On the measured lifetime of light hypernuclei $_\Lambda^3$H and $_\Lambda^4$H

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A B S T R A C T

A statistical combination of the experimental lifetime estimations available in the literatures is performed for $_\Lambda^3$H and $_\Lambda^4$H, including several recent measurements. The combined average values of the lifetime for $_\Lambda^3$H and $_\Lambda^4$H are respectively 216^{+19}_{-16} ps and 192^{+20}_{-18} ps with a reduced χ^2 of 0.89 and 0.48. A new insight into the lifetime estimation of the HypHI Phase 0 experiment by a Bayesian approach is also presented. In this approach, several different prior distributions including the combination of previous lifetime data and a Jeffrey prior are used. The principal mode and the smallest credible interval at 68% of the posterior distribution, given by the prior belief of the previous measurements, are 217^{+19}_{-16} ps and 194^{+20}_{-18} ps respectively for $_\Lambda^3$H and $_\Lambda^4$H. The two employed approaches have revealed that the lifetime of hypernuclei $_\Lambda^3$H and $_\Lambda^4$H can be shorter than the Λ–hyperon lifetime.

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1. Introduction

The study of the lifetime of heavier baryons than the nucleons can reveal the nature of the weak interaction being responsible for the flavor conversion and the strong interaction which governs the structure of the quarks and gluons within the baryons. The lightest one of such baryons is the Λ–hyperon which consists of up-, down- and strange-quarks. The lifetime of the Λ–hyperon has been measured very accurately over the years to be 263 ± 2 ps [1].

Thanks to the long lifetime of Λ in comparison with the typical lifetime of resonance states decaying via the strong interaction, the study of the nucleon–Λ interactions has been possible by studying a Λ–hypernucleus, a bound state of nucleons and the Λ–hyperon, though it has not been practical to study these interactions by

means of scattering experiments. Additionally the study of the deviation of the lifetime of Λ–hypernuclei from the lifetime value of the free Λ–hyperon could exhibit the modification of the Λ wave-function in the nuclear medium. It would especially strike new insight on their fundamental structures, though some light Λ–hypernuclei are considered to have a Λ–hyperon weakly bound to their core nucleus. Full understanding of light Λ–hypernuclei has to be intended, and theoretical models should be able to describe simultaneously the binding energy, the dynamics of the weak decay, the value of the lifetime, and the decay modes.

Several experiments estimated the lifetime of the light hypernuclei, $_\Lambda^3$H [2–9] and $_\Lambda^4$H [2,4,9–13]. At first, the lifetime values were deduced mainly by experiments with emulsion techniques and bubble chambers [2–7,10,11]. Then few more counter experiments were performed improving the interval estimation of the lifetime [8,9,12,13]. Yet, the dispersion of the different estimation to each other did not allow to draw a clear conclusion on the lifetime of those hypernuclei. It was considered that the lifetime of the light hypernuclei should be close to the lifetime of the Λ–hyperon, especially for $_\Lambda^3$H since it is weakly bound. Theoretical calculations do not either provide a clear picture because of

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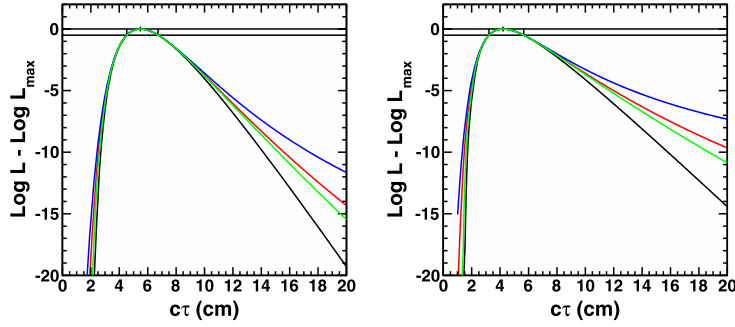
Fig. 1. (Color online.) The likelihood function of the data set shown in [9] with the different extrapolations based on Gaussian function of $^3_\Lambda$H and of $^4_\Lambda$H in left and right panel respectively. The red curves correspond to the real likelihood function obtained from the dataset. The blue and black curves correspond to the Gaussian function with linear standard deviation and with linear variance respectively. The green curve is the average combination of the two curves.

