessary. In [1] we presented a model for the simulation of resistive cooling of ion clouds in cylindrical penning traps, which includes the complete ion-ion interaction. To handle the large computational effort this is done on graphic boards using parallel programming techniques [2]. The induced current is the time derivative of the induced charge and can be calculated by a finite element solver. This allows to model precisely the real geometry. But due to the increasing number of necessary elements, the simulation time increases strongly with the required numerical accuracy, and this method gets inefficient for a time discrete integration scheme in which these calculations must be repeated some billion of times. Based on the solution of the Poisson equation, we therefore worked out an analytical formula for the charge Q which is induced on the electrodes by an ion:

$$Q = \frac{q}{\pi} \int_0^\infty \frac{I_0\left(v \cdot \frac{r}{R}\right)}{v \cdot I_0(v)} \cdot \sum_{j=0}^{1} (-1)^j \sin\left(v \cdot \frac{\Delta z_j}{R}\right) dv \quad (1)$$

Here, I_0 is the modified Bessel function of order zero, R is the trap radius, $\Delta z_j = z - z_j$ with z_0 and z_1 are the left and right axial coordinate of the pick-up electrode and q is the charge of the ion at position $(r, \ z)$. The total charge induced on the pick-up electrode caused by an ion cloud is calculated by superimposing the contributions of all single ions. Since we did not find an analytical solution of (1) some further simplification is needed to ensure a fast evaluation. Approximating now the term $I_0\left(v \cdot \frac{r}{R}\right)/I_0(v)$ in expression (1) leads to an analytical formula which allows fast calculation and provides high accuracy:

$$Q \approx \frac{q}{2} \sum_{k=0}^{k_{max}} \beta_k \sum_{j=0}^{1} (-1)^j \left[\mathrm{erf}\left(\frac{\Delta z_j}{\gamma_k}\right) + \frac{r^2 \Delta z_j}{\sqrt{\pi}\, \gamma_k^3} e^{-\left(\frac{\Delta z_j}{\gamma_k}\right)^2} \right]$$

Here erf is the error function, k_{max} the order of approximation, $\gamma_k = 2R\alpha_k$, α_k and β_k are numerical parameters.

The calculated charge induced on single electrodes is shown in figure 1. The approximation error depends on
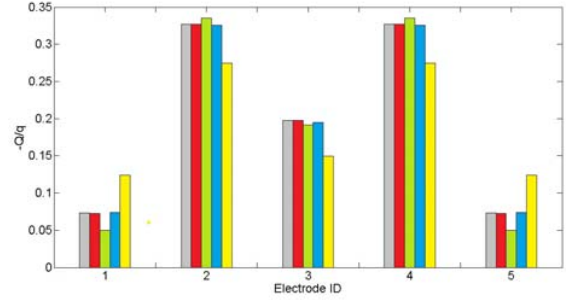

Figure 1: Induced charged at five cylindrical electrodes. The bins from left to right: Analytical solution (gray), approx. $k_{max} = 2$ (red), approx. $k_{max} = 1$ (green), Comsol (blue), calculation scheme of Ref.[3] (yellow). Electrode 2 and 4 are the pick-up electrodes

the choice of k_{max}, see figure 2. Already at $k_{max} = 2$ the results of (1) and the analytical approximation are in good agreement. The resulting absolute error for the pick-up electrodes of less than $5 \cdot 10^{-3}$ will be sufficiently accurate for our simulations.
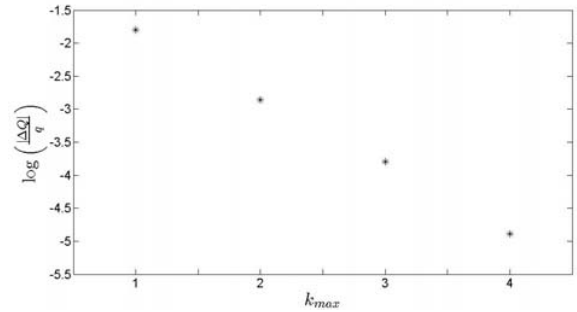

Figure 2: Approximation error per unit charge versus the order of approximation k_{max}

References

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