scattered predicted lifetime values [14–22]. However, Outa et al. measured the lifetime of $^4_\Lambda$H by observing the non-mesonic weak decay mode [13], which revealed that the lifetime of $^4_\Lambda$H should be significantly shorter than Λ. Recently, the lifetime of $^3_\Lambda$H, and anti-hypertriton, $^3_{\bar\Lambda}\bar{\text{H}}$ was measured at RHIC [8]. The lifetime values of $^3_\Lambda$H and $^4_\Lambda$H were also measured very recently by the HypHI Collaboration [9], indicating that the lifetime of these hypernuclei is shorter than Λ.

In the present Letter, we report on the combined analysis of the world data of the lifetime of $^3_\Lambda$H and $^4_\Lambda$H in order to deduce the combined interval estimation of their lifetime values. We also revisit the lifetime values of the HypHI experiment by means of the Bayesian analysis. The results of these two independent analyses infer that the lifetime of $^3_\Lambda$H and $^4_\Lambda$H could be shorter than that of the Λ–hyperon.

2. Combinations of the lifetime world data

In order to summarize the lifetime values of $^3_\Lambda$H and $^4_\Lambda$H deduced in the different experiments, a good practice would be to construct the combination of the estimations [1]. Generally the statistical combination of measurements is to combine the likelihood functions used to estimate the lifetime values that were reported by the experiments.

The lifetime values are usually reported as the inferred mean value, the asymmetric errors corresponding to one standard deviation and occasionally including the systematic uncertainties. In the introduction section of the PDG review [1], the procedure to produce the combined averages and fitting is explained for reported estimations with symmetric errors.

The situation is not as convenient when a parameter estimation is reported with asymmetric errors. One reasonable procedure is to use directly likelihood functions used for the lifetime estimation and then to combine them. However, publications on the lifetime measurements usually do not include deduced likelihood functions, therefore, one has to parametrize the likelihood functions from the deduced mean values and their asymmetric errors. A way to extrapolate expressions of the likelihood function from the reported measurements with asymmetric errors was discussed in [23], and it has been applied for the lifetime observables except for the values reported by the HypHI Collaboration. For the lifetime estimations by the HypHI Collaboration, the likelihood functions were provided and used for the combination. In the following, the procedures to extrapolate the likelihood functions are discussed. The validity of the method is also examined based on the data and deduced likelihood functions by the HypHI Collaboration. The procedures to combine the world data and results are then presented.

2.1. Extrapolation of likelihood functions

First, the method to extrapolate the likelihood function was tested with the data by the HypHI Collaboration [9] since likelihood functions were available. The form of the extrapolated likelihood function was built by applying the methods discussed in [23] and then was compared to the known real likelihood function of interest. Several functions were applied for better or worse extrapolations in the case study presented in [23].

In the following the extrapolation form which was adopted is the *variable Gaussian* form (*Forms 6 and 7* in [23]). Those forms are equivalent to a Gaussian function in which the variance or the standard deviation is a linear function of the parameter of interest. Let us write the extrapolated functions for a likelihood function depending on the parameter of interest x, having its maximum at $\hat{x}$ and with the one standard deviation errors, σ_+ and σ_- obtained by the likelihood ratio $\triangle \ln \mathcal{L} = -1/2$, where the $+$ and $-$ subscripts indicate the upper and lower asymmetric errors, respectively.

For the Gaussian function with a variable variance the extrapolation of the likelihood function is:

$$\ln \mathcal{L} = -1/2 \frac{(\hat{x} - x)^2}{V + V'(x - \hat{x})} \tag{1}$$

in which $V = \sigma_- \sigma_+$ and $V' = \sigma_+ - \sigma_-$. When the Gaussian function has instead a linear variance, the extrapolation function is written as:

$$\ln \mathcal{L} = -1/2 \left(\frac{(\hat{x} - x)}{s + s'(x - \hat{x})} \right)^2 \tag{2}$$

with $s = 2\sigma_- \sigma_+/(\sigma_+ + \sigma_-)$ and $s' = (\sigma_+ - \sigma_-)/(\sigma_+ + \sigma_-)$. By applying this method, the approximation of the likelihood functions of the HypHI data was calculated and is shown in Fig. 1 with the real likelihood function (in red line) of the $^3_\Lambda$H proper decay time $c\tau$ in the left panel and of $^4_\Lambda$H in the right panel. The two forms Eqs. (1) and (2) of a variable Gaussian function are represented in black and blue lines, respectively. The third extrapolation form, which is the arithmetic average of the two forms Eqs. (1) and (2), was also investigated and is represented by the green line in Fig. 1.

A fair agreement between the extrapolations defined by Eqs. (1) and (2) and the real likelihood functions (in red line) is observed at the small $\triangle \ln \mathcal{L}$ values, however, beyond 3 standard deviation ($\triangle \ln \mathcal{L} = -4.5$), the extrapolation forms deviate from the real likelihood function. On the other hand, the arithmetic average of the two forms agrees better with the real likelihood function up to 5 standard deviation ($\triangle \ln \mathcal{L} = -12.5$) as shown in Fig. 1. For the further combination studies, the employed extrapolation form is the arithmetic average of the forms Eqs. (1) and (2).
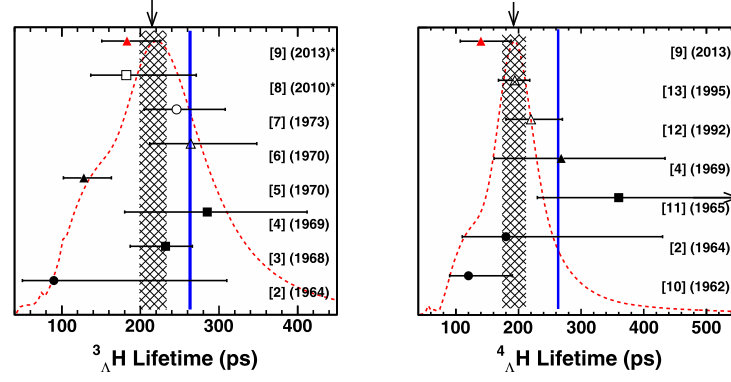
Fig. 2. (Color online.) World data comparison of $^3_\Lambda$H and $^4_\Lambda$H lifetimes. The combined average is represented by the arrow at the top, while the width of the hatched band corresponds to the one standard deviation of the average. The vertical line at 263.2 ps with width of ± 2 ps shows the known lifetime of Λ hyperon. References to counter experiments is marked by an asterisk.

2.2. Application to former measurements and combinations

The extrapolated likelihood functions of the former experiments were calculated with the manner which was discussed above. The combination of the measurements was performed and the weight factor of each measurement was obtained as explained in [1]. When the quoted errors of the measurements are symmetric, the likelihood function is considered to be a Gaussian function with a fixed standard deviation. In this case, the combined average $\hat{x}$ of those measurements x_i is:

$$\hat{x} \pm \delta\hat{x} = \frac{\sum_i w_i x_i}{\sum_i w_i} \pm \left(\sum_i w_i \right)^{-1/2} \tag{3}$$

where, $w_i = 1/(\delta x_i)^2$, and δx is the quoted one standard deviation of the corresponding measurement. This expression is presented in [1] and is straightforwardly obtained from the maximization of the combined likelihood function of the Gaussian likelihood functions of the measurements.

In the case of the measured values with asymmetric errors, the combined likelihood function is also maximized to obtain an expression similar to Eq. (3), though the variable w_i differs from the one of Eq. (3). The obtained expression is not linear as Eq. (3), therefore an iterative numerical calculation has to be performed. The interval estimation for the one standard deviation of the combined average value is determined via the profile likelihood ratio $\Delta \ln \mathcal{L} = -1/2$.

A comparison of the published lifetime values of $^3_\Lambda$H and $^4_\Lambda$H hypernuclei is shown respectively in the left and right panels of Fig. 2. The combination of all the measurements was calculated as described above to obtain an unconstrained average of the lifetime of $^3_\Lambda$H and $^4_\Lambda$H hypernuclei. An ideogram was built in the same manner of the PDG reviews [1,24] to display an overall pattern of the measurements. It is represented by dotted red lines in Fig. 2. The lifetime values of the combined average with the one standard deviation for $^3_\Lambda$H and $^4_\Lambda$H are respectively 216^{+19}_{-16} ps and 192^{+20}_{-18} ps. These values are represented by arrow at the top and the hashed regions in Fig. 2. The reduced χ^2 values are 0.89 for $^3_\Lambda$H and 0.48 for $^4_\Lambda$H. The exclusion band at 95% confidence level, useful for discarding theoretical models, was also deduced. Theoretical values outside of the ranges of [186 ps, 254 ps] for $^3_\Lambda$H and [158 ps, 233 ps] for $^4_\Lambda$H can be excluded with 95% confidence level.

3. Bayesian approach

The second approach was formulated by applying a Bayesian approach, instead of combining the measurements to obtain an av-

erage value of the $^3_\Lambda$H and $^4_\Lambda$H lifetime estimations. The aim is to deduce understanding from the data of the proper decay time measured by the HypHI Collaboration with several prior information. The HypHI data were chosen since the likelihood functions for the lifetime extraction were provided. The Bayesian analysis was carried out within the *Bayesian Analysis Toolkit*, BAT [25], and RooStats [26] framework. The Bayesian inference was obtained via the Markov Chain Monte Carlo method in order to evaluate the complete posterior probability density function given the proper decay time measurements for $^3_\Lambda$H and $^4_\Lambda$H and an assumed prior distribution.

3.1. The likelihood function

The likelihood function, corresponding to the model of the lifetime τ given the proper decay time $l/(\beta\gamma c)$ of the HypHI data, was already introduced in [9] as:

$$\mathcal{L} = \prod_i \left[\frac{1}{c\tau} e^{-l_i/(\beta_i \gamma_i c\tau)} \right]^{w'_i} \tag{4}$$

in which the coefficient w'_i stands for the normalized efficiency correction. More details on the fit model can be found in [9].

3.2. Description of the selected priors

The selection of the prior is an important part of the Bayesian analysis. Several prior functions were employed in order to investigate the sensitivity and influence of the choice of the priors on the resultant posterior distribution of the lifetime observable.

First a uniform density function $\pi^u(\tau)$ was used over a large interval enough to avoid any boundary issues. It is the simplest prior distribution which is assimilated as *no known prior lifetime information*, however the use of the uniform prior is not necessarily the best uninformative prior. The second prior, an objective prior, $\pi^j(\tau)$, so called Jeffreys prior defined by:

$$\pi^j(\tau) = -\sqrt{\mathbb{E}_l \left[-\frac{\partial^2}{\partial \tau^2} \mathcal{L}(\tau|l) \right]} \tag{5}$$

was built using the Fisher information which follows the invariance property [27]. It was used in order to obtain a posterior distribution from a prior reflecting the information about the lifetime observable from the data themselves [27]. The third implemented prior function, $\pi^p(\tau)$, was based on the available measurements of $^3_\Lambda$H and $^4_\Lambda$H lifetime older than the HypHI data. With the available extrapolated likelihood functions of previous measurements discussed in Section 2.1, this prior distribution is defined by:

Table 1
Summary of the parameter estimations.

	$\pi^u(\tau)$	$\pi^p(\tau)$	$\pi^j(\tau)$	$\pi^\Lambda(\tau)$
$^3_\Lambda$H				
Median $\pm68\%$ Central interval (ps)	$193.2^{+45.1}_{-34.5}$	$217.4^{+18.8}_{-16.2}$	$181.7^{+40.4}_{-31.2}$	$263.1^{+2.0}_{-2.0}$
Mode $\pm$ smallest interval (ps)	$185.5^{+42.2}_{-36.2}$	$215.3^{+18.7}_{-16.7}$	$173.0^{+40.5}_{-30.4}$	$263.0^{+3.1}_{-1.7}$
95% upper limit (ps)	275.5	250.0	255.0	266.4
$^4_\Lambda$H				
Median $\pm68\%$ Central interval (ps)	$156.8^{+56.8}_{-38.5}$	$193.6^{+19.9}_{-18.3}$	$139.9^{+46.0}_{-32.2}$	$263.1^{+2.0}_{-2.0}$
Mode $\pm$ smallest interval (ps)	$138.4^{+56.4}_{-33.5}$	$192.3^{+20.2}_{-18.6}$	$125.5^{+46.4}_{-28.9}$	$262.8^{+3.6}_{-2.0}$
95% upper limit (ps)	264.8	227.4	227.1	266.4

$$\pi^p(\tau) = \prod_i \mathcal{L}_i \Big/ \int \prod_i \mathcal{L}_i \qquad (6)$$

where the index i corresponds to the ith measurement. It was determined numerically and was employed to obtain the posterior probability within the BAT framework. It should be comparable to the combination procedure presented in Section 2. Finally, the fourth prior distribution, $\pi^\Lambda(\tau)$, based on the belief that the lifetime of the $^3_\Lambda$H and $^4_\Lambda$H should be similar to the Λ–hyperon lifetime was also considered as a Gaussian distribution with 263.3 ± 2 ps as parameters.

3.3. Results

Once the posterior probability density function, $p(\tau|\text{data}) \propto \mathcal{L}(\tau|\text{data})\pi(\tau)$, is computed for a given prior function and data set, one can extract several information from the posterior. The main information from the Bayesian inference is the principal mode (corresponding to the most probable value) and the smallest interval containing the principal mode and at least 68% of the posterior integral:

$$\int_{\tau_-}^{\tau_+} p(\tau|\text{data})\,d\tau = 0.68 \qquad (7)$$

where τ_- and τ_+ are the interval bounds of interest. The median value (value at 50% quantile) and the central interval at 68% are also noteworthy and are defined by:

$$\int_{-\infty}^{\tau_-} p(\tau|\text{data})\,d\tau = \int_{\tau_+}^{+\infty} p(\tau|\text{data})\,d\tau = 0.16 \qquad (8)$$

The upper limit at 95% for the lifetime of $^3_\Lambda$H and $^4_\Lambda$H hypernuclei, discussed in the previous section, is also employed to calculate the possible upper boundary of the lifetime values with this Bayesian confidence level. The different values extracted from the Bayesian analysis for the lifetime estimation are summarized in Table 1. It includes the median value, the mode position, the central interval at 68%, the smallest interval and the upper limit at 95%.

Figs. 3 and 4 show the resultant posterior distribution (back line) and their given prior functions (red line) for the $^3_\Lambda$H and $^4_\Lambda$H data set, respectively. The mean (open blue diamond), median (dot green line) and principal mode (red triangle) are represented in each panel with the central interval at 68% (gray filled area).

The results for $^3_\Lambda$H and $^4_\Lambda$H obtained with the uniform prior π^u are similar to the results previously reported in [9]. The median value and the smallest interval are the information being comparable with the profile likelihood interval estimation. It is expected since using the uniform prior π^u results to have the likelihood function as posterior distribution. It can be noted that the upper

limits at 95% confidence level with the uniform prior is approximately 275.5 ps and 264.8 ps for $^3_\Lambda$H and $^4_\Lambda$H respectively, that cover the Λ–hyperon lifetime. The Bayesian inference using the objective Jeffreys prior π^j indicates similar values to the uniform prior, however, the posterior distribution is more constrained as shown in the panel (c) of Fig. 3 and Fig. 4. It leads to the upper limit at 95% for the $^3_\Lambda$H and $^4_\Lambda$H lifetime of 255 ps and 227 ps respectively, being below the Λ lifetime. When the prior function π^p based on the previous measurements is employed, the posterior distribution of the lifetime observable is narrower and the influence of the lifetime estimation from the HypHI data set is comparable to the combined likelihood function of the previous measurements as shown in the panel (b) of Fig. 3 and Fig. 4. For this case, the deduced values in Table 1 are close to the value of Section 2.2. The principal mode and the smallest interval give lifetime estimations of 217^{+19}_{-16} ps for $^3_\Lambda$H and 194^{+20}_{-18} ps for $^4_\Lambda$H. The upper limits at 95% are approximately 250 ps and 227 ps respectively. In the case of the Λ lifetime as a prior function, π^Λ, it can be seen in panel (d) of Fig. 3 and Fig. 4 that the posterior distributions do not differ from the prior function. The information from the data is dominated by the information contained in the prior, and only the prior information determines the posterior distribution.

Furthermore, the Bayes factor was calculated in order to determine which assumption is supported by the HypHI data set of the proper decay time of $^3_\Lambda$H and $^4_\Lambda$H. The two tested models, M_1 and M_0, correspond to two posterior distribution functions given the prior belief π^p based on the previous measurements and the prior π^Λ based on the belief on the Λ lifetime respectively. The Bayes factor B_{10} defined by:

$$B_{10} = \frac{\int \mathcal{L}(\tau|\text{data})\pi^p(\tau)\,d\tau}{\int \mathcal{L}(\tau|\text{data})\pi^\Lambda(\tau)\,d\tau} \qquad (9)$$

is the most important quantity since it would help to judge which model would represent better the data set. The Bayes factor for $^3_\Lambda$H and $^4_\Lambda$H are about 2.7 and 3.8 respectively, which suggests more substantial support to the lifetime values obtained by the combined measurements than to the Λ–hyperon lifetime. Additionally the posterior probability of the π^p model is about 72% and 79% respectively for $^3_\Lambda$H and $^4_\Lambda$H.

4. Conclusion

The combined analysis of the world data for the lifetime of $^3_\Lambda$H and $^4_\Lambda$H was performed by using a similar manner to the PDG but with a modification for asymmetric errors. The combined average of $^3_\Lambda$H and $^4_\Lambda$H lifetime was determined to be respectively 216^{+19}_{-16} ps and 192^{+20}_{-18} ps with a respective reduced χ^2 of 0.89 and 0.48. The confidence interval at 95% confidence level was also employed to estimate the exclusion bands of [186 ps, 254 ps] for $^3_\Lambda$H
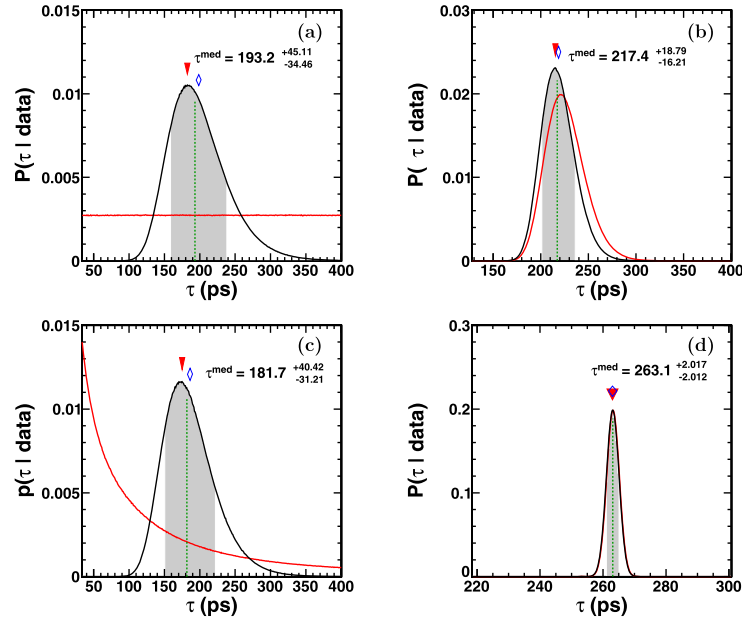
Fig. 3. (Color online.) Posterior distributions for the $^{3}_{\Lambda}$H data set, panel (a) for the uniform prior, panel (b) for the previous measurements prior, panel (c) for the Jeffreys prior and panel (d) for the Λ prior. In each panel the posterior distribution is displayed in black line while the prior distribution is in red line.
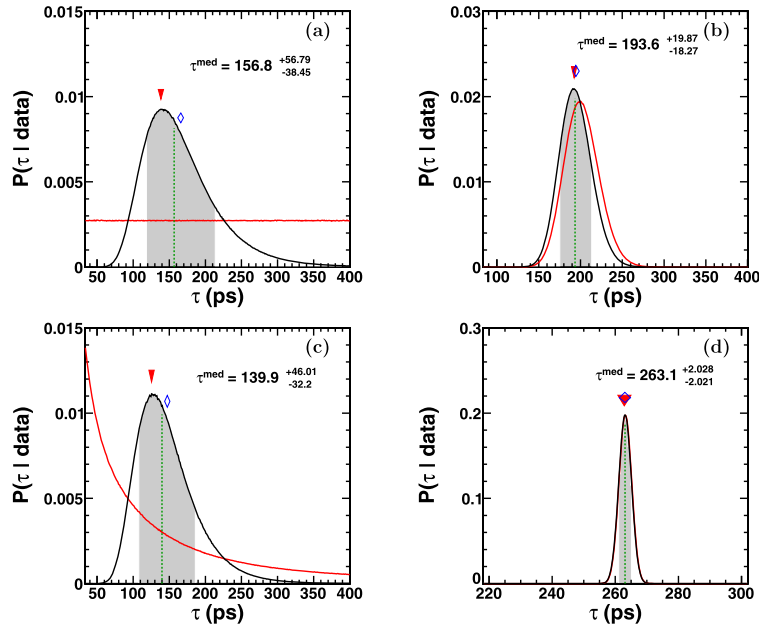

Fig. 4. (Color online.) Posterior distributions for the $^{4}_{\Lambda}$H data set with the same configuration of Fig. 3.

and [158 ps, 233 ps] for $^{4}_{\Lambda}$H. Theoretical values outside of these intervals can be excluded with 95% confidence level. This combination was only possible by taking into account the extrapolation expression of the likelihood functions to represent the quoted asymmetric errors.

Additionally the Bayesian analysis of the HypHI data set of the proper decay time measurement of $^{3}_{\Lambda}$H and $^{4}_{\Lambda}$H was performed by means of Markov Chain Monte Carlo method. The sensitivity of the obtained posterior distribution function to the choice between several priors was studied. Subjective and objective priors, respectively uniform distribution and the Jeffreys prior were employed. The Bayesian analysis with the previous measurements as a prior belief was also performed, and the resultant posterior function was compared to the one obtained from the prior belief of the

Λ–hyperon lifetime. Several estimations for the lifetime of $^{3}_{\Lambda}$H and $^{4}_{\Lambda}$H were deduced from those posterior probability density functions given by the chosen prior distribution. The notable conclusion from the Bayesian analysis was on the smallest interval at 68% with the prior belief from the previous measurements, and results were in good agreement with the estimated values from the combined average of the lifetime. The Bayesian analysis on the HypHI data set estimated lifetime values of $^{3}_{\Lambda}$H and $^{4}_{\Lambda}$H to be approximately 217^{+19}_{-16} ps and 194^{+20}_{-18} ps respectively. Furthermore, the upper limit at 95% was also deduced to be 250 ps and 227 ps respectively for $^{3}_{\Lambda}$H and $^{4}_{\Lambda}$H from the Bayesian approach, excluding the possible theoretical prediction above this limit with 95% confidence level.

With these studies, it has been revealed that the measured lifetime of $^3_\Lambda$H and $^4_\Lambda$H can be significantly shorter than that of the Λ–hyperon. Theoretical studies of the lifetime of those light hypernuclei have to be further carried on in order to gather the full understanding of those light baryon systems, as well as further precise measurements on the lifetime have to be performed.

Acknowledgements

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